World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:1, No:6, 2007

A New Method for Estimation of the Source Coherency Structure of Wideband Sources

Yong-jun Zhao, Heng-li Zhang, and Zong-yun Hu

***Abstract*—**Based on the sources' smoothed rank profile (SRP) and modified minimum description length (MMDL) principle, a method for estimation of the source coherency structure (SCS) and the number of wideband sources is proposed in this paper. Instead of focusing, we first use a spatial smoothing technique to pre-process the array covariance matrix of each frequency for de-correlating the sources and then use smoothed rank profile to determine the SCS and the number of wideband sources. We demonstrate the availability of the method by numerical simulations.

***Keywords*—**Wideband sources, source coherency structure (SCS), smoothed rank profile (SRP).

I. Introduction

IN signal processing area, there is increasing interest to investigate some properties of signals radiated by sources and received by sensor arrays. Direction of arrival (DOA) estimation is particularly important. Many high-resolution approaches [1,2,3] for DOA estimation have been developed. However, all high-resolution estimation algorithms in modern array processing either depend on or, in the case of higher dimensional search methods, benefit from explicit knowledge of the number of incident signals. The ability to detect and enumerate correctly all incident signals independent of their extent of correlation is crucial to the correct application of any so-called eigenstructure-based methods such as MUSIC, Min-Norm, ESPRIT, Cadzow's signal eigenvector method, etc. This is because the number of "large" (signal) eigenvalues of the array covariance matrix must equal the number of signals in order for the algorithm to produce valid estimates of the desired signal parameters. Several methods for detection the number of narrow-band sources have been proposed. Schmidt [4] proposed a subjective threshold that gives a rough procedure when applied to a covariance matrix obtained from a finite number of sources. Wax-Kailath introduced the information-theoretic techniques [5] based on either Akaike's information-theoretic criterion (AIC) or the minimum description length (MDL) criterion of Risannen. However, none of the above methods can be applied in a fully correlated signals environment, referred to as the coherent signals case. This case appears, for example, in specular multipath propagation and therefore it is of great practical importance. In the following years, a number of related methods have been proposed. These include the extension [6] of the MDL method so that it is now applicable to coherent signals and the smoothed rank profile method of Shan et al.,[7,8,9].

However, above detection methods are based on the narrow-band signal model, and they do not apply to the wideband sources. Kaveh et al [10] proposed the coherent signal subspace (CSS) processing for detection the number and estimation of the DOA of the wideband sources. Unfortunately, the CSS processing requires the preliminary estimates of source locations and focusing which require great computational complexity. When the source locations deviate from the preliminary estimates of locations, inaccurate detection of the number of signal sources may happen and asymptotic peak may increase. In this paper, we present a method which not only can detect the number of wideband sources but also can estimate the source coherency structure without the preliminary estimates of source locations or focusing.

The paper is structured as follows: The wideband signal model and modified information-theory technique based on the minimum description length (MDL) criterion of Risannen (MMDL) [11] is reviewed in Section II, with the smoothed rank profile method introduced in Section III. In Section IV we propose the method for estimation of the source coherency structure and the number of wideband sources. The feasibility of the method is demonstrated by several simulation results in section V. Finally, the conclusion related to the method is drawn in Section VI.

II. The Wideband Signal Model and MMDL [11]

Here we consider a uniform linear array composed of M identical equally spaced omni-directional sensors with sensor spacing d and P ($P < M$) wideband sources impinging on the array from directions $\theta_1, \theta_2, \cdots, \theta_P$. The source signals are characterized as zero mean, stationary stochastic processes over the observation interval T_0, band-limited to a common frequency band with bandwidth B which may be on the same order of magnitude as the center frequency f_0. The source spectral density matrix $P_s(f)$, $P \times P$, is unknown to the processor. The noise is assumed to be independent of the source signals with known noise spectral density matrix $P_n(f)$, but

Manuscript received July 26, 2007. This work was supported by Beijing Institute of Technology and Zhengzhou Information Science and Technology Institute.

Yong-jun Zhao is a professor with Department of Electronics Engineering, School of Information Science and Technology, Beijing Institute of Technology, Beijing 100081, China.

Heng-li Zhang is with the Zhengzhou Information Science and Technology Institute, Zhengzhou, Henan 450002, China (e-mail: zhhl8208@sohu.com).

Open Science Index, Electronics and Communication Engineering Vol:1, No:6, 2007 publications.waset.org/15402.pdf

unknown level $\sigma_n^{\ 2}(f)$. Hence, the spectral density matrix of the array output $x(t)$ is given by

$$P_x(f) = A(f,\theta)P_s(f)A^H(f,\theta) + \sigma_n^{\ 2}(f)P_n(f) \qquad (1)$$

where $A(f,\theta)$ is the $M \times P$ direction matrix at frequency f containing the unknown parameter vector θ of DOA's, and superscript H denotes Hermitian transpose.

The observation time T_0 is sectioned into K snapshots of duration T_d each. For sufficiently large T_d at each snapshot, the frequency-decomposed components of $x(t)$, $X(f_j)$, $j = 1, 2, \cdots, J$ via the discrete Fourier transform can be shown[12] to be approximately uncorrelated and their correlation matrices are given by

$$R_x(f_j) = \frac{1}{T_d}[A(f_j,\theta)P_s(f_j)A^H(f_j,\theta)$$

$$+\sigma_n^{\ 2}(f_j)P_n(f_j)] \qquad j = 1, 2, \cdots, J \qquad (2)$$

Without loss of generality, T_d is assumed to be 1. If we make a further assumption that the noise field is spatially uncorrelated at each frequency, i.e., $P_n(f_j) = I$, $j = 1, 2, \cdots, J$, then equation (2) becomes

$$R_x(f_j) = A(f_j,\theta)P_s(f_j)A^H(f_j,\theta) + \sigma^2{}_n(f_j)I$$

$$j = 1, 2, \cdots, J \qquad (3)$$

Thus, given the data set $X_k(f_j)$ $k = 1, 2, \cdots, K$, $j = 1, 2, \cdots, J$, our objective is to determine the number of sources. For simplicity of notation, in the sequel, all references to $R_x(f_j)$ and $\sigma_n^{\ 2}(f_j)$ will be indicated by R_j and $\sigma_j^{\ 2}$, respectively.

We define the correlation matrix of the cleaned data P_j as

$$P_j = R_j - \sigma_j^{\ 2}I \qquad (4)$$

By the cleaned data, we mean the output of a preprocessing step that decreases the effect of the noise.

If λ_{j_i} and u_{j_i} ($i = 1, 2, \cdots, M$) are eigenvalues and the corresponding normalized eigenvectors of P_j, then λ_{j_i} are shown in descending order. If the sources are not perfectly correlated, then the eigenvalues of P_j are given by[13]

$$U_j^{\ H} P_j U_j = \Gamma_j = diag(\lambda_{j_1}, \lambda_{j_2}, \cdots, \lambda_{j_M}) \qquad (5)$$

$$\lambda_{j_1} \geq \lambda_{j_2} \geq \cdots \geq \lambda_{j_P} > \lambda_{j_{P+1}} = \cdots \lambda_{j_M} = 0 \qquad (6)$$

where $U_j = [u_{j_1}, u_{j_2}, \cdots, u_{j_M}]$.

Based on equation (6), we can detect the number of wideband sources by counting the multiplicity of non-zero eigenvalues.

For practical reasons, the matrix P_j and its eigenvalues must be estimated from a finite set snapshots, i.e.,

$$\hat{P}_j = \hat{R}_j - \hat{\sigma}_j^{\ 2}I \qquad (7)$$

where

$$\hat{R}_j = \frac{1}{K}\sum_{k=1}^{K} X_k(f_j)X_k^{\ H}(f_j), \; \hat{\sigma}_j^{\ 2} = \frac{1}{M-P}\sum_{i=P+1}^{M} \lambda_i(\hat{R}_j),$$

and $\lambda_i(B)$ is the ith eigenvalue of B. In general, as an alternative to equation (7) and to guarantee the non-negativeness of the estimated source correlation matrix, the noise power can be estimated from $\hat{\sigma}_j^{\ 2} = \lambda_M(\hat{R}_j)$, where $\lambda_M(\hat{R}_j)$ is the smallest eigenvalue of $\hat{R}_j$.

Due to the noisy nature and finite data, equation (6) is not satisfied. As a result, we can not detect the number of wideband sources by counting the multiplicity of non-zero eigenvalues. Therefore, the eigenvalues become

$$\hat{\lambda}_{j_1} \geq \hat{\lambda}_{j_2} \geq \cdots \geq \hat{\lambda}_{j_P} \geq \hat{\lambda}_{j_{P+1}} \geq \cdots \geq \hat{\lambda}_{j_M} \qquad (8)$$

Then we can calculate the average eigenvalues of every frequency correlation matrix $\hat{P}_j$, i.e.,

$$\hat{\lambda}_i = \frac{1}{J}\sum_{j=1}^{J} \hat{\lambda}_{j_i} \quad i = 1, 2, \cdots, M \qquad (9)$$

So we can substitute equation (9) into the information-theory techniques [5] to detect the number of wideband sources. However, the information-theory techniques [5] based on AIC tends to overestimate even at high signal-to-noise ratio (SNR) and MDL criterion tends to underestimate at low or moderate SNR. Risannen [11] proposed the MMDL that moderates the overestimation of the AIC and the underestimation of the MDL criterion.

$$MMDL(k) = L_1 + L_2 + L_3 + P_a + P_c + \frac{1}{2}k\log K \qquad (10)$$

$$\text{where } L_1 = -K\ln\left[\frac{\prod_{i=k+1}^{M}\hat{\lambda}_i}{\left(\frac{1}{M-k}\sum_{i=k+1}^{M}\hat{\lambda}_i\right)^{(M-k)}}\right] \qquad (11)$$

$$L_2 = \sum_{\substack{i,j=1 \\ i<j}}^{k} \log(\hat{\lambda}_i - \hat{\lambda}_j)^2 \qquad (12)$$

$$L_3 = \sum_{i=1}^{k} \log(\hat{\lambda}_i - \tilde{\sigma}_\upsilon^2)^{M-k} \quad \tilde{\sigma}_\upsilon^2 = \frac{1}{M-k}\sum_{i=k+1}^{M}\hat{\lambda}_i \qquad (13)$$

$$p_a = \frac{1}{2}k(2M-k-1)\ln K \qquad (14)$$

$$p_c = -\sum_{i=M-k+1}^{M} \ln \Gamma(i) \tag{15}$$

The estimation of the number of wideband signal source $\hat{k}$ is

$$\hat{k} = \underset{k=\{0,1,\cdots,M-1\}}{\arg\min}\ MMDL(k) \tag{16}$$

The above method can only be used to detect the number of wideband sources that are not coherent. When wideband sources are coherent, the eigenvalues of correlation matrix P_j do not satisfy equation (6). Therefore, one can not use the above method to estimate the number of signal sources. In the following section, based on the SRP and MMDL, we propose a new method that can estimate the source coherency structure and the number of wideband sources.

III. Smoothed Rank Profile [7,8,9]

From [7,8,9], we can see that the smoothed rank profile (SRP)[7,8,9] can estimate the source coherency structure and the number of narrowband sources.

Whenever the narrowband signals are not perfectly correlated, the eigenvalues of the correlation matrix R of the sensor outputs, or array covariance matrix satisfy

$$\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_P > \lambda_{P+1} = \cdots \lambda_M = \sigma_v^{\ 2} \tag{17}$$

where $\sigma_v^{\ 2} I$ is the covariance matrix of the additive noise, which is assumed to be uncorrelated from sensor to sensor.

However, if the sources are coherent with respect to each other, the above properties (17) are not available in the eigenstructure of R. A solution of this problem is referred to as spatial smoothing, which is based on a preprocessing scheme that essentially de-correlates the signals by averaging the covariance matrices across several identical overlapping sub-arrays in order to restore the rank of a smaller sub-array covariance matrix to the number of the sources P. If there are several groups of coherent signals, the minimum number of sub-arrays required equals the size of the largest group of coherent signals.

Shan et al. [7,8,9] have also suggested a procedure called smoothed rank profile (SRP) to deduce the source coherent structure. Let $\{g_i, i = 1,2,\cdots,L\}$ denote the number of coherent groups of degree i and let L be the highest degree of source coherency present (i.e., maximum number of sources that are coherent with one another in any group). Note that $\{g_i, i = 1,2,\cdots,L\}$ is a characterization of SCS. The total number of coherent groups Q is then

$$Q = \sum_{i=1}^{L} g_i \tag{18}$$

The total number of the signal source D is

$$D = \sum_{i=1}^{L} i g_i \tag{19}$$

Let us define the $(M-k)\times(M-k)$ matrices $R^{(k)}$ as follows:

$$R^{(k)} = \frac{1}{k+1}\sum_{i=0}^{k} I_{M-k,i} R I^T_{\ M-k,i} \qquad k = 0,1,\cdots,M-1 \tag{20}$$

where $I_{M-k,j} = [0,0,\cdots,0,I_{M-k},0,0,\cdots,0]$, which has first j and last $k-j$ columns equal to zero and an $(M-k)\times(M-k)$ identity matrix I_{M-k} between them.

From equation (20), one can see that $R^{(k)}$ is the $(M-k)\times(M-k)$ smoothed covariance matrix, obtained by averaging $(k+1)$ sub-array covariance matrices, corresponding to a sub-array of length $(M-k)$.

The smoothed rank profile is then defined as the set of ranks $\gamma(R^{(i)})$, i.e.,

$$SRP(R) = \{\gamma(R^{(i)}), i = 0,1,\cdots,M-1\} \tag{21}$$

where it can be shown that

$$\gamma(R_f^{\ (i)}) = \begin{cases} \sum_{j=1}^{L} g_j & i = 0 \\ \min(M-i, \sum_{j=1}^{i} j g_j + (i+1)\sum_{j=i+1}^{L} g_j) & i = 1,2,\cdots,M-1 \end{cases} \tag{22}$$

The SCS is determined as the set of negative second-order differences in the increasing and the stationary segments of SRP, i.e. ,

$$g_i = -\{\gamma(R^{(i-2)}) - 2\gamma(R^{(i-1)}) + \gamma(R^{(i)})\}$$
$$i = 1,2,\cdots,L \ \text{ where } \gamma(R^{(-1)}) = 0 \tag{23}$$

It has also been shown by Shan et al. [7,8,9] that in order to obtain the SCS, the minimum number of required sensors is equal to the total number of sources plus the highest degree of source coherency present.

Finally one can calculate $\gamma(R^{(i)})$ by MMDL criterion, i.e.,

$$\gamma(R^{(i)}) = \min MMDL(k) \quad k = 0,1,2,\cdots,M-i-1$$

IV. Estimation of the Source Coherency Structure and the Number of Wideband Sources

The method for estimation of the source coherency structure and the number of wideband sources based on SRP and MMDL is summarized as follows:

1. Calculate $\hat{R}_j = \frac{1}{K}\sum_{k=1}^{K} X_k(f_j) X_k^{\ H}(f_j)$ and $\hat{P}_j = \hat{R}_j - \hat{\sigma}_j^{\ 2} I$, where $\hat{\sigma}_j^{\ 2} = \lambda_M(\hat{R}_j)$.
2. For each j, $j = 1,2,\cdots,J$ calculate the smoothed covariance matrices $\{\hat{P}_j^{\ (k)}, k = 0,1,\cdots,M-1\}$ and their corresponding eigenvalues $\hat{\lambda}_{j_m}^{(k)}$, $m = 1,2,\cdots,M-k$

3. For a special k, calculate the average of $\hat{\lambda}_{j_m}{}^{(k)}$, i.e., $\hat{\lambda}_m^{(k)} = \frac{1}{J}\sum_{j=1}^{J}\hat{\lambda}_{j_m}^{(k)}$

4. Obtain the SRP by the MMDL.

5. Then SCS is $g_i = -\{\gamma(R^{(i-2)}) - 2\gamma(R^{(i-1)}) + \gamma(R^{(i)})\}$ and the the total number of the signal source D is $D = \sum_{i=1}^{L} i g_i$.

It can be seen that the proposed method does not require the preliminary estimates of source locations or focusing, thereby it requires much lower computational cost than CSS algorithm.

V. Simulation Results

To illustrate the performance of the method proposed in the previous section, we shall perform several simulation experiments.

In all cases, the following hold:

1. The array is assumed to be uniform and linear composed of eight sensors, having identical, omni-directional with unit gain.
2. The signals are assumed to be wideband linear frequency modulated signals.
3. The additive noise $n_i(t)$ is assumed to be white Gaussian with common variance σ^2 and uncorrelated with the signal.
4. The SNR is $10dB$

In the first experiment, we simulated three uncorrelated signals impinging on the array from 10°¡20°¢40°, namely, the sources are divided into three groups with $g_1 = 3$. The result is shown in Fig. 1. As shown in Fig. 1, the number of uncorrelated wideband sources can be correctly estimated without smoothing and the SCS and the number of wideband sources in this case can be correctly determined by the proposed method.

In the second experiment, we simulated three wideband signals among which two signals are coherent and another one is not correlated with them, namely, the sources are comprised of groups with $g_1 = 1$£ $g_2 = 1$. The result of the method is shown in Fig. 2. In this case, the method can not correctly detect the number of signals without smoothing. From Fig. 2, it can be also obtained that $g_1 = 1$£ $g_2 = 1$ and the rank profile of matrix sequence $\{\hat{P}^{(k)}, k = 0,1,\cdots,M-1\}$ is stationary for two points (when $k = 2$, the matrix $\hat{P}^{(k)}$ become stationary or full rank).

In the final experiment, we simulated three coherent wideband signals, i.e., the sources are divided into one group with $g_3 = 1$. As shown in Fig. 3, we can get that $g_3 = 1$ i.e., we can use the proposed method to estimate the source coherency structure, and the rank profile of matrix sequence $\{\hat{P}^{(k)}, k = 0,1,\cdots,M-1\}$ is stationary for three points.

The above examples have shown that the method proposed in the previous section can correctly determine the SCS and the number of wideband signal sources without any pre-knowledge or assumption of the SCS of wideband sources.

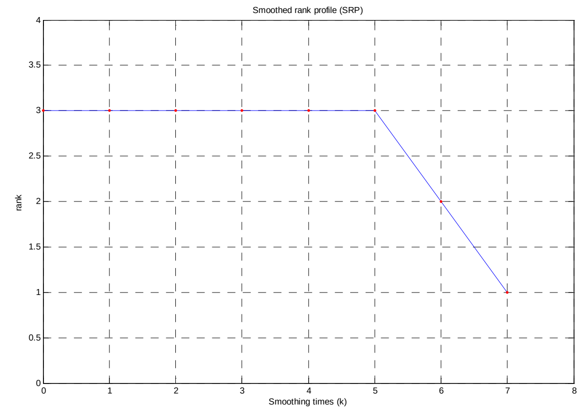

Fig. 1 From the above SRP, it can be calculated that $g_1 = 3$ and $D_1 = 3$

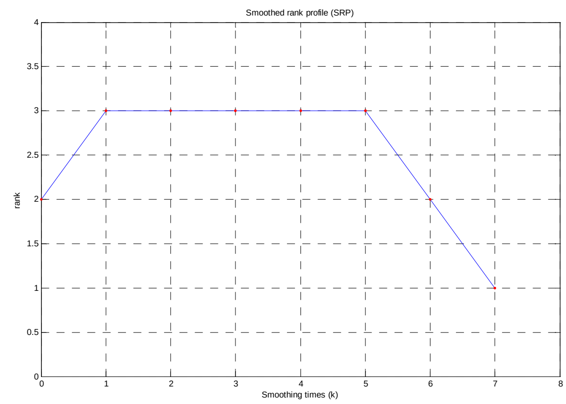

Fig. 2 From the above SRP, it can be calculated that $g_1 = 1, g_2 = 1$

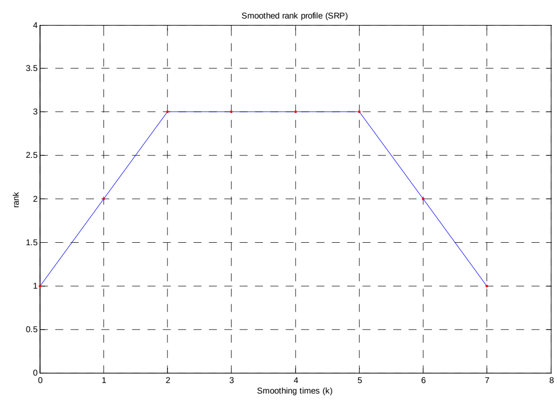

Fig. 3 From the above SRP, it can be calculated that $g_3 = 1$

VI. Conclusion

In summary, we have introduced a new method for estimation of the SCS and the number of wideband signals using an uniform and linear array of sensors. Our method is based on smoothed rank profile and MMDL criteria. Without any pre-knowledge or assumption of the SCS, the proposed method can determine the SCS and the number of wideband sources. Unlike the CSS algorithm, this method does not require the preliminary estimates of source locations or focusing, and can reduce computation dramatically. Simulations on several examples show that the proposed method work well in the presence of uncorrelated or coherent wideband sources for high SNR. However, like the smoothed rank profile, this method can only be used in the uniform linear arrays.

REFERENCES

[1] Schmidt R O. Multiple emitter location and signal parameter estimation[J]. IEEE Trans. On AP, 1986, 34(3): 276-280.
[2] Ziskind L, Wax M. Maximum likelihood localization of multiple sources by alternating projection[J]. IEEE Trans. On ASSP, 1988, 36(10): 1553-1559.
[3] Ottersten B, Viberg M, Kailath T. Analysis of subspace fitting and ML techniques for parameter estimation from sensor array data[J]. IEEE Trans. On SP, 1992, 40(3): 590–600.
[4] Schmidt R O. Multiple emitter location and signal parameter estimation. IEEE Trans., 1986,AP-34(3): 276-280.
[5] M. Wax and T. Kailath. Detection of signals by information theoretic criteria. IEEE Trans. Acoust. Speech Signal Processing. 1985. vol. ASSP-33: 387-392.
[6] M. Wax. Detection and localization of multiple sources via the stochastic signals model. IEEE Trans. Signal Processing, vol. 39, pp. 2450-2455, Nov. 1991.
[7] T. J. Shan, A. Paulraj, and T. Kailath. On smoothed rank profile tests in eigenstructure methods for directions-of-arrival estimation.IEEE Trans. Acoust.Speech Signal Processing, vol. ASSP-35, no, pp. 1377-1385, 1987.
[8] S. Prasad and B. Chandna. An augmented smoothed rank profile algorithm for determination of source coherency structure. IEEE Trans. Acoust. Speech Signal Processing. 1989. vol. 37(7): 1144-1146.
[9] Cozzens J H, Sousa M J. Source enumeration in a correlated signal environment. IEEE Trans. on SP, 1994, 42(2): 304-317.
[10] H. Wang and M. Kaveh. Coherent signal-subspace processing for detection and estimation of angles of arrival of multiple wideband sources. IEEE Trans. Acoust. Speech. Signal Processing. , 1985. vol. ASSP-33: 823-831.
[11] K. M. Wong, Q. T. Zhang, J. P. Reilly, and P. C. Yip. On information theoretic criteria for determining the number of signals in high resolution array processing. IEEE Trans. Acoust. Speech Signal Processing, . 1990. vol. 38: 1959-1970.
[12] D. R. Brillinger. Time series: Data Analysis and Theory. New York: Holt, Rinehart and Winston, 1975.
[13] Valaee S, Kabal P. Wideband array processing using a two-sided correlation transformation. IEEE Trans. on SP, 1995, 43(1): 160-172.

Yong-jun Zhao was born in HeNan, China, in 1964. He received master degree in electrical engineering with honors from Shanghai JiaoTong University.

He is a professor with Information Science and Technology Institute of Zhengzhou. His current research interests are mainly in the areas of signal processing and communications.