

# Delay-dependent stability analysis for uncertain switched neutral system

Lianglin Xiong, Shouming Zhong, and Mao Ye

*Abstract*—This paper considers the robust exponential stability issues for a class of uncertain switched neutral system which delays switched according to the switching rule. The system under consideration includes both stable and unstable subsystems. The uncertainties considered in this paper are norm bounded, and possibly time varying. Based on multiple Lyapunov functional approach and dwell-time technique, the time-dependent switching rule is designed depend on the so-called average dwell time of stable subsystems as well as the ratio of the total activation time of stable subsystems and unstable subsystems. It is shown that by suitably controlling the switching between the stable and unstable modes, the robust stabilization of the switched uncertain neutral systems can be achieved. Two simulation examples are given to demonstrate the effectiveness of the proposed method.

*Keywords*—Switched neutral system, exponential stability, multiple Lyapunov functional, dwell time technique, time-dependent switching rule.

## I. INTRODUCTION

A Switched system is a dynamical system that consists of a finite number of subsystems and a logical rule which orchestrates switching between these subsystems. Such system has gained a great deal of attention mainly because various real-world systems, such as chemical processing [1], communication networks, traffic control [2]-[4], control of manufacturing systems [5]-[6], automotive engine control and aircraft control [7] can be modeled as switched systems. In the past, large number of excellent papers and monographs on the stability of switched systems have been published[8]-[13], and the reference therein. Dwell time technique is an effective tool for analyzing the stability of switched systems.

On the other hand, time-delay is a common phenomenon in engineering control design. During recent decades, great efforts have been made by mathematicians as well as engineers to study the stability of neutral systems. Various analysis techniques have been utilized to derive asymptotical stability criteria for neutral systems [14]-[22]. Generally speaking, the current results for this time-delay systems can be classified into two categories: delay-independent and delay-dependent conditions. Delay-independent criteria do not employ any information on the size of the delay; while delay-dependent criteria make use of such information at different levels. Delay-dependent stability conditions are generally less conservative

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than delay-independent ones especially when the delay is small [15]. To the best of our knowledge, it seems that few people have studied the problem of stability for switched neutral control system besides [23]-[25]. With single Lyapunov approach and multiple approach, delay-independent stabilization conditions for the switched neutral system were obtained in [23]. Lately, with single Lyapunov approach, delay-dependent stability conditions for the switched neutral system are derived in [24] and [25]. All switching rules designed in these papers are trajectory dependent. However, it is very practical to obtain the time-controlled switching rule for the switched neutral systems. Moreover, the delays are remained the same according to the switching rule in [23]-[25]. All of those have motivated our research.

In this paper, we are interested in the robust exponential stabilization synthesis for switched neutral system that consists of stable and unstable modes. The delays in the systems are switched according to the switching law. New classes of multiple Lyapunov functionals are constructed for new delay-dependent exponential stability condition of the switched system, and dwell-time technique is used to analyze the stability property, as a result the criterion on delay-dependent stability is derived in terms of linear matrix inequalities. The switching rule designed in this paper depends on not only the ratio of the total activation time of stable modes and unstable modes but also the so-called average dwell time of stable modes. This paper is organized as follows. Section 2 describes the switched neutral system and introduces some notations and lemmas that will be used in the rest of this paper. Section 3 gives our main results in this paper. Simulation examples are given to demonstrate the effectiveness of our theoretical results in section 4. Some conclusions are drawn in Section 5.

### A. Problem statement and preliminaries

#### Nomenclature

- $R^n$  n-dimensional real space
- $R^{n \times n}$  set of all real n by n matrices
- $x^T$  or  $A^T$  transpose of vector x (or matrix A)
- $P > 0$  (respectively,  $P < 0$ ) matrix  $P$  is symmetric positive (respectively, negative) definite
- $P \geq 0$  (respectively,  $P \leq 0$ ) matrix  $P$  is symmetric positive (respectively, negative) semi-definite
- \* the elements below the main diagonal of a symmetric block matrix

Consider the following switched uncertain neutral system:

$$\Xi_0 : \begin{cases} \dot{x}(t) - C_{\sigma(t)}(t)\dot{x}(t - \tau_{\sigma(t)}) = \\ A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t - r_{\sigma(t)}) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0] \end{cases} \quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $r_{\sigma(t)}, \tau_{\sigma(t)} > 0$  are constant time delays,  $\tau = \max\{\tau_i : i \in \sigma(t)\}$ ,  $r = \max\{r_i : i \in \sigma(t)\}$ ,  $\rho = \max\{\tau, r\}$  and  $\varphi(\theta)$  is the initial condition function.  $\sigma(t) \in M = \{1, 2, \dots, m\}$  is piecewise constant switching signal. This means that the matrices  $(A_{\sigma(t)}(t), B_{\sigma(t)}(t), C_{\sigma(t)}(t))$  are allowed to take values, at an arbitrary time, in the finite set

$$(A_{\sigma(t)}(t), B_{\sigma(t)}(t), C_{\sigma(t)}(t)) \in \{(A_1(t), B_1(t), C_1(t)), \dots, (A_m(t), B_m(t), C_m(t))\}. \quad (2)$$

The system matrices are assumed to be uncertain and satisfy

$$[A_i(t), B_i(t), C_i(t)] = [A_i, B_i, C_i] + DF(t)[E_{ai}, E_{bi}, E_{ci}], \quad (3)$$

where  $A_i, B_i, C_i, D, E_{ai}, E_{bi}, E_{ci}$  are constant matrices with appropriate dimensions for  $i \in M$ , and  $F(t)$  is an unknown, real, and possibly time-varying matrix with Lebesgue measurable elements, satisfying

$$F^T(t)F(t) \leq I. \quad (4)$$

The following definitions are necessary.

**Definition 1:** ([11]) The equilibrium of system (1) is said to be exponentially stable under the switching rules  $\sigma(t)$ , if there exist scalars  $\alpha > 0$  and  $\gamma \geq 1$  such that for all  $x(t)$  the following inequality holds:

$$\|x(t)\| \leq \gamma e^{-\alpha(t-t_0)} \|\phi\|_{1\rho},$$

where  $\|\cdot\|$  denote the Euclidean norm and  $\|\varphi\|_{1\rho} = \max\left\{\max_{-\rho \leq s \leq 0} \|\varphi(s)\|, \max_{-\rho \leq s \leq 0} \|\varphi'(s)\|\right\}$ .

**Definition 2:** ([11]) For any  $T_2 > T_1 \geq 0$ , let  $N_{\sigma}(T_1, T_2)$  denote the number of switching of  $\sigma(t)$  over  $(T_1, T_2)$ . If  $N_{\sigma}(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a$  holds for  $T_a > 0, N_0 \geq 0$ , then  $T_a$  is called average dwell time. As commonly used in the literature, we choose  $N_0 = 0$ .

Before presenting the main result, we first state the following lemmas which will be used in the proof of our main result.

**Lemma 1:** ([26]) Given matrices  $Q = Q^T, H$  and  $E$  of appropriate dimensions, then

$$Q + HFE + E^T F^T H^T < 0$$

for all  $F$  satisfying  $F^T(t)F(t) \leq I$ , if and only if exists an  $\varepsilon > 0$  such that

$$Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$$

**Lemma 2:** ([27]) For given matrices  $A_{11}, A_{12}, A_{22}$  with appropriate dimensions,

$$\begin{bmatrix} A_{11} & A_{12} \\ * & A_{22} \end{bmatrix} < 0$$

holds if and only if  $A_{22} < 0, A_{11} - A_{12}A_{22}^{-1}A_{12}^T < 0$ .

**Lemma 3:** For any constant matrices  $Q_{11}, Q_{12}, Q_{22} \in R^{n \times n}$ ,  $\begin{pmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{pmatrix} > 0$ , scalar  $h > 0$ , and vector function

$\dot{x} : [-h, 0] \rightarrow R^n$  such that the following integration is well defined, then

$$\begin{aligned} & -h \int_{t-h}^t \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \\ & \leq - \begin{bmatrix} \int_{t-h}^t x(s) ds \\ x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} & -Q_{12} \\ * & Q_{22} & -Q_{22} \\ * & * & Q_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h}^t x(s) ds \\ x(t) \\ x(t-h) \end{bmatrix}. \end{aligned} \quad (5)$$

**Proof.** Following Jensen's integral inequality [28], one can obtain

$$\begin{aligned} & -h \int_{t-h}^t \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \\ & \leq - \begin{bmatrix} \int_{t-h}^t x(s) ds \\ \int_{t-h}^t \dot{x}(s) ds \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h}^t x(s) ds \\ \int_{t-h}^t \dot{x}(s) ds \end{bmatrix} \\ & = - \begin{bmatrix} \int_{t-h}^t x(s) ds \\ x(t) - x(t-h) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h}^t x(s) ds \\ x(t) - x(t-h) \end{bmatrix}. \end{aligned} \quad (6)$$

Re-arranging some terms of (6) yields (5). This completes the proof.

From (1), the system discussed in this paper is a general form of that considered in [23]-[25]. In addition, the switched neutral system consists of both stable modes and unstable modes. The aim of this paper is to find a new strategy for the exponential stabilization of the switched neutral system.

## B. Stability analysis

In this section, we establish exponential stability of switched neutral systems incorporating stable and unstable modes. Let  $i \in S_s$  and  $j \in S_u$  be respectively the set of indices of stable and unstable modes.

Firstly consider the nominal and stable switched neutral subsystems

$$\begin{cases} \dot{x}(t) - C_i \dot{x}(t - \tau_i) = A_i x(t) + B_i x(t - r_i) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0], i \in S_s \end{cases} \quad (7)$$

Choose a new class of Lyapunov functionals candidate for systems (7) as following

$$V_i(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t), \quad (8)$$

where

$$\begin{aligned} V_{i1}(t) &= x^T(t) P_i x(t), \\ V_{i2}(t) &= r_i \int_{-r_i}^0 \int_{t+\theta}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T e^{-\alpha_i(t-s)} \\ & \quad \times \begin{bmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds d\theta, \\ V_{i3}(t) &= \int_{t-\tau_i}^t \dot{x}^T(s) e^{-\alpha_i(t-s)} R_i x(s) ds, \end{aligned}$$

scalars  $\alpha_i > 0$ , matrices  $P_i > 0, R_i > 0, Q_{i11} > 0, Q_{i22} > 0, Q_{i12}$  with appropriate dimensions to be determined. The following lemma gives a decay estimation of  $V_i(t)$  along the state trajectory of systems (7).

**Lemma 4:** Given scalars  $\alpha_i > 0$ , if there exist matrices  $P_i > 0, Q_{i11} > 0, Q_{i22} > 0, Q_{i12}$  and  $N_{il}^T (l = 1, \dots, 5)$

are any matrices with appropriate dimensions, such that the following LMIs hold:

$$Q_i = \begin{pmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{pmatrix} > 0, i \in S_s \quad (9)$$

$$\phi_i = \begin{pmatrix} \phi_{i11} & \phi_{i12} & \phi_{i13} & \phi_{i14} & \phi_{i15} \\ * & \phi_{i22} & \phi_{i23} & \phi_{i24} & \phi_{i25} \\ * & * & \phi_{i33} & \phi_{i34} & \phi_{i35} \\ * & * & * & \phi_{i44} & \phi_{i45} \\ * & * & * & * & \phi_{i55} \end{pmatrix} < 0, \quad (10)$$

where

$$\begin{aligned} \phi_{i11} &= \alpha_i P_i + P_i A_i + A_i^T P_i + r_i^2 Q_{i11} \\ &\quad - e^{-\alpha_i r_i} Q_{i22} + N_{i1}^T A_i + A_i^T N_{i1}, \\ \phi_{i12} &= r_i^2 Q_{i12} - N_{i1}^T A_i + A_i^T N_{i2}, \\ \phi_{i13} &= P_i C_i + N_{i1}^T C_i + A_i^T N_{i3}, \\ \phi_{i14} &= P_i B_i + N_{i1}^T B_i + A_i^T N_{i4} + e^{-\alpha_i r_i} Q_{i22}, \\ \phi_{i15} &= -e^{-\alpha_i r_i} Q_{i12}^T + A_i^T N_{i5}, \\ \phi_{i22} &= r_i^2 Q_{i22} - N_{i2}^T - N_{i2}, \\ \phi_{i23} &= N_{i2}^T C_i - N_{i3}, \quad \phi_{i55} = -e^{-\alpha_i r_i} Q_{i11}, \\ \phi_{i24} &= N_{i2}^T B_i - N_{i4}, \quad \phi_{i25} = -N_{i5}, \\ \phi_{i35} &= C_i^T N_{i5}, \quad \phi_{i45} = B_i^T N_{i5} + e^{-\alpha_i r_i} Q_{i12}^T, \\ \phi_{i33} &= -e^{-\alpha_i \tau_i} R_i + N_{i3}^T C_i + C_i^T N_{i3}, \\ \phi_{i34} &= N_{i3}^T B_i + C_i^T N_{i4}, \\ \phi_{i44} &= -e^{-\alpha_i r_i} Q_{i22} + N_{i4}^T B_i + B_i^T N_{i4}, \end{aligned}$$

then along the trajectory of the systems (7), it follows that

$$\dot{V}_i(t) \leq e^{-\alpha_i(t-t_0)} V_i(t_0), i \in S_s. \quad (11)$$

**Proof.** Along the trajectories of systems (7), with lemma 3, it hold that

$$\dot{V}_{i1}(t) = 2x^T(t)P_i[A_i x(t) + B_i x(t-r_i) + C_i \dot{x}(t-\tau_i)], \quad (12)$$

$$\begin{aligned} \dot{V}_{i2}(t) &= r_i^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &\quad - \alpha_i V_{i2}(t) - r_i \int_{t-r_i}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T e^{-\alpha_i(t-s)} \\ &\quad \times \begin{bmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ &\leq r_i^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \alpha_i V_{i2}(t) \\ &\quad - e^{-\alpha_i r_i} \begin{bmatrix} \int_{t-r_i}^t x(s) ds \\ x(t) \\ x(t-r_i) \end{bmatrix}^T \begin{bmatrix} Q_{i11} & Q_{i12} & -Q_{i12} \\ * & Q_{i22} & -Q_{i22} \\ * & * & Q_{i22} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \int_{t-r_i}^t x(s) ds \\ x(t) \\ x(t-r_i) \end{bmatrix}, \quad (13) \end{aligned}$$

$$\begin{aligned} \dot{V}_{i3}(t) &= \dot{x}^T(t)R_i \dot{x}(t) - \dot{x}^T(t-\tau_i)e^{-\alpha_i \tau_i} R_i \dot{x}(t-\tau_i) \\ &\quad - \alpha_i V_{i3}(t). \quad (14) \end{aligned}$$

For any matrices  $N_{il}(l = 1, \dots, 5)$ , it follows from the systems (7) that

$$\begin{aligned} &[x^T(t)N_{i1}^T + \dot{x}^T(t)N_{i2}^T + \dot{x}^T(t-\tau_i)N_{i3}^T + x^T(t-r_i)N_{i4}^T \\ &\quad + (\int_{t-r_i}^t x(s)ds)^T N_{i5}^T] \times [A_i x(t) - \dot{x}(t) \\ &\quad + B_i x(t-r_i) + C_i \dot{x}(t-\tau_i)] = 0. \quad (15) \end{aligned}$$

Then, from (12)-(15), one can obtain that

$$\begin{aligned} \dot{V}_i(t) + \alpha_i V_i(t) &\leq \alpha_i x^T(t)P_i x(t) + 2x^T(t)P_i[A_i x(t) \\ &\quad + B_i x(t-r_i) + C_i \dot{x}(t-\tau_i)] + \dot{x}^T(t)R_i \dot{x}(t) \\ &\quad + r_i^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &\quad - e^{-\alpha_i r_i} \begin{bmatrix} \int_{t-r_i}^t x(s) ds \\ x(t) \\ x(t-r_i) \end{bmatrix}^T \begin{bmatrix} Q_{i11} & Q_{i12} & -Q_{i12} \\ * & Q_{i22} & -Q_{i22} \\ * & * & Q_{i22} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \int_{t-r_i}^t x(s) ds \\ x(t) \\ x(t-r_i) \end{bmatrix} - \dot{x}^T(t-\tau_i)e^{-\alpha_i \tau_i} R_i \dot{x}(t-\tau_i) \\ &\quad + [x^T(t)N_{i1}^T + \dot{x}^T(t)N_{i2}^T + \dot{x}^T(t-\tau_i)N_{i3}^T \\ &\quad + x^T(t-r_i)N_{i4}^T + (\int_{t-r_i}^t x(s)ds)^T N_{i5}^T] \\ &\quad \times [A_i x(t) - \dot{x}(t) + B_i x(t-r_i) + C_i \dot{x}(t-\tau_i)] \\ &= \xi^T(t)\phi_i \xi(t) < 0, \end{aligned}$$

where

$$\xi^T(t) = [x^T(t) \quad \dot{x}^T(t) \quad \dot{x}^T(t-\tau_i) \quad x^T(t-r_i) \quad (\int_{t-r_i}^t x(s)ds)^T]$$

Thus,  $\dot{V}_i(t) + \alpha_i V_i(t) \leq 0$ . Integrating these inequalities gives the inequalities (11). This completes the proof.  $\square$

With the uncertainty described by (3) and (4), the following corollary is obtain for the switched neutral subsystems as

$$\begin{cases} \dot{x}(t) - C_i(t)\dot{x}(t-\tau_i) = A_i(t)x(t) + B_i(t)x(t-r_i), \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0], i \in S_s \end{cases} \quad (16)$$

**Corollary 1:** Given scalars  $\alpha_i > 0$ , if there exist scalars  $\varepsilon_i > 0$  and matrices  $P_i > 0, Q_{i11} > 0, Q_{i22} > 0, Q_{i12}, N_{il}^T \{l \in (1, \dots, 5)\}$  with appropriate dimensions, such that the following LMIs hold

$$Q_i = \begin{pmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{pmatrix} > 0, \quad (17)$$

$$\phi_i = \begin{pmatrix} \varphi_{i11} & \varphi_{i12} & \varphi_{i13} & \varphi_{i14} & \varphi_{i15} & N_{i1}^T D & P_i D \\ * & \varphi_{i22} & \varphi_{i23} & \varphi_{i24} & \varphi_{i25} & N_{i2}^T D & 0 \\ * & * & \varphi_{i33} & \varphi_{i34} & 0 & N_{i3}^T D & 0 \\ * & * & * & \varphi_{i44} & 0 & N_{i4}^T D & 0 \\ * & * & * & * & \varphi_{i55} & N_{i5}^T D & 0 \\ * & * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & * & -\varepsilon_i I \end{pmatrix} < 0 \quad (18)$$

where

$$\begin{aligned} \varphi_{i11} &= \phi_{i11} + 2\varepsilon_i E_{ai}^T E_{ai}, & \varphi_{i13} &= \phi_{i13} + 2\varepsilon_i E_{ai}^T E_{ci}, \\ \varphi_{i14} &= \phi_{i14} + \varepsilon_i E_{ai}^T E_{bi}, & \varphi_{i33} &= \phi_{i33} + 2\varepsilon_i E_{ci}^T E_{ci}, \\ \varphi_{i34} &= \phi_{i34} + \varepsilon_i E_{ci}^T E_{bi}, & \varphi_{i44} &= \phi_{i44} + \varepsilon_i E_{bi}^T E_{bi}, \end{aligned}$$

with  $\phi_{ikl}(i \in S_s, k, l = (1, \dots, 5))$  are defined in lemma 4, then along the trajectory of the systems (16), it follows that

$$V_i(t) \leq e^{-\alpha_i(t-t_0)} V_i(t_0), i \in S_s. \quad (19)$$

**Proof.** Replaced  $A_i, B_i, C_i$  in LMIs (9) respectively with  $A_i(t), B_i(t), C_i(t)$  which described in (3) and (4).  $\phi_i$  are changed into  $\tilde{\phi}_i$  as following

$$\begin{aligned} \tilde{\phi}_i = & \phi_i + H \begin{pmatrix} F(t) & 0 \\ & F(t) \end{pmatrix} E \\ & + E^T \begin{pmatrix} F(t) & 0 \\ & F(t) \end{pmatrix}^T H^T < 0. \end{aligned} \quad (20)$$

where

$$E = \begin{pmatrix} E_{ai} & 0 & E_{ci} & E_{bi} & 0 \\ E_{ai} & 0 & E_{ci} & 0 & 0 \end{pmatrix}$$

$$H^T = \begin{pmatrix} D^T N_{i1} & D^T N_{i2} & D^T N_{i3} & D^T N_{i4} & D^T N_{i5} \\ D^T P_i & 0 & 0 & 0 & 0 \end{pmatrix}$$

From lemma 1, the above inequality (20) holds if and only if there exist some scalars  $\varepsilon_i > 0$  such that

$$\phi_i + \varepsilon_i E^T E + \varepsilon_i^{-1} H H^T < 0. \quad (21)$$

With lemma 2, the aforementioned inequality (21) is equivalent to LMIs (18). This complete the proof.  $\square$

The following lemma will be given for the nominal and unstable switched neutral subsystems such that

$$\begin{cases} \dot{x}(t) - C_j \dot{x}(t - \tau_j) = A_j x(t) + B_j x(t - r_j) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0], j \in S_u \end{cases} \quad (22)$$

Choose another class of Lyapunov functionals candidate of the following form

$$V_j(t) = V_{j1}(t) + V_{j2}(t) + V_{j3}(t), \quad (23)$$

where

$$\begin{aligned} V_{j1}(t) &= x^T(t) P_j x(t), \\ V_{j2}(t) &= r_j \int_{-r_j}^0 \int_{t+\theta}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T e^{\beta_j(t-s)} \\ &\quad \times \begin{bmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds d\theta, \\ V_{j3}(t) &= \int_{t-\tau_j}^t \dot{x}^T(s) e^{\beta_j(t-s)} R_j x(s) ds, \end{aligned}$$

scalars  $\beta_j > 0$ ,  $P_j, R_j, Q_{j11}, Q_{j22}$  are positive definite symmetric,  $Q_{j12}$  are any matrices with appropriate dimensions to be determined.

**Lemma 5:** Given scalars  $\beta_j > 0$ , if there exist matrices  $P_j > 0, Q_{j11} > 0, Q_{j22} > 0, Q_{j12}$  and  $N_{jl}^T \{l = 1, \dots, 5\}$  are any matrices with appropriate dimensions, such that the following LMIs hold:

$$Q_j = \begin{pmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{pmatrix} > 0, j \in S_u, \quad (24)$$

$$\phi_j = \begin{pmatrix} \phi_{j11} & \phi_{j12} & \phi_{j13} & \phi_{j14} & \phi_{j15} \\ * & \phi_{j22} & \phi_{j23} & \phi_{j24} & \phi_{j25} \\ * & * & \phi_{j33} & \phi_{j34} & \phi_{j35} \\ * & * & * & \phi_{j44} & \phi_{j45} \\ * & * & * & * & \phi_{j55} \end{pmatrix} < 0, \quad (25)$$

where

$$\begin{aligned} \phi_{j11} &= -\beta_j P_j + P_j A_j + A_j^T P_j + r_j^2 Q_{j11} \\ &\quad - Q_{j22} + N_{j1}^T A_j + A_j^T N_{j1}, \\ \phi_{j12} &= r_j^2 Q_{j12} - N_{j1}^T + A_j^T N_{j2}, \\ \phi_{j13} &= P_j C_j + N_{j1}^T C_j + A_j^T N_{j3}, \\ \phi_{j14} &= P_j B_j + N_{j1}^T B_j + A_j^T N_{j4} + Q_{j22}, \\ \phi_{j15} &= -Q_{j12}^T + A_j^T N_{j5}, \\ \phi_{j22} &= r_j^2 Q_{j22} - N_{j2}^T - N_{j2}, \\ \phi_{j23} &= N_{j2}^T C_j - N_{j3}, \\ \phi_{j24} &= N_{j2}^T B_j - N_{j4}, \\ \phi_{j25} &= -N_{j5}, \quad \phi_{j35} = C_j^T N_{j5}, \\ \phi_{j33} &= -e^{\beta_j \tau_j} R_j + N_{j3}^T C_j + C_j^T N_{j3}, \\ \phi_{j34} &= N_{j3}^T B_j + C_j^T N_{j4}, \quad \phi_{j45} = B_j^T N_{j5} + Q_{j12}^T, \\ \phi_{j44} &= -Q_{j22} + N_{j4}^T B_j + B_j^T N_{j4}, \quad \phi_{j55} = -Q_{j11}, \end{aligned}$$

then along the trajectory of the systems (22), it follows that

$$V_j(t) \leq e^{\beta_j(t-t_0)} V_j(t_0), j \in S_u. \quad (26)$$

**Proof.** Similar to the proof of lemma 4, any matrices  $N_{jk}(k = 1, \dots, 5)$  with appropriate dimensions, it follows that

$$\begin{aligned} \dot{V}_j(t) - \beta_j V_j(t) &\leq -\beta_j x^T(t) P_j x(t) + 2x^T(t) P_j [A_j x(t) \\ &\quad + B_j x(t - r_j) + C_j \dot{x}(t - \tau_j)] + \dot{x}^T(t) R_j \dot{x}(t) \\ &\quad + r_j^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &\quad - \begin{bmatrix} \int_{t-r_j}^t x(s) ds \\ x(t) \\ x(t-r_j) \end{bmatrix}^T \begin{bmatrix} Q_{j11} & Q_{j12} & -Q_{j12} \\ * & Q_{j22} & -Q_{j22} \\ * & * & Q_{j22} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \int_{t-r_j}^t x(s) ds \\ x(t) \\ x(t-r_j) \end{bmatrix} - \dot{x}^T(t - \tau_j) e^{\beta_j \tau_j} R_j \dot{x}(t - \tau_j) \\ &\quad + [x^T(t) N_{j1}^T + \dot{x}^T(t) N_{j2}^T + \dot{x}^T(t - \tau_j) N_{j3}^T \\ &\quad + x^T(t - r_j) N_{j4}^T + (\int_{t-r_j}^t x(s) ds)^T N_{j5}^T] \\ &\quad \times [A_j x(t) - \dot{x}(t) + B_j x(t - r_j) + C_j \dot{x}(t - \tau_j)] \\ &= \xi^T(t) \phi_j \xi(t) < 0, \end{aligned}$$

where  $\xi^T(t)$  are defined in lemma 4. Thus,  $\dot{V}_j(t) - \beta_j V_j(t) \leq 0$ . Integrating these inequalities give inequalities (26). This is the end of proof.  $\square$

Similar to corollary 3.1, the following corollary is given for the switched neutral subsystems (22) with uncertain structure satisfy (3) and (4), such that

$$\begin{cases} \dot{x}(t) - C_j(t) \dot{x}(t - \tau_j) = A_j(t) x(t) + B_j(t) x(t - r_j) \\ x(t_0 + \theta) = \varphi(\theta), \forall \theta \in [-\rho, 0], j \in S_u \end{cases} \quad (27)$$

**Corollary 2:** Given scalars  $\beta_j > 0$ , if there exist scalars  $\varepsilon_j > 0$  and matrices  $P_j > 0, Q_{j11} > 0, Q_{j22} > 0, Q_{j12}, N_{jl}^T \{l \in (1, \dots, 5)\}$  with appropriate dimensions, such that

the following LMIs hold

$$Q_j = \begin{pmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{pmatrix} > 0, \quad (28)$$

$$\varphi_j = \begin{pmatrix} \varphi_{j11} & \varphi_{j12} & \varphi_{j13} & \varphi_{j14} & \varphi_{j15} & N_{j1}^T D & P_j D \\ * & \varphi_{j22} & \varphi_{j23} & \varphi_{j24} & \varphi_{j25} & N_{j2}^T D & 0 \\ * & * & \varphi_{j33} & \varphi_{j34} & 0 & N_{j3}^T D & 0 \\ * & * & * & \varphi_{j44} & 0 & N_{j4}^T D & 0 \\ * & * & * & * & \varphi_{j55} & N_{j5}^T D & 0 \\ * & * & * & * & * & -\varepsilon_j I & 0 \\ * & * & * & * & * & * & -\varepsilon_j I \end{pmatrix} < 0 \quad (29)$$

where

$$\begin{aligned} \varphi_{j11} &= \phi_{j11} + 2\varepsilon_j E_{aj}^T E_{aj}, & \varphi_{j13} &= \phi_{j13} + 2\varepsilon_j E_{aj}^T E_{cj}, \\ \varphi_{j14} &= \phi_{j14} + \varepsilon_j E_{aj}^T E_{bj}, & \varphi_{j33} &= \phi_{j33} + 2\varepsilon_j E_{cj}^T E_{cj}, \\ \varphi_{j34} &= \phi_{j34} + \varepsilon_j E_{cj}^T E_{bj}, & \varphi_{j44} &= \phi_{j44} + \varepsilon_j E_{bj}^T E_{bj}, \end{aligned}$$

with  $\phi_{jkl} (j \in S_u, k, l = (1, \dots, 5))$  are defined in (24), then along the trajectory of the systems (27), it follows that

$$V_j(t) \leq e^{\beta_j(t-t_0)} V_j(t_0), j \in S_u. \quad (30)$$

*Remark 1:* Obviously, lemma 4 and corollary 1 also imply exponential stability of each subsystems in (7) and (16), while lemma 5 and corollary 2 do not guarantee stability of each subsystems in (22) and (27) since they provide sufficient condition of growth estimation of Lyapunov functionals candidate (23).

Now, we are in the position to give the main result of this paper.

*Theorem 1:* The trivial solution of systems (1) is globally exponentially stable and the state decay estimation is given as

$$\|x(t)\| \leq \sqrt{\frac{b}{a}} e^{-\frac{1}{2}\lambda^*(t-t_0)} \|x_0\|_{c1}, \quad (31)$$

if the following assumptions hold:

**I1.** For  $i \in S_s$ ,

$$V_i(t) \leq e^{-\alpha_i(t-t_0)} V_i(t_0), \quad (32)$$

$\alpha_i$  and  $V_i(t)$  are determined by corollary 3.1.

**I2.** For  $j \in S_u$ ,

$$V_j(t) \leq e^{\beta_j(t-t_0)} V_j(t_0), \quad (33)$$

$\beta_j$  and  $V_j(t)$  are determined by corollary 3.2.

**II.** Let  $\alpha = \min\{\alpha_i : \forall i \in S_s\}$  and  $\beta = \max\{\beta_j : \forall j \in S_u\}$  with  $\alpha_i$  and  $\beta_j$  being respectively the decay rates of stable modes and the growth rates of the unstable modes,  $T^+(t_0, t)$  and  $T^-(t_0, t)$  denote the total activation times of the unstable and stable modes over  $(t_0, t)$ , respectively. Assume that for any  $t_0$ , the switching law guarantees that

$$\frac{T^+(t_0, t)}{T^-(t_0, t)} \leq \frac{\alpha - \lambda^* - \ln \mu_1 / T_s}{\beta + \lambda^* + \ln \mu_2 / T_u} \quad (34)$$

where  $\lambda^* \in (0, \alpha)$ ,  $T_s$  denote the average dwell time of the stable subsystems over  $T^-(t_0, t)$ , and  $T_u$  denote the average dwell time of the unstable subsystems over  $T^+(t_0, t)$ . The average dwell time  $T_s$  satisfying

$$T_s > T_s^* = \frac{\ln \mu_1}{\alpha - \lambda^*} \quad (35)$$

Moreover,  $\mu_1, \mu_2 \geq 1$  satisfies

$$\begin{aligned} P_i &\leq \mu_1 P_j, R_i \leq \mu_1 R_j, i \in S_s, j \in S_u, \\ Q_i &= \begin{pmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{pmatrix} \leq \mu_1 \begin{pmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{pmatrix} = \mu_1 Q_j, \\ P_j &\leq \mu_2 P_i, R_j \leq \mu_2 R_i, i \in S_s, j \in S_u, \\ Q_j &= \begin{pmatrix} Q_{j11} & Q_{j12} \\ * & Q_{j22} \end{pmatrix} \leq \mu_2 \begin{pmatrix} Q_{i11} & Q_{i12} \\ * & Q_{i22} \end{pmatrix} = \mu_2 Q_i, \end{aligned} \quad (36)$$

while the other case is different as following

$$\begin{aligned} P_k &\leq \mu_m P_l, R_k \leq \mu_m R_l, m = 1, 2 \\ Q_k &= \begin{pmatrix} Q_{k11} & Q_{k12} \\ * & Q_{k22} \end{pmatrix} \leq \mu_m \begin{pmatrix} Q_{l11} & Q_{l12} \\ * & Q_{l22} \end{pmatrix} = \mu_m Q_l, \end{aligned} \quad (37)$$

where,  $m = 1$  when both  $k$  and  $l$  are in  $S_s$ , and  $m = 2$  when both  $k$  and  $l$  are in  $S_u$ .

$$\begin{aligned} a &= \lambda_{\min}(P_i), \\ b &= \lambda_{\max}(P_i) + \tau_i \lambda_{\max}(R_i) + \frac{r^2}{2} \lambda_{\max}(Q_i) \end{aligned} \quad (38)$$

**Proof.** Just for the sake of our discussion, we assume that the unstable subsystems active during  $[t_{2n}, t_{2n+1})$ ,  $V_{2n+1}(t)$  and  $\beta_{n+1}$  belong to this interval; while the stable subsystems works during  $[t_{2n+1}, t_{2n+2})$ ,  $V_{2n+2}(t)$  and  $\alpha_{n+1}$  belong to this interval. In fact, the order of these switching sequences has no influence to our discussion.

Now we just discuss the case that  $t \in [t_{2n}, t_{2n+1})$ , while the other case is similar to this case. As  $t \in [t_{2n}, t_{2n+1})$ , with the conditions (32), (33) (36) and (37), it holds that

$$\begin{aligned} V_{2n+1}(t) &\leq e^{\beta_{n+1}(t-t_{2n})} V_{2n+1}(t_{2n}) \\ &\leq e^{\beta_{n+1}(t-t_{2n})} \mu_2 V_{2n}(t_{2n}^-) \\ &\leq e^{\beta_{n+1}(t-t_{2n})} \mu_2 e^{-\alpha_n(t_{2n}-t_{2n-1})} V_{2n}(t_{2n-1}) \\ &\leq e^{\beta_{n+1}(t-t_{2n})} \mu_2 e^{-\alpha_n(t_{2n}-t_{2n-1})} \mu_1 V_{2n-1}(t_{2n-1}^-) \\ &\leq \dots \\ &\leq \mu_2^{n+1} \mu_1^n V_1(t_0) \exp[\beta_{n+1}(t-t_{2n}) \\ &\quad + \sum_{k=1}^n \beta_k(t_{2k-1} - t_{2k-2}) - \alpha_k(t_{2k} - t_{2k-1})]. \end{aligned} \quad (39)$$

Let  $N_s(T^-)$  denote the number of switching of  $\sigma(t)$  in the total activation times of stable subsystems, and  $N_u(T^+)$  denote the number of switching of  $\sigma(t)$  in the total activation times of unstable subsystems. According to the definition 2, the above inequality (39) becomes

$$\begin{aligned} V_{2n+1}(t) &\leq e^{N_u(T^+) \ln \mu_2 + N_s(T^-) \ln \mu_1} e^{\beta T^+ - \alpha T^-} V_1(t_0) \\ &\leq e^{\frac{\ln \mu_2}{T_u} T^+ + \frac{\ln \mu_1}{T_s} T^-} e^{\beta T^+ - \alpha T^-} V_1(t_0) \end{aligned} \quad (40)$$

Combining (39)-(40) with (34)-(35) yields

$$V_{2n+1}(t) \leq e^{-\lambda^*(t-t_0)} V_1(t_0) \quad (41)$$

Also notice that

$$V_1(t_0) \leq b \|x_0\|_{c1}^2, \quad a \|x(t)\|^2 \leq V(t) \quad (42)$$

Combining (41) and (42), leads to

$$a \|x(t)\|^2 \leq b e^{-\lambda^*(t-t_0)} \|x_0\|_{c1}^2$$

which implies (31). This completes the proof.  $\square$

**Remark 2:** Assumption I is made to estimate the growth rate of unstable subsystems or the decay rate of stable subsystems, while Assumption II means that the total activation time of unstable subsystems is smaller than that of the stable subsystems, and it also implies that the dwell time  $T_s$  should not less than the specified value.

**Remark 3:** There is no other restriction on the dwell time  $T_u$ , however, the condition (34) implies that the ratio of the total activation time of unstable subsystems to that of the stable subsystems should decrease as the dwell time  $T_u$  increase. In addition, it also means that the ratio can be raised as  $T_s$  increase from condition (34).

**Remark 4:** It is easy to see that our delay-dependent results consider not only the information on the size of the discrete delays but also such information on neutral delays. However, according to the meaning of single Lyapunov approach, [24] and [25] could not obtain the delay-dependent conditions on neutral delays at all.

### C. Simulation examples

In order to show the effectiveness of the conditions presented in Section 3, in this section, two examples are provided.

**Example 1.** Consider the nominal switched neutral systems as following,

$$\begin{cases} \dot{x}(t) - C_1 \dot{x}(t - \tau_1) = A_1 x(t) + B_1 x(t - r_1), & 1 \in S_s, \\ \dot{x}(t) - C_2 \dot{x}(t - \tau_2) = A_2 x(t) + B_2 x(t - r_2), & 2 \in S_u, \end{cases} \quad (43)$$

the parameters of the system are specified as follows

$$\begin{aligned} A_1 &= \begin{pmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{pmatrix}, & A_2 &= \begin{pmatrix} 1 & 1 \\ 1.5 & -2 \end{pmatrix}, \\ B_1 &= \begin{pmatrix} -1.1 & -0.2 \\ 0.1 & -1.1 \end{pmatrix}, & B_2 &= \begin{pmatrix} -1 & -0.6 \\ 0.5 & -1.2 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{pmatrix}, & C_2 &= \begin{pmatrix} 0.2 & 0.1 \\ -0.4 & 0.8 \end{pmatrix}, \end{aligned}$$

with  $r_1 = 0.245, \tau_1 = 0.2455, r_2 = 0.4372, \tau_2 = 0.2526$ . Seeing from Figs. 1 and 2, we can easily find that subsystem 1 is stable while subsystem 2 is unstable. However, for  $\alpha_1 = 2.3, \beta_2 = 1.9, \lambda^* = 1.6, \mu_1 = 1.3532$  and  $\mu_2 = 1.1242$ , we have feasible solution of LMIs in lemma 4 and lemma 5. Theorem 3.1 gives  $T_s^* = \frac{\ln \mu_1}{\alpha - \lambda^*} = 0.4321$ . Taking  $T_s > T_s^* = \frac{\ln \mu_1}{\alpha - \lambda^*} = 0.4321$  and  $T_0 = 1.4732$  as a period time of both subsystem 1 and subsystem 2 are activated, Subsystem 1 activated on  $nT_0 \leq t < nT_0 + 1.4321 (n = 0, 1, \dots)$ , while subsystem 2 activated on  $nT_0 + 1.4321 \leq t < (n+1)T_0 (n = 0, 1, \dots)$ . From (38), we have  $a = 16.6045, b = 1356.9$ . Using (31), one can obtain

$$\|x(t)\| \leq 1.7986e^{-0.8(t-t_0)} \|x_0\|_{c1},$$

which means that the switched neutral systems is exponentially stable by Theorem 3.1. Let  $(4, -5)$  be the initial point. Fig3 show the state trajectories of the switched neutral system, and Fig4 show the the phase map of the switched neutral systems. From these figures, one can see that the switched neutral

systems consist of unstable subsystems and stable subsystems can reach to stability rapidly using the above time-dependent switching rule.

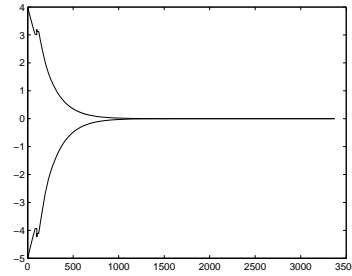


Fig. 1. Behavior of the state component of the switched neutral subsystem 1.

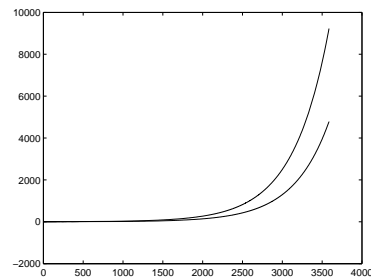


Fig. 2. Behavior of the state component of the switched neutral subsystem 2.

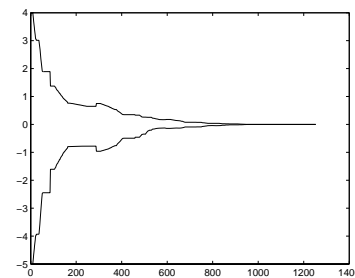


Fig. 3. Behavior of the solution  $x(t)$  of the switched neutral systems.

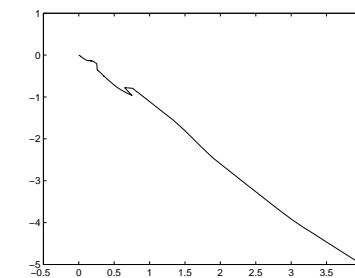


Fig. 4. The phase map of the switched neutral systems.

**Example 2.** Consider the systems discussed in Example. 1 with uncertain structure described by (3) and (4) as following

$$\begin{cases} \dot{x}(t) - C_1(t) \dot{x}(t - \tau_1) = A_1(t)x(t) + B_1(t)x(t - r_1), \\ \dot{x}(t) - C_2(t) \dot{x}(t - \tau_2) = A_2(t)x(t) + B_2(t)x(t - r_2), \end{cases} \quad (44)$$

where  $D = 0.1459I, E_{ai} = E_{bi} = E_{ci} = I (i = 1, 2)$ . The

uncertainty matrix is chosen as

$$F(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}$$

For  $\alpha_1 = 2.3, \beta_2 = 1.9, \lambda^* = 1.6, \mu_1$  and  $\mu_2$  are as same as it in example 1. Choose  $\varepsilon_1 = 0.1, \varepsilon_2 = 0.05$ , we have feasible solution of the LMIs in Corollary 3.1 and Corollary 3.2 as following

$$P_1 = \begin{pmatrix} 1.1409 & -0.6198 \\ -0.6198 & 1.8927 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 0.8973 & -0.5463 \\ -0.5463 & 1.5553 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 420.5677 & 5.9623 \\ 5.9623 & 410.9918 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 265.0990 & 18.9656 \\ 18.9656 & 268.3814 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 1.4344 & -22.2792 & 0.0574 & -0.2406 \\ 18.0982 & 3.6692 & -0.4080 & 0.8578 \\ 0.0574 & -0.4080 & 2.9957 & -0.2636 \\ -0.2406 & 0.8578 & -1.3419 & 3.4756 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 1.3255 & 1.5421 & -0.1424 & -0.2460 \\ -5.4866 & 3.5090 & -0.2001 & 0.7516 \\ -0.1424 & -0.2001 & 3.0241 & 0.0234 \\ -0.2460 & 0.7516 & -1.1701 & 2.6254 \end{pmatrix}.$$

Using (31), one can obtain

$$\|x(t)\| \leq 17.9430e^{-0.8(t-t_0)} \|x_0\|_{c1},$$

which means that with same switching law given in Example. 1, the uncertain switched neutral systems can be robust exponentially stable.

## II. CONCLUSIONS

A new switching rule for stabilization of a class of uncertain switched neutral systems is achieved in this paper. By employing multiple Lyapunov functional approach and dwell time technique, more flexible time dependent switching rule for stabilization of this systems is given. The robust exponentially stable criterion is derived in terms of linear matrix inequalities. Simulation examples are given to demonstrate our theoretical results.

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