Global exponential stability of impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms

Liping Zhang and Kelin Li

Abstract—In this paper, a class of impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms is formulated and investigated. By establishing a delay differential inequality and M-matrix theory, some sufficient conditions ensuring the existence, uniqueness and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms are obtained. In particular, a precise estimate of the exponential convergence rate is also provided, which depends on system parameters and impulsive perturbation intention. It is believed that these results are significant and useful for the design and applications of BAM fuzzy cellular neural networks. An example is given to show the effectiveness of the results obtained here.

Keywords—global exponential stability; Bidirectional associative memory; fuzzy cellular neural networks; leakage delays; impulses.

I. INTRODUCTION

IN mathematical modelling of real world problems, we encounter inconveniences, namely, the complexity and the uncertainty or vagueness. In order to take vagueness into consideration, fuzzy theory is considered as a suitable setting. Based on traditional CNN, Yang et al. proposed the fuzzy cellular neural networks (FCNN) [1], [2], which integrates fuzzy logic into the structure of the traditional CNN and maintains local connectedness among cells. Unlike previous CNN structures, FCNN has fuzzy logic between its template input and/or output besides the sum of product operation. FCNN is very useful paradigm for image processing problems (e.g., see, [12], [13]), which is a cornerstone in image processing and pattern recognition. In such applications, the stability of networks plays an important role, it is significant and necessary to investigate the stability. It is well known, in both biological and artificial neural networks, that the delays arise because of the processing of information. Time delays may lead to oscillation, divergence, or instability which may be harmful to a system. Therefore, study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. In recent years, there have been many analytical results for FCNNs with various axonal signal transmission delays, for example, see [3]-[13] and references therein. However, except various axonal signal transmission delays, time delay in the leakage term also has great impact on the dynamics of neural

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networks. As pointed out by Gopalsamy [14], [15], time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Recently, some authors have paid attention to stability analysis of neural networks with time delays in the leakage (or"forgetting") terms [14]-[20].

On the other hand, in respect of the complexity, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as economics, mechanics, medicine and biology, electronics and telecommunications, etc. Many interesting results on impulsive effect have been gained, e.g., Refs. [31]-[40]. As artificial electronic systems, neural networks such as CNN, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the system just as time delays. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of neural networks.

The bidirectional associative memory (BAM) neural network models were first introduced by Kosko [21]-[23]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers, the X-layer and Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer. Through iterations of forward and backward information flows between the two layers, it performs a two-way associative search for stored bipolar vector pairs and generalizes the single-layer autoassociative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks possesses good application prospects in some fields such as pattern recognition, signal and image processing, artificial intelligence. Many researchers have done extensive works on this subject due to their comprehensive applications [24]-[30]. To the best of our knowledge, few authors have considered impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms.

Motivated by the above discussions, by dint of the idea of BAM neural networks, the objective of this paper is to formulate and study impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms. Under quite general conditions, by establishing a delay differential inequality and M-matrix theory, some sufficient conditions ensuring the existence, uniqueness and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms are obtained.

The paper is organized as follows. In Section II, the

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new neural network model is formulated, and the necessary knowledge is provided. The main results and their proofs are presented in Section III. In Section IV, an example is given to show the effectiveness of the results obtained here. Finally, in Section V we give the conclusion.

II. MODEL DESCRIPTION AND PRELIMINARIES

In this section, we will consider the model of impulsive BAM fuzzy cellular neural networks with time-varying delays, it is described by the following functional differential equations

$$
\begin{cases}
\dot{x}_i(t) = -a_i x_i(t-\delta_i) + \sum_{j=1}^{m} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{m} \tilde{a}_{ij} w_j \\
\quad + \bigwedge_{j=1}^{m} \alpha_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\quad + \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\quad + \bigwedge_{j=1}^{m} T_{ij} w_j + \bigvee_{j=1}^{m} H_{ij} w_j + I_i, \quad t \geq 0, \quad t \neq t_k, \\
x_i(t^+) = x_i(t^-) + P_{ik}(x_i(t^-)), \quad t = t_k, \quad k \in N, \\
\dot{y}_j(t) = -b_j y_j(t-\theta_j) + \sum_{i=1}^{n} b_{ji} f_i(x_i(t)) + \sum_{i=1}^{n} \tilde{b}_{ji} \tilde{w}_i \\
\quad + \bigwedge_{i=1}^{n} \beta_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\quad + \bigvee_{i=1}^{n} \tilde{\beta}_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\quad + \bigwedge_{i=1}^{n} \tilde{T}_{ji} \tilde{w}_i + \bigvee_{i=1}^{n} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \geq 0, \quad t \neq t_k, \\
y_j(t^+) = y_j(t^-) + Q_{ik}(y_j(t^-)), \quad t = t_k, \quad k \in N
\end{cases}
\tag{1}
$$

for $i \in \mathscr{I} \triangleq \{1, 2, \cdots, n\}$, $j \in \mathscr{J} \triangleq \{1, 2, \cdots, m\}$, where $N \triangleq \{1, 2, \cdots\}$, $x_i(t)$ and $y_j(t)$ are the states of the ith neuron and the jth neuron at time t, respectively; $\delta_i \geq 0$ and $\theta_j \geq 0$ denote the leakage delays, respectively; f_i and g_j denote the signal functions of the ith neuron and the jth neuron at time t, respectively; $\tilde{w}_i$ and w_j denote inputs of the ith neuron and the jth neuron at the time t, respectively; and I_i and J_j denote bias of the ith neuron and the jth neuron at the time t, respectively; $a_i > 0, b_j > 0, a_{ij}, \tilde{a}_{ij}, \alpha_{ij}, \tilde{\alpha}_{ij}, b_{ji}, \tilde{b}_{ji}, \beta_{ji}, \tilde{\beta}_{ji}$ are constants, a_i and b_j represent the rates with which the ith neuron and the jth neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; a_{ij}, b_{ji} and $\tilde{a}_{ij}, \tilde{b}_{ji}$ denote connection weights of feedback template and feedforward template, respectively; α_{ij}, β_{ji} and $\tilde{\alpha}_{ij}, \tilde{\beta}_{ji}$ denote connection weights of the delays fuzzy feedback MIN template and the delays fuzzy feedback MAX template, respectively; $T_{ij}, \tilde{T}_{ji}$ and $H_{ij}, \tilde{H}_{ji}$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; $\tau_{ij}(t)$ $(0 \leq \tau_{ij}(t) \leq \tau_{ij})$ and $\sigma_{ji}(t)$ $(0 \leq \sigma_{ji}(t) \leq \sigma_{ji})$ correspond to the transmission delays at time t, respectively; t_k is called the impulsive moment, and satisfies $0 < t_1 < t_2 < \cdots$, $\lim_{k \to +\infty} t_k = +\infty$; $x_i(t_k^-)$ and $x_i(t_k^+)$ denote the left-hand and right-hand limits at t_k, respectively; P_{ik} and Q_{jk} show the impulsive perturbations of the ith neuron and jth neuron at

time t_k, respectively. We always assume $x_i(t_k^+) = x_i(t_k)$ and $y_j(t_k^+) = y_j(t_k)$, $k \in N$.

The initial conditions are given by

$$
\begin{cases}
x_i(s) = \phi_i(s), & -\tau \leq t \leq 0, \\
y_j(s) = \varphi_j(s), & -\sigma \leq t \leq 0,
\end{cases}
\tag{2}
$$

where $\tau = \max\limits_{1 \leq i \leq n, 1 \leq j \leq m} \{\tau_{ij}\}$, $\sigma = \max\limits_{1 \leq i \leq n, 1 \leq j \leq m} \{\sigma_{ji}\}$, and $\phi_i(s)$ and $\varphi_j(s)$ $(i \in \mathscr{I}, j \in \mathscr{J})$ are continuous on $[-\tau, 0]$ and $[-\sigma, 0]$, respectively.

If the impulsive operators $P_{ik}(x_i) = 0$, $Q_{jk}(y_j) = 0$, $i \in \mathscr{I}$, $j \in \mathscr{J}$, $k \in N$, then system (1) may reduce to the following model

$$
\begin{cases}
\dot{x}_i(t) = -a_i x_i(t-\delta_i) + \sum_{j=1}^{m} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{m} \tilde{a}_{ij} w_j \\
\quad + \bigwedge_{j=1}^{m} \alpha_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\quad + \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\quad + \bigwedge_{j=1}^{m} T_{ij} w_j + \bigvee_{j=1}^{m} H_{ij} w_j + I_i, \quad t \geq 0, \quad i \in \mathscr{I}, \\
\dot{y}_j(t) = -b_j y_j(t-\theta_j) + \sum_{i=1}^{n} b_{ji} f_i(x_i(t)) + \sum_{i=1}^{n} \tilde{b}_{ji} \tilde{w}_i \\
\quad + \bigwedge_{i=1}^{n} \beta_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\quad + \bigvee_{i=1}^{n} \tilde{\beta}_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\quad + \bigwedge_{i=1}^{n} \tilde{T}_{ji} \tilde{w}_i + \bigvee_{i=1}^{n} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \geq 0, \quad j \in \mathscr{J}.
\end{cases}
\tag{3}
$$

System (3) is called the continuous system of model (1).

Throughout this paper, we make the following assumptions:
(H1) For neuron activation functions f_i and g_j $(i \in \mathscr{I}, j \in \mathscr{J})$, there exist two positive diagonal matrices $F = \mathrm{diag}(F_1, F_2, \cdots, F_n)$ and $G = \mathrm{diag}(G_1, G_2, \cdots, G_m)$ such that

$$
F_i = \sup_{x \neq y} \left| \frac{f_i(x) - f_i(y)}{x - y} \right|, \quad G_j = \sup_{x \neq y} \left| \frac{g_j(x) - g_j(y)}{x - y} \right|
$$

for all $x, y \in R$ $(x \neq y)$.
(H2) Let $\bar{P}_k(x) = x + P_k(x)$ and $\bar{Q}_k(y) = y + Q_k(y)$, where
$$
\begin{aligned}
&\bar{P}_k(x) = (\bar{P}_{1k}(x_1), \bar{P}_{2k}(x_2), \cdots, \bar{P}_{nk}(x_n))^T, \\
&\bar{Q}_k(x) = (\bar{Q}_{1k}(y_1), \bar{Q}_{2k}(y_2), \cdots, \bar{Q}_{mk}(y_m))^T, \\
&P_k(x) = (P_{1k}(x_1), P_{2k}(x_2), \cdots, P_{nk}(x_n))^T, \\
&Q_k(y) = (Q_{1k}(y_1), Q_{2k}(y_2), \cdots, Q_{mk}(y_m))^T.
\end{aligned}
$$
For $k \in N$ there exist nonnegative diagnose matrices $\Gamma_k = \mathrm{diag}(\gamma_{1k}, \gamma_{2k}, \cdots, \gamma_{nk})$ and $\bar{\Gamma}_k = \mathrm{diag}(\bar{\gamma}_{1k}, \bar{\gamma}_{2k}, \cdots, \bar{\gamma}_{mk})$ such that

$$
\gamma_{ik} = \sup_{x \neq y} \left| \frac{\bar{P}_{ik}(x) - \bar{P}_{ik}(y)}{x - y} \right|,
$$

$$
\bar{\gamma}_{ik} = \sup_{x \neq y} \left| \frac{\bar{Q}_{jk}(x) - \bar{Q}_{jk}(y)}{x - y} \right|
$$

for all $x, y \in R$ $(x \neq y)$, $i \in \mathscr{I}$, $j \in \mathscr{J}$, $k \in N$.

To begin with, we introduce some notation and recall some basic definitions.

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$PC[J, R^n] \triangleq \{z(t) : J \to R^l \,|\, z(t) \text{ is continuous at } t \neq t_k,$ $z(t_k^+) = z(t_k), \text{ and } z(t_k^-) \text{ exists for } t, t_k \in J, k \in N\}$, where $J \subset R$ is an interval.

$PC_\tau \triangleq \{\phi : [-\tau, 0] \to R^n| \phi(s^+) = \phi(s) \text{ for } s \in [-\tau, 0), \phi(s^-) \text{ exists for } s \in (-\tau, 0], \phi(s^-) = \phi(s) \text{ for all but at most a finite number of points } s \in (-\tau, 0]\}$.

$PC_\sigma \triangleq \{\varphi : [-\sigma, 0] \to R^m| \varphi(s^+) = \varphi(s) \text{ for } s \in [-\sigma, 0), \varphi(s^-) \text{ exists for } s \in (-\sigma, 0], \varphi(s^-) = \varphi(s) \text{ for all but at most a finite number of points } s \in (-\tau, 0]\}$.

For an $m \times n$ matrix A, $|A|$ denotes the absolute value matrix given by $|A| = (|a_{ij}|)_{m \times n}$. For $A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n} \in R^{m \times n}$, $A \geq B$ ($A > B$) means that each pair of corresponding elements of A and B such that the inequality $a_{ij} \geq b_{ij}$ ($a_{ij} > b_{ij}$).

Definition 1: A function $(x(t), (y(t))^T$ ($x : [-\tau, +\infty) \to R^n$, $y : [-\sigma, +\infty) \to R^m$) is said to be the solution of impulsive system (1) with the initial condition (2), if the following two conditions are satisfied:

(i) $(x, y)^T$ is piecewise continuous with first kind discontinuity at the points t_k, $k \in N$. Moreover, $(x, y)^T$ is right continuous at each discontinuity point.

(ii) $(x, y)^T$ satisfies system (1) for $t \geq 0$, and $x(t) = \phi(t)$ ($t \in [-\tau, 0]$), $y(t) = \varphi(t)$ ($t \in [-\sigma, 0]$).

Especially, a point $(x^*, y^*)^T$ ($x^* \in R^n, y^* \in R^m$) is called an equilibrium point of system (1), if $(x(t), y(t))^T = (x^*, y^*)^T$ is a solution of system (1).

Throughout this paper, we always assume that the impulsive operators P_k and Q_k satisfy (referring to [27]-[40])

$$P_k(x^*) = 0 \text{ and } Q_k(y^*) = 0, \quad k \in N,$$

i.e.,

$$\bar{P}_k(x^*) = x^* \text{ and } \bar{Q}_k(y^*) = y^*, \quad k \in N, \quad (4)$$

where $(x^*, y^*)^T$ is the equilibrium point of continuous systems (3). That is, if $(x^*, y^*)^T$ is an equilibrium point of continuous system (3), then $(x^*, y^*)^T$ is also the equilibrium point of impulsive system (1).

Definition 2: The equilibrium point $(x^*, y^*)^T$ of system (1) is said to be globally exponentially stable, if there exist constants $\lambda > 0$ and $M \geq 1$ such that

$$\sum_{i=1}^n |x_i(t) - x_i^*| + \sum_{j=1}^m |y_j(t) - y_j^*|$$
$$\leq M(\| \phi - x^* \| + \| \varphi - y^* \|)e^{-\lambda t}$$

for all $t \geq 0$, where $(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ is any solution of system (1) with the initial condition (2), $x^* = (x_1^*, \cdots, x_n^*)^T$, $y^* = (y_1^*, \cdots, y_m^*)^T$, $\phi = (\phi_1, \cdots, \phi_n)^T$, $\varphi = (\varphi_1, \cdots, \varphi_m)^T$, and $\|\phi - x^*\| = \sup_{-\tau \leq t \leq 0} \sum_{i=1}^n |\phi_i(t) - x_i^*|$, $\|\varphi - y^*\| = \sup_{-\sigma \leq t \leq 0} \sum_{j=1}^m |\varphi_j(t) - y_j^*|$.

Definition 3: A real matrix $D = (d_{ij})_{n \times n}$ is said to be a nonsingular M-matrix if $d_{ij} \leq 0$, $i, j = 1, 2, \cdots, n$, $i \neq j$, and all successive principal minors of D are positive.

Lemma 1: [41] Let $D = (d_{ij})_{n \times n}$ with $d_{ij} \leq 0 \, (i \neq j)$, then the following statements are true:

(i) D is a nonsingular M-matrix if and only if D is inverse-positive; that is, D^{-1} exists and D^{-1} is a nonnegative matrix.

(ii) D is a nonsingular M-matrix if and only if there exists a positive vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T$ such that $D\xi > 0$.

Lemma 2: [2] For any positive integer l, let $h_j : R \to R$ be a function ($j = 1, 2, \cdots, l$), then we have

$$\left| \bigwedge_{j=1}^l \alpha_j h_j(z_j) - \bigwedge_{j=1}^l \alpha_j h_j(\bar{z}_j) \right| \leq \sum_{j=1}^l \left|\alpha_j\right| \cdot \left| h_j(z_j) - h_j(\bar{z}_j) \right|,$$

$$\left| \bigvee_{j=1}^l \alpha_j h_j(z_j) - \bigvee_{j=1}^l \alpha_j h_j(\bar{z}_j) \right| \leq \sum_{j=1}^l \left|\alpha_j\right| \cdot \left| h_j(z_j) - h_j(\bar{z}_j) \right|$$

for all $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_l)^T$, $z = (z_1, z_2, \cdots, z_l)^T$, $\bar{z} = (\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_l)^T \in R^l$.

III. MAIN RESULTS

In this section, we will discuss the existence and global exponential stability of the equilibrium point of impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms, and give their proofs. In order to prove our main result, we need the following lemma.

Lemma 3: Let $a < b \leq +\infty$, and $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in PC[[a, b), R^n]$ and $v(t) = (v_1(t), v_2(t), \cdots, v_m(t))^T \in PC[[a, b), R^m]$ satisfy the following delay differential inequalities with the initial conditions $u(a + s) \in PC_\tau$ and $v(a + s) \in PC_\sigma$:

$$\begin{cases} D^+u_i(t) \leq -r_i u_i(t - \delta_i) + \sum_{j=1}^m p_{ij} v_j(t) \\ \quad + \sum_{j=1}^m q_{ij} v_j(t - \tau_{ij}(t)), \quad i \in \mathscr{I}, \\ D^+v_j(t) \leq -\bar{r}_j v_j(t - \theta_j) + \sum_{i=1}^n \bar{p}_{ji} u_i(t) \\ \quad + \sum_{i=1}^n \bar{q}_{ji} u_i(t - \sigma_{ji}(t)), \quad j \in \mathscr{J}, \end{cases} \quad (5)$$

where $r_i > 0$, $p_{ij} > 0$, $q_{ij} > 0$, $\bar{r}_j > 0$, $\bar{p}_{ji} > 0$, $\bar{q}_{ji} > 0$, $i \in \mathscr{I}, j \in \mathscr{J}$. If the initial conditions satisfy

$$\begin{cases} u(s) \leq \kappa \xi e^{-\lambda(s-a)}, & s \in [-\tau, 0], \\ v(s) \leq \kappa \eta e^{-\lambda(s-a)}, & s \in [-\sigma, 0], \end{cases} \quad (6)$$

in which $\lambda > 0$, $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$ and $\eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ satisfy

$$\begin{cases} (\lambda - r_i e^{\lambda \delta_i})\xi_i + \sum_{j=1}^m (p_{ij} + q_{ij} e^{\lambda \tau_{ij}})\eta_j < 0, & i \in \mathscr{I}, \\ (\lambda - \bar{r}_j e^{\lambda \theta_j})\eta_j + \sum_{i=1}^n (\bar{p}_{ji} + \bar{q}_{ji} e^{\lambda \sigma_{ji}})\xi_i < 0, & j \in \mathscr{J}. \end{cases} \quad (7)$$

Then

$$\begin{cases} u(t) \leq \kappa \xi e^{-\lambda(t-a)}, & t \in [-a, b), \\ v(t) \leq \kappa \eta e^{-\lambda(t-a)}, & t \in [-a, b). \end{cases}$$

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Proof. For $i \in \mathscr{I}$, $j \in \mathscr{J}$ and arbitrary $\varepsilon > 0$, set $z_i(t) \triangleq (\kappa + \varepsilon)\xi_i e^{-\lambda(t-a)}$, $\bar{z}_j(t) \triangleq (\kappa + \varepsilon)\eta_j e^{-\lambda(t-a)}$, we prove that

$$\begin{cases} u_i(t) \leq z_i(t) = (\kappa + \varepsilon)\xi_i e^{-\lambda(t-a)}, & t \in [a,b), \quad i \in \mathscr{I}, \\ v_j(t) \leq \bar{z}_j(t) = (\kappa + \varepsilon)\eta_j e^{-\lambda(t-a)}, & t \in [a,b), \quad j \in \mathscr{J}. \end{cases} \tag{8}$$

If this is not true, no loss of generality, suppose that there exist i_0 and $t^* \in [a,b)$ such that

$$u_{i_0}(t^*) = z_{i_0}(t^*), \quad D^+ u_{i_0}(t^*) \geq \dot{z}_{i_0}(t^*),$$
$$u_i(t) \leq z_i(t), \quad v_j(t) \leq \bar{z}_j(t), \quad t \in [a, t^*] \tag{9}$$

for $i \in \mathscr{I}$, $j \in \mathscr{J}$.

However, from (5) and (8), we get

$$\begin{aligned} D^+ u_{i_0}(t^*) \quad &\leq \quad -r_{i_0} u_{i_0}(t^* - \delta_{i_0}) + \sum_{j=1}^{m} p_{ji_0} v_j(t^*) \\ &\quad + \sum_{j=1}^{m} q_{ji_0} v_j(t^* - \tau_{ji_0}(t^*)) \\ &\leq \quad -r_{i_0}(\kappa + \varepsilon)\xi_{i_0} e^{-\lambda(t^* - \delta_{i_0} - a)} \\ &\quad + \sum_{j=1}^{m} p_{ji_0} \eta_j (\kappa + \varepsilon)\eta_j e^{-\lambda(t^* - a)} \\ &\quad + \sum_{j=1}^{m} q_{ji_0}(\kappa + \varepsilon)\eta_j e^{-\lambda(t^* - \tau_{ji_0}(t^*) - a)} \\ &= \quad \Big[-r_{i_0} e^{\lambda \delta_{i_0}} \xi_{i_0} + \sum_{j=1}^{m} (p_{ji_0} + q_{ji_0} e^{\lambda \tau_{ji_0}(t^*)})\eta_j \Big] \\ &\quad \times (\kappa + \varepsilon) e^{-\lambda(t^* - a)} \\ &\leq \quad \Big[-r_{i_0} e^{\lambda \delta_{i_0}} \xi_{i_0} + \sum_{j=1}^{m} (p_{ji_0} + q_{ji_0} e^{\lambda \tau_{ji_0}})\eta_j \Big] \\ &\quad \times (\kappa + \varepsilon) e^{-\lambda(t^* - a)}. \end{aligned}$$

Since (7) holds, it follows that $-r_{i_0} e^{\lambda \delta_{i_0}} \xi_{i_0} + \sum_{j=1}^{m} (p_{ji_0} + q_{ji_0} e^{\lambda \tau_{ji_0}})\eta_j < -\lambda \xi_{i_0} < 0$. Therefore, we have

$$D^+ u_{i_0}(t^*) < -\lambda \xi_{i_0}(\kappa + \varepsilon) e^{-\lambda(t^* - a)} = \dot{z}_{i_0}(t^*),$$

which contradicts the inequality $D^+ u_{i_0}(t^*) \geq \dot{z}_{i_0}(t^*)$ in (9). Thus (8) holds for all $t \in [a,b)$. Letting $\varepsilon \to 0$, we have

$$\begin{cases} u_i(t) \leq \kappa \xi_i e^{-\lambda(t-a)}, & t \in [a,b), \quad i \in \mathscr{I} \\ v_j(t) \leq \kappa \eta_j e^{-\lambda(t-a)}, & t \in [a,b), \quad j \in \mathscr{J}. \end{cases}$$

The proof is completed.

Theorem 1: Under assumptions (H1) and (H2), if the following conditions hold:

(C1) there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$, $\eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ such that

$$\begin{cases} 0 > (\lambda - r_i e^{\lambda \delta_i})\xi_i \\ \quad + \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)e^{\lambda \tau_{ij}} \Big] G_j \eta_j, \\ 0 > (\lambda - \bar{r}_j e^{\lambda \theta_j})\eta_j \\ \quad + \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|)e^{\lambda \sigma_{ji}} \Big] F_i \xi_i \end{cases}$$

for $i \in \mathscr{I}$, $j \in \mathscr{J}$;

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \leq i \leq n, 1 \leq j \leq m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}$, $k \in N$;

then system (1) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate approximatively equals $\lambda - \mu$.

Proof. By M-matrix theory ([41]), Condition (C1) can guarantee that system (1) has exactly an equilibrium point (in detail, see, [10]). Let $(x^*, y^*)^T$ be an equilibrium point of system (1), $(x(t), y(t))^T$ is an arbitrary solution of system (1). Now let $u_i(t) = x_i(t) - x_i^*$, $i \in \mathscr{I}$, $v_j(t) = y_j(t) - y_j^*$, $j \in \mathscr{J}$. It is easy to see that system (1) can be transformed into the following system:

$$\begin{cases} \dot{u}_i(t) = -a_i u_i(t - \delta_i) + \sum_{j=1}^{m} a_{ij}(g_j(v_j(t) + y_j^*) - g_j(y_j^*)) \\ \quad + \bigwedge_{j=1}^{m} \alpha_{ij} g_j(v_j(t - \tau_{ij}(t)) + y_j^*) - \bigwedge_{j=1}^{m} \alpha_{ij} g_j(y_j^*) \\ \quad + \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(v_j(t - \tau_{ij}(t)) + y_j^*) - \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(y_j^*), \\ \qquad t \geq 0, \quad t \neq t_k, \\ u_i(t_k^+) = \tilde{P}_{ik}(u_i(t_k^-)), \quad k \in N \\ \dot{v}_j(t) = -b_j v_j(t - \theta_j) + \sum_{i=1}^{n} b_{ji}(f_i(u_i(t) + x_i^*) - f_i(x_i^*)) \\ \quad + \bigwedge_{i=1}^{n} \beta_{ji} f_i(u_i(t - \sigma_{ji}(t)) + x_i^*) - \bigwedge_{i=1}^{n} \beta_{ji} f_i(x_i^*) \\ \quad + \bigvee_{i=1}^{n} \tilde{\beta}_{ji} f_i(u_i(t - \sigma_{ji}(t)) + x_i^*) - \bigvee_{i=1}^{n} \tilde{\beta}_{ji} f_i(x_i^*), \\ \qquad t \geq 0, \quad t \neq t_k \\ v_j(t_k^+) = \tilde{Q}_{jk}(v_j(t_k^-)), \quad k \in N, \end{cases} \tag{10}$$

where $\tilde{P}_{ik}(u_i(t)) = \bar{P}_{ik}(u_i(t) + x_i^*) - \bar{P}_{ik}(x_i^*)$, $\tilde{Q}_{jk}(v_j(t)) = \bar{Q}_{jk}(v_j(t) + y_j^*) - \bar{Q}_{jk}(y_j^*)$, and initial conditions of (10) are

$$\begin{cases} u(s) = \bar{\phi}(s) = x(s) - x^* = \phi(s) - x^*, & s \in [-\tau, 0], \\ v(s) = \bar{\varphi}(s) = y(s) - y^* = \varphi(s) - y^*, & s \in [-\sigma, 0]. \end{cases} \tag{11}$$

From (H1) and Lemma 2, we calculate the upper right derivative along the solutions of first equation equation of (10), we can obtain

$$\begin{aligned} &D^+ |u_i(t)| \\ &= \text{sgn}(u_i(t)) \frac{du_i(t)}{dt} \\ &= \text{sgn}(u_i(t)) \Big\{ -a_i u_i(t - \delta_i) \\ &\quad + \sum_{j=1}^{m} a_{ij} \Big[g_j(v_j(t) + y_j^*) - g_j(y_j^*) \Big] \\ &\quad + \Big[\bigwedge_{j=1}^{m} \alpha_{ij} g_j(v_j(t - \tau_{ij}(t)) + y_j^*) - \bigwedge_{j=1}^{m} \alpha_{ij} g_j(y_j^*) \Big] \\ &\quad + \Big[\bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(v_j(t - \tau_{ij}(t)) + y_j^*) - \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(y_j^*) \Big] \Big\} \\ &\leq -a_i |u_i(t - \delta_i)| + \sum_{j=1}^{m} |a_{ij}| \Big| g_j(v_j(t) + y_j^*) - g_j(y_j^*) \Big| \end{aligned}$$

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$$+\left|\bigwedge_{j=1}^{m}\alpha_{ij}g_j(v_j(t-\tau_{ij}(t))+y_j^*)-\bigwedge_{j=1}^{m}\alpha_{ij}g_j(y_j^*)\right|$$

$$+\left|\bigvee_{j=1}^{m}\tilde{\alpha}_{ij}g_j(v_j(t-\tau_{ij}(t))+y_j^*)-\bigvee_{j=1}^{m}\tilde{\alpha}_{ij}g_j(y_j^*)\right|$$

$$\leq\quad -a_i|u_i(t-\delta_i)|+\sum_{j=1}^{m}|a_{ij}|G_j|v_j(t)|$$

$$+\sum_{j=1}^{m}|\alpha_{ij}|G_j|v_j(t-\tau_{ij}(t))|$$

$$+\sum_{j=1}^{m}|\tilde{\alpha}_{ij}|G_j|v_j(t-\tau_{ij}(t))|$$

$$=\quad -a_i|u_i(t-\delta_i)|+\sum_{j=1}^{m}|a_{ij}|G_j|v_j(t)|$$

$$+\sum_{j=1}^{m}(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j|v_j(t-\tau_{ij}(t))|. \quad (12)$$

By the same way, we calculate the upper right derivative along the solutions of third equation equation of (10), we can obtain

$$D^+|v_j(t)|\quad\leq\quad -b_j|v_j(t-\theta_j)|+\sum_{i=1}^{n}|b_{ji}||u_i(t)|$$
$$+\sum_{j=i}^{n}(|\beta_{ji}|+|\tilde{\beta}_{ji}|)F_i|u_i(t-\sigma_{ji}(t))|. \ (13)$$

From (12) and (13), we have

$$\begin{cases} D^+|u_i(t)|\leq -a_i|u_i(t-\delta_i)|+\sum_{j=1}^{m}|a_{ij}|G_j|v_j(t)| \\ \qquad +\sum_{j=1}^{m}(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j|v_j(t-\tau_{ij}(t))|, \quad i\in\mathscr{I}, \\ D^+|v_j(t)|\leq -b_j|v_j(t-\theta_j)|+\sum_{i=1}^{n}|b_{ji}|F_i|u_i(t)| \\ \qquad +\sum_{i=1}^{n}(|\beta_{ji}|+|\tilde{\beta}_{ji}|)F_i|u_i(t-\sigma_{ji}(t))|, \quad j\in\mathscr{J}. \end{cases}$$

Let $u_i(t)=|u_i(t)|$, $v_j(t)=|v_j(t)|$, $r_i=a_i$, $p_{ij}=|a_{ij}|G_j$, $q_{ij}=(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j$, $\bar{r}_j=b_j$, $\bar{p}_{ji}=|b_{ji}|F_i$, $\bar{q}_{ji}=(|\beta_{ji}|+|\tilde{\beta}_{ji}|)F_i$ $(i\in\mathscr{I},j\in\mathscr{J})$, then we have

$$\begin{cases} D^+u_i(t)\leq -r_iu_i(t-\delta_i)+\sum_{j=1}^{m}p_{ij}v_j(t) \\ \qquad +\sum_{j=1}^{m}q_{ij}v_j(t-\tau_{ij}(t)), \quad i\in\mathscr{I}, \\ D^+v_j(t)\leq -\bar{r}_jv_j(t-\theta_j)+\sum_{i=1}^{n}\bar{p}_{ji}u_i(t) \\ \qquad +\sum_{i=1}^{n}\bar{q}_{ji}u_i(t-\sigma_{ji}(t)), \quad j\in\mathscr{J}, \end{cases} \quad (14)$$

and from (C1), there exist vectors $\xi=(\xi_1,\xi_2,\cdots,\xi_n)^T>0,\eta=(\eta_1,\eta_2,\cdots,\eta_m)^T>0$ and positive number $\lambda>0$ such that

$$\begin{cases} (\lambda-r_ie^{\lambda\delta_i})\xi_i+\sum_{j=1}^{m}\left[p_{ij}+q_{ij}e^{\lambda\tau_{ij}}\right]G_j\eta_j<0, \quad i\in\mathscr{I}, \\ (\lambda-\bar{r}_je^{\lambda\theta_j})\eta_j+\sum_{i=1}^{n}\left[\bar{p}_{ji}+\bar{q}_{ji}e^{\lambda\sigma_{ji}}\right]F_i\xi_i<0. \quad j\in\mathscr{J}. \end{cases} \quad (15)$$

Taking $\kappa=\frac{\|\bar{\phi}\|+\|\bar{\varphi}\|}{\min\limits_{1\leq i\leq n,1\leq j\leq m}\{\xi_i,\eta_j\}}$, it is easy to prove that

$$\begin{cases} u(t)\leq\kappa\xi e^{-\lambda t}, \quad -\tau\leq t\leq 0=t_0, \\ v(t)\leq\kappa\eta e^{-\lambda t}, \quad -\sigma\leq t\leq 0=t_0. \end{cases} \quad (16)$$

From Lemma 3, we obtain that

$$\begin{cases} u(t)\leq\kappa\xi e^{-\lambda t}, \quad t_0\leq t<t_1, \\ v(t)\leq\kappa\eta e^{-\lambda t}, \quad t_0\leq t<t_1. \end{cases} \quad (17)$$

Suppose that for $l\leq k$, the inequalities

$$\begin{cases} u(t)\leq\kappa\mu_0\mu_1\cdots\mu_{l-1}\xi e^{-\lambda t}, \quad t_{l-1}\leq t<t_l, \\ v(t)\leq\kappa\mu_0\mu_1\cdots\mu_{l-1}\eta e^{-\lambda t}, \quad t_{l-1}\leq t<t_l \end{cases} \quad (18)$$

hold, where $\mu_0=1$. When $l=k+1$, we note that

$$\begin{aligned} u(t_k) &= |\tilde{P}_k(u(t_k^-))|\leq\Gamma_k u(t_k^-) \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\Gamma_k\xi\lim_{t\to t_k^-}e^{-\lambda t} \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t_k}, \quad (19) \end{aligned}$$

and

$$\begin{aligned} v(t_k) &= |\tilde{Q}_k(v(t_k^-))|\leq\bar{\Gamma}_k v(t_k^-) \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\bar{\Gamma}_k\eta\lim_{t\to t_k^-}e^{-\lambda t} \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t_k}. \quad (20) \end{aligned}$$

From (19), (20) and $\mu_k\geq 1$, we have

$$\begin{cases} u(t)\leq\kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t}, \quad -\tau\leq t\leq t_k, \\ v(t)\leq\kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t}, \quad -\sigma\leq t\leq t_k. \end{cases} \quad (21)$$

Combining (14), (15), (21) and Lemma 3, we obtain that

$$\begin{cases} u(t)\leq\kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, \quad t_k\leq t<t_{k+1}, \\ v(t)\leq\kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, \quad t_k\leq t<t_{k+1}. \end{cases} \quad (22)$$

Applying the mathematical induction, we can obtain the following inequalities

$$\begin{cases} u(t)\leq\kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, \quad t\in[t_k,t_{k+1}), \quad k\in N, \\ v(t)\leq\kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, \quad t\in[t_k,t_{k+1}), \quad k\in N. \end{cases} \quad (23)$$

According to (C2), we have $\mu_k\leq e^{\mu(t_k-t_{k-1})}<e^{\lambda(t_k-t_{k-1})}$, it follows that

$$\begin{aligned} u(t) &\leq \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\xi e^{-\lambda t} \\ &= \kappa\xi e^{\mu t_{k-1}}e^{-\lambda t}\leq\kappa\xi e^{-(\lambda-\mu)t}, \quad t\in[t_{k-1},t_k), \end{aligned}$$

and

$$\begin{aligned} v(t) &\leq \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\eta e^{-\lambda t} \\ &= \kappa\xi e^{\mu t_{k-1}}e^{-\lambda t}\leq\kappa\eta e^{-(\lambda-\mu)t}, \quad t\in[t_{k-1},t_k) \end{aligned}$$

for $k\in N$. That is

$$\begin{cases} u(t)\leq\kappa\xi e^{-(\lambda-\mu)t}, \quad t\in[-\tau,t_k), \quad k\in N, \\ v(t)\leq\kappa\eta e^{-(\lambda-\mu)t}, \quad t\in[-\sigma,t_k), \quad k\in N, \end{cases} \quad (24)$$

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It follows that

$$\sum_{i=1}^{n} |x_i(t) - x_i^*| + \sum_{j=1}^{m} |y_j(t) - y_j^*|$$

$$= \sum_{i=1}^{n} u_i(t) + \sum_{j=1}^{m} v_j(t)$$

$$\leq \sum_{i=1}^{n} \kappa \xi_i e^{-(\lambda-\mu)t} + \sum_{j=1}^{m} \kappa \eta_j e^{-(\lambda-\mu)t}$$

$$= \frac{\sum_{i=1}^{n} \xi_i + \sum_{j=1}^{m} \eta_j}{\min_{1 \leq i \leq n, 1 \leq j \leq m} \{\xi_i, \eta_j\}} (\|\bar{\phi}\| + \|\bar{\varphi}\|) e^{-(\lambda-\mu)t}$$

$$= \frac{\sum_{i=1}^{n} \xi_i + \sum_{j=1}^{m} \eta_j}{\min_{1 \leq i \leq n, 1 \leq j \leq m} \{\xi_i, \eta_j\}} (\|\phi - x^*\| + \|\varphi - y^*\|) e^{-(\lambda-\mu)t}.$$

Let $M = \frac{\sum_{i=1}^{n} \xi_i + \sum_{j=1}^{m} \eta_j}{\min_{1 \leq i \leq n, 1 \leq j \leq m} \{\xi_i, \eta_j\}}$, then we have

$$\sum_{i=1}^{n} |x_i(t) - x_i^*| + \sum_{j=1}^{m} |y_j(t) - y_j^*|$$

$$\leq M \Big(\|\phi - x^*\| + \|\varphi - y^*\| \Big) e^{-(\lambda-\mu)t}.$$

The proof is completed.

Remark 1: In Theorem 1, the parameters μ_k and μ depend on the impulsive perturbation of system (1), and λ is actually an estimate of the exponential convergence rate of continuous system (3), which depends on delays and system parameters. Condition (C2) shows the fact that the exponential stability of system (1) still remains when the impulsive perturbation intensity $\mu \in [0, \lambda)$. Thus, Theorem 1 actually characterizes the robustness of stability for the impulsive BAM fuzzy cellular neural networks (1).

Remark 2: In order to obtain more precise estimate of the exponential convergence rate of system (1) (or system (3)), we suggest the following optimization problem

$$(OP) \quad \begin{cases} \max \lambda, \\ s.t. \text{ (C1) } holds. \end{cases}$$

Henceforth, $\hat{\lambda}$ denotes the optimal solution of this optimization problem.

Corollary 1: Under assumptions (H1) and (H2), if the following conditions hold:

(C1′) there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ such that

$$\begin{cases} 0 > (\lambda - a_i e^{\lambda \delta_i}) \xi_i \\ \quad + \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|) e^{\lambda \tau} \Big] G_j \eta_j, \\ 0 > (\lambda - b_j e^{\lambda \theta_j}) \eta_j \\ \quad + \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|) e^{\lambda \sigma} \Big] F_i \xi_i \end{cases}$$

for all $i \in \mathscr{I}, j \in \mathscr{J}$;

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \leq i \leq n, 1 \leq j \leq m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}, k \in N$;

then system (1) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\hat{\lambda} - \mu$.

Proof. Since $e^{\lambda \tau_{ij}} \leq e^{\lambda \tau}, e^{\lambda \sigma_{ji}} \leq e^{\lambda \sigma}$ for $i \in \mathscr{I}, j \in \mathscr{J}$, $\sum_{j=1}^{m} (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|) e^{\lambda \tau_{ij}} \leq \sum_{j=1}^{m} (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|) e^{\lambda \tau}, \sum_{i=1}^{n} (|\beta_{ji}| + |\tilde{\beta}_{ji}|) e^{\lambda \sigma_{ji}} \leq \sum_{i=1}^{n} (|\beta_{ji}| + |\tilde{\beta}_{ji}|) e^{\lambda \sigma}$. That is, condition (C1′) can guarantee (C1). This completes the proof.

If the leakage delays satisfy $\delta_i = 0$ and $\theta_j = 0$ for $i \in \mathscr{I}, j \in \mathscr{J}$, then system (1) may reduce to the following system [10]:

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{m} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{m} \tilde{a}_{ij} w_j \\ \quad + \bigwedge_{j=1}^{m} \alpha_{ij} g_j(y_j(t - \tau_{ij}(t))) \\ \quad + \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} g_j(y_j(t - \tau_{ij}(t))) \\ \quad + \bigwedge_{j=1}^{m} T_{ij} w_j + \bigvee_{j=1}^{m} H_{ij} w_j + I_i, \quad t \geq 0, \quad t \neq t_k, \\ x_i(t^+) = x_i(t^-) + P_{ik}(x_i(t^-)), \quad t = t_k, \quad k \in N, \\ \dot{y}_j(t) = -b_j y_j(t) + \sum_{i=1}^{n} b_{ji} f_i(x_i(t)) + \sum_{i=1}^{n} \tilde{b}_{ji} \tilde{w}_i \\ \quad + \bigwedge_{i=1}^{n} \beta_{ji} f_i(x_i(t - \sigma_{ji}(t))) \\ \quad + \bigvee_{i=1}^{n} \tilde{\beta}_{ji} f_i(x_i(t - \sigma_{ji}(t))) \\ \quad + \bigwedge_{i=1}^{n} \tilde{T}_{ji} \tilde{w}_i + \bigvee_{i=1}^{n} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \geq 0, \quad t \neq t_k, \\ y_j(t^+) = y_j(t^-) + Q_{ik}(y_j(t^-)), \quad t = t_k, \quad k \in N \end{cases}$$ (25)

for $i \in \mathscr{I}, j \in \mathscr{J}$.

For model (25), it is easy to obtain the following corollary.

Corollary 2: Under assumptions (H1) and (H2), if the following conditions hold:

(C1″) there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ such that

$$\begin{cases} 0 > (\lambda - a_i) \xi_i \\ \quad + \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|) e^{\lambda \tau_{ij}} \Big] G_j \eta_j, \\ 0 > (\lambda - b_j) \eta_j \\ \quad + \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|) e^{\lambda \sigma_{ji}} \Big] F_i \xi_i \end{cases}$$

for $i \in \mathscr{I}, j \in \mathscr{J}$;

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \leq i \leq n, 1 \leq j \leq m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}, k \in N$;

then system (25) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate approximatively equals $\hat{\lambda} - \mu$.

Remark 3: Corollary 2 is actually Theorem 1 in [10], so, Theorem 1 in this paper generalizes some existing results.

Remark 4: Note that Lemma 2 transforms the fuzzy AND ($\bigwedge$) and the fuzzy OR ($\bigvee$) operations into the SUM operation ($\sum$). So above results can be applied to the following classical

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impulsive BAM neural networks with time delay in the leakage term:

$$
\begin{cases}
\dot{x}_i(t) = -a_i x_i(t-\delta_i) + \sum_{j=1}^{m} a_{ij} g_j(y_j(t)) \\
\qquad + \sum_{j=1}^{m} \alpha_{ij} g_j(y_j(t-\tau_{ij}(t))) + I_i, \quad t \geq 0, \ t \neq t_k, \\
x_i(t^+) = x_i(t^-) + P_{ik}(x_i(t^-)), \quad t = t_k, \quad k \in N, \\
\dot{y}_j(t) = -b_j y_j(t-\theta_j) + \sum_{i=1}^{n} b_{ji} f_i(x_i(t)) \\
\qquad + \sum_{i=1}^{n} \beta_{ji} f_i(x_i(t-\sigma_{ji}(t))) + J_j, \quad t \geq 0, \ t \neq t_k, \\
y_j(t^+) = y_j(t^-) + Q_{ik}(y_j(t^-)), \quad t = t_k, \quad k \in N
\end{cases}
\tag{26}
$$

for $i \in \mathscr{I}$, $j \in \mathscr{J}$.

For model (26), it is easy to obtain the following result.

Theorem 2: Under assumptions (H1) and (H2), if the following conditions hold:

$(\bar{C}1)$ there exist vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ and positive number $\lambda > 0$ such that

$$
\begin{cases}
0 > (\lambda - a_i e^{\lambda \delta_i})\xi_i + \sum_{j=1}^{m} \Big[|a_{ij}| + |\alpha_{ij}| e^{\lambda \tau_{ij}} \Big] G_j \eta_j, \\
0 > (\lambda - b_j e^{\lambda \theta_j})\eta_j + \sum_{i=1}^{n} \Big[|b_{ji}| + |\beta_{ji}| e^{\lambda \sigma_{ji}} \Big] F_i \xi_i
\end{cases}
$$

for all $i \in \mathscr{I}$, $j \in \mathscr{J}$;

$(C2)$ $\mu = \sup\limits_{k \in N}\left\{ \dfrac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max\limits_{1 \leq i \leq n, 1 \leq j \leq m}\{1, \gamma_{ik}, \bar{\gamma}_{jk}\}, k \in N$;

then system (26) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\hat{\lambda} - \mu$.

IV. AN ILLUSTRATE EXAMPLE

Consider the following impulsive BAM fuzzy neural networks with time-varying delays:

$$
\begin{cases}
\dot{x}_i(t) = -a_i x_i(t-\delta_i) + \sum_{j=1}^{2} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{2} \tilde{a}_{ij} w_j \\
\qquad + \bigwedge_{j=1}^{2} \alpha_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\qquad + \bigvee_{j=1}^{2} \tilde{\alpha}_{ij} g_j(y_j(t-\tau_{ij}(t))) \\
\qquad + \bigwedge_{j=1}^{2} T_{ij} w_j + \bigvee_{j=1}^{2} H_{ij} w_j + I_i, \quad t \neq t_k, \\
x_i(t_k) = -e^{0.05k} x_i(t_k^-), \quad t = t_k, \quad k \in N, \\
\dot{y}_j(t) = -b_j y_j(t-\theta_j) + \sum_{i=1}^{2} b_{ji} f_i(x_i(t)) + \sum_{i=1}^{2} \tilde{b}_{ji} \tilde{w}_i \\
\qquad + \bigwedge_{i=1}^{2} \beta_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\qquad + \bigvee_{i=1}^{2} \tilde{\beta}_{ji} f_i(x_i(t-\sigma_{ji}(t))) \\
\qquad + \bigwedge_{i=1}^{2} \tilde{T}_{ji} \tilde{w}_i + \bigvee_{i=1}^{2} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \neq t_k, \\
y_j(t_k) = -e^{0.05k} y_j(t_k^-), \quad t = t_k, \quad k \in N
\end{cases}
\tag{27}
$$

for $i, j = 1, 2$, $t \geq 0$, $t_0 = 0$, $t_k = t_{k-1} + 0.5k$, $k \in N$, where

$\delta_1 = \delta_2 = 0.5, \quad \theta_1 = \theta_2 = 0.5,$
$a_1 = 3, \quad a_2 = 3, \quad a_{11} = \frac{4}{3}, \quad a_{12} = -\frac{1}{2},$
$a_{21} = \frac{1}{2}, \quad a_{22} = \frac{2}{3}, \quad \tilde{a}_{11} = 1, \quad \tilde{a}_{12} = -2,$
$\tilde{a}_{21} = -2, \quad \tilde{a}_{22} = 1, \quad I_1 = 0, \quad I_2 = -1,$
$\alpha_{11} = \frac{1}{3}, \quad \alpha_{12} = -\frac{1}{4}, \quad \alpha_{21} = \frac{1}{4}, \quad \alpha_{22} = \frac{2}{3},$
$\tilde{\alpha}_{11} = \frac{1}{3}, \quad \tilde{\alpha}_{12} = \frac{1}{4}, \quad \tilde{\alpha}_{21} = -\frac{1}{4}, \quad \tilde{\alpha}_{22} = \frac{2}{3},$
$T_{11} = 1, \quad T_{12} = 0, \quad T_{21} = 0, \quad T_{22} = 1,$
$H_{11} = 1, \quad H_{12} = 0, \quad H_{21} = 0, \quad H_{22} = 1,$
$w_1 = 1, \quad w_2 = 2, \quad \tau_{11}(t) = |\sin t|,$
$\tau_{12}(t) = 1.5 \sin^2 t, \ \tau_{21}(t) = |\cos 2t|, \ \tau_{22}(t) = |\cos t|,$
$g_j(s) = \frac{|s+1|-|s-1|}{2}, \quad j = 1, 2;$
$b_1 = 3, \quad b_2 = 3, \quad b_{11} = \frac{1}{3}, \quad b_{12} = -\frac{2}{3},$
$b_{21} = \frac{4}{3}, \quad b_{22} = \frac{1}{3}, \quad \tilde{b}_{11} = -1, \quad \tilde{b}_{12} = 3,$
$\tilde{b}_{21} = 2, \quad \tilde{b}_{22} = -2, \quad J_1 = -2, \quad J_2 = -1,$
$\beta_{11} = \frac{1}{3}, \quad \beta_{12} = -\frac{1}{6}, \quad \beta_{21} = \frac{1}{3}, \quad \beta_{22} = \frac{1}{3},$
$\tilde{\beta}_{11} = \frac{1}{3}, \quad \tilde{\beta}_{12} = \frac{1}{6}, \quad \tilde{\beta}_{21} = \frac{1}{3}, \quad \tilde{\beta}_{22} = \frac{1}{3},$
$\tilde{T}_{11} = 1, \quad \tilde{T}_{12} = 0, \quad \tilde{T}_{21} = 0, \quad \tilde{T}_{22} = 1,$
$\tilde{H}_{11} = 1, \quad \tilde{H}_{12} = 0, \quad \tilde{H}_{21} = 0, \quad \tilde{H}_{22} = 1,$
$\tilde{w}_1 = -1, \quad \tilde{w}_2 = 1, \quad \sigma_{11}(t) = 1 + 0.5\sin t,$
$\sigma_{12} = |\cos t|, \ \sigma_{21}(t) = |\sin 3t|, \ \sigma_{22}(t) = 1 + \cos t,$
$f_i(s) = \tanh(s), \quad i = 1, 2.$

It is easy to verify that assumptions (H1) and (H2) are satisfied, and it is easy to calculate that $F_1 = F_2 = 1$, $G_1 = G_2 = 1$, $(\tau_{11}) = 1$, $\tau_{12} = 1.5$, $\tau_{21} = 1$, $\tau_{22} = 1$, $\sigma_{11} = 1.5$, $\sigma_{12} = 1$, $\sigma_{21} = 1$, $\sigma_{22} = 1.5$, $\Gamma_k = \begin{pmatrix} e^{0.05k} & \\ & e^{0.05k} \end{pmatrix}$, $\bar{\Gamma}_k = \begin{pmatrix} e^{0.05k} & \\ & e^{0.05k} \end{pmatrix}$.

Solving the following optimization problem

$$
\begin{cases}
\max \lambda \\
0 > (\lambda - a_1 e^{\lambda \delta_1})\xi_1 + \Big(|a_{11}| + (|\alpha_{11}| + |\tilde{\alpha}_{11}|)e^{\lambda \tau_{11}} \Big) G_1 \eta_1 \\
\qquad + \Big(|a_{12}| + (|\alpha_{12}| + |\tilde{\alpha}_{12}|)e^{\lambda \tau_{12}} \Big) G_2 \eta_2, \\
0 > (\lambda - a_2 e^{\lambda \delta_2})\xi_1 + \Big(|a_{21}| + (|\alpha_{21}| + |\tilde{\alpha}_{21}|)e^{\lambda \tau_{21}} \Big) G_1 \eta_1 \\
\qquad + \Big(|a_{22}| + (|\alpha_{22}| + |\tilde{\alpha}_{22}|)e^{\lambda \tau_{22}} \Big) G_2 \eta_2, \\
0 > (\lambda - b_1 e^{\lambda \theta_1})\eta_1 + \Big(|b_{11}| + (|\beta_{11}| + |\tilde{\beta}_{11}|)e^{\lambda \sigma_{11}} \Big) F_1 \xi_1 \\
\qquad + \Big(|b_{12}| + (|\beta_{12}| + |\tilde{\beta}_{12}|)e^{\lambda \sigma_{12}} \Big) F_2 \xi_2, \\
0 > (\lambda - b_2 e^{\lambda \theta_2})\eta_2 + \Big(|b_{21}| + (|\beta_{21}| + |\tilde{\beta}_{21}|)e^{\lambda \sigma_{21}} \Big) F_1 \xi_1 \\
\qquad + \Big(|b_{22}| + (|\beta_{22}| + |\tilde{\beta}_{22}|)e^{\lambda \sigma_{22}} \Big) F_2 \xi_2, \\
\lambda > 0, \quad \xi = (\xi_1, \xi_2)^T > 0, \quad \eta = (\eta_1, \eta_2)^T > 0,
\end{cases}
$$

we get $\hat{\lambda} \approx 0.224428 > 0$, $\xi = (11372499, 1310864)^T > 0$ and $\eta = (9136837, 13174078)^T > 0$, so (C1) holds. From Theorem 1, we know that system (27) has a unique equilibrium point, this equilibrium point is $(0, 0, 0, 0)^T$. Also,

$$
\mu_k = \max_{1 \leq i \leq 2, 1 \leq j \leq 2}\{1, \gamma_{ik}, \bar{\gamma}_{jk}\} = e^{0.05k},
$$

$$
\mu = \sup_{k \in N} \frac{\ln \mu_k}{t_k - t_{k-1}} = \frac{0.05k}{0.5k} = 0.1 < 0.224428 = \hat{\lambda}.
$$

That is, (C2) holds. From Theorem 1, the unique equilibrium point $(0, 0, 0, 0)^T$ of system (27) is globally exponentially stable, and its exponential convergence rate is 0.124428.

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V. CONCLUSIONS

A class of impulsive BAM fuzzy cellular neural networks with time delays in the leakage terms has been formulated and investigated. Some new criteria on the existence, uniqueness and global exponential stability of the equilibrium point for the networks have been derived by using M-matrix theory and the impulsive delay differential inequality. Our stability criteria are delay-dependent and impulse-dependent. The neuronal output activation functions and the impulsive operators only need to satisfy (H1) and (H2), respectively; but need not be bounded and monotonically increasing. It is worthwhile to mention that our technical methods are practical, in the sense that all new stability conditions are stated in simple algebraic forms and provided a more precise estimate of the exponential convergence rate, so their verification and applications are straightforward and convenient. The effectiveness of our results has been demonstrated by the convenient numerical example.

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