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Periodic solutions in a delayed competitive system with the effect of toxic substances on time scales

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Abstract—In this paper, the existence of periodic solutions of a delayed competitive system with the effect of toxic substances is investigated by using the Gaines and Mawhin's continuation theorem of coincidence degree theory on time scales. New sufficient conditions are obtained for the existence of periodic solutions. The approach is unified to provide the existence of the desired solutions for the continuous differential equations and discrete difference equations. Moreover, The approach has been widely applied to study existence of periodic solutions in differential equations and difference equations.

Keywords—time Scales; competitive system; periodic solution; coincidence degree; topological degree

I. INTRODUCTION

After the work of Lotka[1] and Volterra[2], A great many realistic continuous and discrete predator-prey models have been proposed and investigated by many authors[3-7]. In 2009, Song and Chen[8] proposed a delay two-species competitive system in which two species have toxic inhibitory effects on each other:

$$\begin{cases} \frac{dx_1}{dt} = x_1(t)[K_1(t) - \alpha_1(t)x_1(t) - \beta_1(t)x_2(t) \\ \qquad - \gamma_1(t)x_1(t)x_2(t-\tau_1(t))], \\ \frac{dx_2}{dt} = x_2(t)[K_2(t) - \alpha_2(t)x_2(t) - \beta_2(t)x_1(t) \\ \qquad - \gamma_2(t)x_1(t-\tau_2(t))x_2(t)], \end{cases} \tag{1}$$

where $x_1(t), x_2(t)$ stand for the population densities of two competing species, respectively. $K_i(t)(i = 1,2)$ are the intrinsic growth rates of two competing species; $\alpha_i(t)(i = 1,2)$ denote the coefficients of interspecific competition; $K_i(t)/\alpha_i(t)(i = 1,2)$ are the environmental carrying capacities of two competing species; γ_1 and γ_2 stand for, respectively, the rates of toxic inhibition of the species x_1 by the species x_2 and vice versa. More details about the model, one can see [8]. By applying the theory of coincidence degree theory, Song and Chen[8] established the existence of positive periodic solution for system (1).

It is worth to point out that although there are numerous papers studying the existence of positive periodic solutions of differential or difference equations by using the coincidence degree theory in mathematical ecology, one often deal with these types of equations in a different way to prove the existence results. It is natural for us to think wether we can explore such an existence problem in an unified way. In order to unify continuous and discrete analysis, the theory of time scales(measure chain), which has recently received a great many attention, was introduced by Stefan Hilger in his PhD thesis in 1998. After that, people have done a lot of research about dynamic equations on time scales. Moreover, many results on the existence of periodic solutions of dynamic equations have been reported[9-13]. Motivated by papers[9-13], the principle object of this article is to explore the existence of periodic solutions of the following delayed competitive system with the effect of toxic substances on time scales:

$$\begin{cases} x_1^{\Delta}(t) = K_1(t) - \alpha_1(t)\exp(x_1(t)) - \beta_1(t)\exp(x_2(t)) \\ \qquad - \gamma_1(t)\exp(x_1(t))\exp(x_2(t-\tau_1(t))), \\ x_2^{\Delta}(t) = K_2(t) - \alpha_2(t)\exp(x_2(t)) - \beta_2(t)\exp(x_1(t)) \\ \qquad - \gamma_2(t)\exp(x_1(t-\tau_2(t)))\exp(x_2(t)). \end{cases} \tag{2}$$

To the best of our knowledge, it is the first time to deal with the existence problem of periodic solution for system (2) on time scales. In order to obtain the main results of our paper, throughout this paper, we assume
(H1) $K_i(t), \alpha_i(t), \beta_i(t), \gamma_i(t)$ and $\tau_i(t)$ are positive continuous ω-periodic functions, where $i = 1,2$.
(H2) sign$\{\bar{K_1}\bar{\alpha_2} - \bar{K_2}\bar{\beta_1}\}$ = sign$\{\bar{K_1}\bar{\beta_2} - \bar{K_2}\bar{\alpha_1}\}$ = sign $\{\bar{\alpha_1}\bar{\alpha_2} - \bar{\beta_2}\bar{\beta_2}\} \neq 0$.

The remainder of the paper is organized as follows: in Section 2, we present some preliminary definitions, notations and some basic knowledge for dynamic system on time scales. In Section 3, a easily verifiable sufficient condition for the existence of positive solutions of system (2) is obtained.

II. PRELIMINARY RESULTS ON TIME SCALES

In order to make an easy and convenient reading of this paper, we present some definitions and notations on time scales which can be found in the literatures[9,10].

Definition 2.1. *A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$, the real numbers. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.*

Definition 2.2. *The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0,\infty)$ are defined, respectively, by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$, $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$, $\mu(t) = \sigma(t) - t$ for $t \in \mathbb{T}$.*
If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

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Definition 2.3. *A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sides limits exists(finite) at left-dense points in $\mathbb{T}$. The set rd-continuous functions is shown by $C^1_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.*

Definition 2.4. *For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{R}$, we define $f^{\Delta}(t)$, the delta-derivative of f at t, to be the number(provided it exists) with the property that, given any $\varepsilon > 0$, there is a neighborhood U of t in $\mathbb{T}$ such that*

$$|[f(\sigma(t))-f(s)]-f^{\Delta}(t)[\sigma(t)-s]| \leq \varepsilon|\sigma(t)-s| \; for\ all\ s \in U.$$

Thus f is said to be delta-differentiable if its delta-derivative exists. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.5. *A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$, for all $t \in \mathbb{T}$. Then we write $\int_r^s f(t)\Delta t := F(s)-F(r)$ $for\ all\ s, t \in \mathbb{T}$.*

For the usual time scales $\mathbb{T} = \mathbb{R}$, rd-continuous coincides with the usual continuity in calculus. Moreover, every rd-continuous function on $\mathbb{T}$ has a delta-antiderivative[9]. For more information about the above definitions and their related concepts, one can see [9-13].

III. Existence of periodic solutions

For convenience and simplicity in the following discussion, we always use the notations below throughout the paper. Let $\mathbb{T}$ be ω-periodic, that is, $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$, $\kappa = \min\{\mathbb{R}^+ \cap \mathbb{T}\}$, $I_\omega = [\kappa, \kappa+\omega] \cap \mathbb{T}$, $g^l = \inf_{t\in\mathbb{T}} g(t)$, $g^u = \max_{t\in\mathbb{T}} g(t)$, $\bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} g(s)\Delta s$, where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, i.e., $g(t+\omega) = g(t)$ for all $t \in \mathbb{T}$.

In order to explore the existence of positive periodic solutions of (2) and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing from [14].

Let X, Y be normed vector spaces, $L : \mathrm{Dom}L \subset X \to Y$ is a linear mapping, $N : X \to Y$ is a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$, It follows that $L \mid \mathrm{Dom}L \cap \mathrm{Ker}P : (I-P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exist isomorphisms $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 3.1. ([14]Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be $L-$compact on $\bar{\Omega}$. Suppose*
(a) For each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;
(b) $QNx \neq 0$ for each $x \in KerL \bigcap \partial\Omega$, and $\deg\{JQN, \Omega \bigcap \partial KerL, 0\} \neq 0$;
Then the equation $Lx = Nx$ has at least one solution lying in $DomL \bigcap \bar{\Omega}$.

Lemma 3.2. [12] *Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R}$ is ω-periodic, then*

$$g(t) \leq g(t_1) + \int_\kappa^{\kappa+\omega} |g^{\Delta}(s)|\Delta s,$$

and

$$g(t) \leq g(t_2) - \int_\kappa^{\kappa+\omega} |g^{\Delta}(s)|\Delta s.$$

Lemma 3.3. *If condition (H2) is satisfied, then the following equation*

$$\begin{cases} \bar{K_1} - \bar{\alpha_1}\exp(x_1) - \bar{\beta_1}\exp(x_2) = 0, \\ \bar{K_2} - \bar{\alpha_2}\exp(x_2) - \bar{\beta_2}\exp(x_1) = 0 \end{cases} \tag{3}$$

has a unique solution $(x_1^*, x_2^*)^T$.

The proofs of Lemma 3.3 are trivial, so we omitted the details here.

Theorem 3.1. *Let S_1, S_3 be defined by (12) and (20), respectively. In addition to (H1) and (H2). Suppose that*

$$(H3)\ \bar{K_2} > \bar{\gamma_2}\exp(S_1), \bar{K_1} > \bar{\beta_1}\exp(S_3)$$

hold, then (2) has at least one ω-periodic solution.

Proof. Define

$$X = Z = \{(x_1, x_2)^T \in C(\mathbb{T}, \mathbb{R}^2) | x_i \in C_{rd},$$

$$x_i(t+\omega) = x_i(t), i = 1, 2\},$$

$$||(x_1, x_2)^T|| = \sum_{i=1}^{2} \max_{t\in I_\omega} |x_i(t)|, (x_1, x_2)^T \in X(\text{or } Z).$$

$$\mathrm{Dom}L = \{x = (x_1, x_2)^T \in X | x_i \in C_{rd}, i = 1, 2\}.$$

It is easy to see that X and Z are both Banach spaces if they are endowed with the above norm $||.||$.

For $(x_1, x_2)^T \in X$, we define

$$\begin{aligned} N\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = \begin{bmatrix} x_1^{\Delta} \\ x_2^{\Delta} \end{bmatrix}(t), \\ P\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) &= Q\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = \begin{bmatrix} \frac{1}{\omega}\int_\kappa^{\kappa+\omega} x_1(t)\Delta t \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} x_2(t)\Delta t \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} f_1(t) &= K_1(t) - \alpha_1(t)\exp(x_1(t)) - \beta_1(t)\exp(x_2(t)) \\ &\quad -\gamma_1(t)\exp(x_1(t))\exp(x_2(t-\tau_1(t))), \\ f_2(t) &= K_2(t) - \alpha_2(t)\exp(x_2(t)) - \beta_2(t)\exp(x_1(t)) \\ &\quad -\gamma_2(t)\exp(x_1(t-\tau_2(t)))\exp(x_2(t)). \end{aligned}$$

Then

$$\begin{aligned}\mathrm{Ker}L &= \{(x_1,x_2)^T\in X|(x_1(t),x_2(t))^T=(h_1,h_2)^T\in\mathbb{R}^2\\ &\quad \text{for } t\in\mathbb{T}\},\\ \mathrm{Im}L &= \{(x_1,x_2)^T\in X|\int_\kappa^{\kappa+\omega}x_1(t)\Delta t=0,\\ &\quad \int_\kappa^{\kappa+\omega}x_2(t)\Delta t=0,\ \text{for } t\in\mathbb{T}\}.\end{aligned}$$

Then dim Ker$L = 2 =$ codim ImL. Since ImL is closed in Z, L is a Fredholm mapping of index zero, it is easy to show that P and Q are continuous projections and Im$P =$ Ker$L$, Im$L =$ Ker$Q$ = Im$(I-Q)$. Clearly, QN and $K_p(I-Q)N$ are continuous. It can be shown that N is L-compact on $\bar{\Omega}$ for every open bounded set, $\Omega\subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $L(x_1,x_2)^T=\lambda N(x_1,x_2)^T,\lambda\in(0,1)$, we have

$$\begin{cases} x_1^\Delta(t)=\lambda f_1(t),\\ x_2^\Delta(t)=\lambda f_2(t).\end{cases}\tag{4}$$

Suppose that $x(t)=(x_1(t),x_2(t))^T\in X$ is an arbitrary solution of system (4) for a certain $\lambda\in(0,1)$, Integrating (4) over the set I_ω, we obtain

$$\begin{cases}\bar{K_1}\omega=\int_\kappa^{\kappa+\omega}\alpha_1(t)\exp(x_1(t))\Delta t\\ \qquad+\int_\kappa^{\kappa+\omega}\beta_1(t)\exp(x_2(t))\Delta t\\ \qquad+\int_\kappa^{\kappa+\omega}\gamma_1(t)\exp(x_1(t))\exp(x_2(t-\tau_1(t)))\Delta t,\\ \bar{K_2}\omega=\int_\kappa^{\kappa+\omega}\alpha_2(t)\exp(x_2(t))\Delta t\\ \qquad+\int_\kappa^{\kappa+\omega}\beta_2(t)\exp(x_1(t))\Delta t\\ \qquad+\int_\kappa^{\kappa+\omega}\gamma_2(t)\exp(x_1(t-\tau_2(t)))\exp(x_2(t))\Delta t.\end{cases}\tag{5}$$

Since $(x_1,x_2,)^T\in X$, there exists $\xi_i,\eta_i\in[\kappa,\kappa+\omega],i=1,2$ such that

$$x_i(\xi_i)=\min_{t\in[\kappa,\kappa+\omega]}\{x_i(t)\},x_i(\eta_i)=\max_{t\in[\kappa,\kappa+\omega]}\{x_i(t)\}.$$

It follows from (5) that

$$\int_\kappa^{\kappa+\omega}|x_1^\Delta(t)|\Delta t<2\bar{K_1}\omega.\tag{6}$$

$$\int_\kappa^{\kappa+\omega}|x_2^\Delta t|\Delta(t)<2\bar{K_2}\omega.\tag{7}$$

From the first equation of (5), it follows that

$$\bar{K_1}\omega>\bar{\alpha_1}\omega\exp(x_1(\xi_1)),\bar{K_1}\omega>\bar{\beta_1}\omega\exp(x_2(\xi_2)).$$

Then

$$x_1(\xi_1)<\ln\frac{\bar{K_1}}{\bar{\alpha_1}}:=m_1,x_2(\xi_2)<\ln\frac{\bar{K_1}}{\bar{\beta_1}}:=m_2.\tag{8}$$

In the sequel, we consider two cases.
(a) If $x_1(\eta_1)\geq x_2(\eta_2)$, then it follows from (5) that

$$\overline{(\alpha_1+\beta_1)}\exp(x_1(\eta_1))+\bar{\gamma_1}\exp(2x_1(\eta_1))\geq\bar{K_1},$$

which leads to

$$x_1(\eta_1)>\ln\left[\frac{-\overline{(\alpha_1+\beta_1)}+\sqrt{\overline{(\alpha_1+\beta_1)}^2+4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right]:=M_1.\tag{9}$$

Based on (6), (8) and (9), using the Lemma 3.2, we get

$$x_1(t)\leq x_1(\xi_1)+\int_\kappa^{\kappa+\omega}|x_1^\Delta(t)|\Delta t\leq m_1+2\bar{K_1}\omega=:B_1,\tag{10}$$

and

$$x_1(t)\geq x_1(\eta_1)-\int_\kappa^{\kappa+\omega}|x_1^\Delta(t)|\Delta t\geq M_1-2\bar{K_1}\omega=:B_2.\tag{11}$$

Thus

$$\max_{t\in I_\omega}|x_1(t)|\leq\max\{|B_1|,|B_2|\}:=S_1.\tag{12}$$

From the first equation of (5), it follows that

$$\bar{\alpha_2}\exp(x_2(\eta_2))+\bar{\beta_2}\exp(S_1)+\bar{\gamma_2}\exp(S_1)\exp(x_2(\eta_2))\geq\bar{K_2}.$$

Then

$$x_2(\eta_2)\geq\ln\left[\frac{\bar{K_2}-\bar{\gamma_2}\exp(S_1)}{\bar{\alpha_2}+\bar{\gamma_2}\exp(S_1)}\right]:=M_2.\tag{13}$$

From (7),(8) and (13) and using the Lemma 3.2, we obtain

$$x_2(t)\leq x_2(\xi_2)+\int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t\leq m_2+2\bar{K_2}\omega=:B_3\tag{14}$$

and

$$x_2(t)\geq x_2(\eta_2)-\int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t\geq M_2-2\bar{K_2}\omega=:B_4.\tag{15}$$

It follows from (14) and (15) that

$$\max_{t\in I_\omega}|x_2(t)|\leq\max\{|B_3|,|B_4|\}:=S_2.\tag{16}$$

(b) If $x_1(\eta_1)<x_2(\eta_2)$, then it follows from (11) that

$$\overline{(\alpha_1+\beta_1)}\exp(x_2(\eta_2))+\bar{\gamma_1}\exp(2x_2(\eta_2))\geq\bar{K_1},$$

which leads to

$$x_2(\eta_2)>\ln\left[\frac{-\overline{(\alpha_1+\beta_1)}+\sqrt{\overline{(\alpha_1+\beta_1)}^2+4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right]:=M_3.\tag{17}$$

From (7),(8) and (17) and using the Lemma 3.2, we obtain

$$x_2(t)\leq x_2(\xi_2)+\int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t\leq m_2+2\bar{K_2}\omega=:B_5,\tag{18}$$

and

$$x_2(t)\geq x_2(\eta_2)-\int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t\geq M_3-2\bar{K_2}\omega=:B_6.\tag{19}$$

It follows from (14) and (15) that

$$\max_{t\in I_\omega}|x_2(t)|\leq\max\{|B_5|,|B_6|\}:=S_3.\tag{20}$$

From the first equation of (5), it follows that

$$\bar{\alpha_1}\exp(x_1(\eta_1))+\bar{\beta_1}2\exp(S_3)+\bar{\gamma_1}\exp(S_3)\exp(x_1(\eta_1))\geq\bar{K_1}.$$

Then

$$x_1(\eta_1)\geq\ln\left[\frac{\bar{K_1}-\bar{\beta_1}\exp(S_3)}{\bar{K_1}+\bar{\gamma_1}\exp(S_3)}\right]:=M_4.\tag{21}$$

From (7),(8) and (21) and using the Lemma 3.2, we obtain

$$x_1(t) \le x_1(\xi_1) + \int_{\kappa}^{\kappa+\omega} |x_1^{\Delta}(t)|\Delta t \le m_1 + 2\bar{K}_1\omega =: B_7, \tag{22}$$

and

$$x_1(t) \ge x_1(\eta_1) - \int_{\kappa}^{\kappa+\omega} |x_1^{\Delta}(t)|\Delta t \ge M_4 - 2\bar{K}_1\omega =: B_8. \tag{23}$$

It follows from (22) and (23) that

$$\max_{t\in I_\omega} |x_1(t)| \le \max\{|B_7|, |B_8|\} := S_4. \tag{24}$$

Obviously, $S_i (i = 1, 2, 3, 4)$ are independent of the choice of $\lambda \in (0, 1)$. Take $M = \max\{S_1, S_4\} + \max\{S_2, S_3\} + S_0$, where S_0 is taken sufficiently large such that $S_0 \ge |m_1| + |m_2| + \max\{|M_1|, |M_4|\} + \max\{|M_2|, |M_3|\}$.

Now we define $\Omega := \{(x_1, x_2)^T \in X : ||x|| < M\}$. It is clear that Ω verifies the requirement (a) of Lemma 3.1. If $(x_1, x_2)^T \in \partial\Omega \bigcap KerL = \partial\Omega \bigcap \mathbb{R}^2$, then $(x_1, x_2)^T$ is a constant vector in $\mathbb{R}^2$ with $||(x_1, x_2)^T|| = |x_1| + |x_2| = M$. Then

$$QN\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \bar{K}_1 - \bar{\alpha_1}\exp(x_1) - \bar{\beta}_1\exp(x_2) \\ -\bar{\gamma_1}\exp(x_1)\exp(x_2) \\ \bar{K}_2 - \bar{\alpha_2}\exp(x_2) - \bar{\beta}_2\exp(x_1) \\ -\bar{\gamma_2}\exp(x_1)\exp(x_2) \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Now let us consider homotopic $\phi(x_1, x_2, \mu) = \mu QNx + (1 - \mu)Gx, \mu \in [0, 1], x = (x_1, x_2)^T$, where

$$Gx = \begin{bmatrix} \bar{K}_1 - \bar{\alpha_1}\exp(x_1) - \bar{\beta}_1\exp(x_2) \\ \bar{K}_2 - \bar{\alpha_2}\exp(x_2) - \bar{\beta}_2\exp(x_1) \end{bmatrix}.$$

Letting J be the identity mapping, according to Lemma 3.3 and condition (H2) and by direct calculation, we get

$$\begin{aligned} & \deg\left[JQN(x_1, x_2)^T; \partial\Omega\bigcap \mathrm{Ker}L; 0\right] \\ = \ & \deg\left[QN(x_1, x_2)^T; \partial\Omega\bigcap \mathrm{Ker}L; 0\right] \\ = \ & \deg\left[\phi(x_1, x_2, 1); \partial\Omega\bigcap \mathrm{Ker}L; 0\right] \\ = \ & \deg\left[\phi(x_1, x_2, 0); \partial\Omega\bigcap \mathrm{Ker}L; 0\right] \\ = \ & \mathrm{sign}\left[(\bar{\alpha_1}\bar{\alpha_2} - \bar{\beta_1}\bar{\beta_2})e^{(x_1^* + x_2^*)}\right] \ne 0. \end{aligned}$$

where $\deg(.,.,.,)$ is the Brower degree. Thus we have proved that Ω verifies all requirements of Lemma 3.1, then it follows that $Lx = Nx$ has at least one solution in $\mathrm{Dom}L \cap \overline{\Omega}$. The proof is complete.

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