Secret Communications Using Synchronized Sixth-Order Chuas's Circuits

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***Abstract*—**In this paper, we use Generalized Hamiltonian systems approach to synchronize a modified sixth-order Chua's circuit, which generates hyperchaotic dynamics. Synchronization is obtained between the master and slave dynamics with the slave being given by an observer. We apply this approach to transmit private information (analog and binary), while the encoding remains potentially secure.

***Keywords*—**hyperchaos synchronization; sixth-order Chua's circuit; observers; simulation; secure communication.

I. Introduction

Nowadays, information transmission plays a crucial role, where an ever-growing capacity for communication services are required. Two of the major requirements in communication systems are privacy and security.

Synchronization of chaotic systems, see e.g. [1]-[9] has been greatly motivated by the possibility of information encoding by using a chaotic carrier. Firstly explored with electronic circuits, see e.g. [10]-[13], where a small signal (the confidential information) was added to a chaotic voltage and transmitted to a receiver circuit.

If chaotic synchronization is achieved between transmitter and receiver circuits, then with the chaotic carrier itself, and subtraction of the synchronized signal from the transmitted signal (carrier plus information signal) results in the recovery of the information.

From synchrony of chaotic systems, there is opened the potential application of this principle to construct systems for encrypt that substitute the complicated conventional algorithms, in order to transmit private information of a safe way. Some techniques of chaotic communication have proposed for this end: additive masking [10], commutation between two chaotic attractors [14], parametric modulation [15], etc. Nevertheless, some posterior works have showed that these techniques have a degree of slightly reliable safety ([16], [17]). Recently some methods to increase the complexity of the dynamics of the chaotic systems have been proposed to do more difficult the information identification. For example, in [18] is used Hamiltonian forms and observer to synchronize two unidirectional coupled hyperchaotic Chua's circuits, in [19] with Chua's oscillator with time-delay, and in [20] with Chua's circuit generating multi-scroll attractors.

The aim of this paper is study the encoding, transmission, and decoding of confidential information, in particular, analog and binary messages. This objective is achieved by synchronizing the Chua's of sixth-order via Hamiltonian form and observer design. We show that the proposed approach is indeed suitable to transmit information encoding.

The rest of this paper is arranged as follows: in Section II a summary on synchronization of chaotic systems in Generalized Hamiltonian forms is given. In Section III, the hyperchaotic Chua's circuit (sixth-order) is described. In Section IV, the synchronization of two hyperchaotic Chua's circuits is shown. In Section V, stability conditions are presented. In the Section VI an application to encoding, transmission, and decoding is given. Finally, in Section VII some concluding remarks are given.

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II. Chaotic synchronization via generalized Hamiltonian Systems

Consider the following n-dimensional system

$$\dot{x} = f(x), \quad x(t) \in \Re^n \tag{1}$$

which represent a model exhibiting hiperchaotic behavior. Following the approach provided in [Sira-Ramirez y Cruz-Hernández, 2001], many physical systems described by Eq. (1) can be written in "Generalized Hamiltonian" canonical form,

$$\dot{x} = \mathsf{J}(x)\frac{\partial H}{\partial x} + \mathsf{S}(x)\frac{\partial H}{\partial x} + \mathsf{F}(x), \quad x \in \Re^n, \tag{2}$$

where $H(x)$ denotes a smooth energy function which is globally positive definite in $\Re^n$. The gradient vector of H, denoted by $\partial H/\partial x$, is asumed to exist everywhere. We use quadratic energy function $H(x) = (1/2)x^T \mathcal{M} x$ with M being a constant, symmetric positive definite matrix; and $\partial H/\partial x$ =M_. The matrices, J(_) and S(_) satisfy, for all $x \in \Re^n$, the properties: J(_) + J ^{T}(_) = 0 and S(_) = S^T(_). The vector field J(_) $\partial H/\partial x$ exhibits the conservative part of the system and it is also referred to as the workless part, or work-less forces of the system; and S(x) depicting the working or nonconservative part of the system. For certain systems, S(_) is negative definite or negative semidefinite. Thus, the vector field is considered as the dissipative part of the system. If, on the other hand, S(_) is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global, and local destabilizing part of the system. In the last case, we can always (although nonuniquely) descompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite, matrix R(_) and a symmetric positive semidefinite matrix N(_). Finally, F(_) represents a locally destabilizing vector field.

In the context of observer design, we consider a *special class* of Generalized Hamiltonian forms with linear output map $y(t)$, given by

$$\dot{x} = \mathcal{J}(y)\frac{\partial H}{\partial x} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial x} + \mathcal{F}(y), \quad x \in \Re^n, \qquad (3)$$
$$y = C\frac{\partial H}{\partial x}, \quad y \in \Re^m,$$

where S is a constant symmetric matrix, not necessarily of definite sign. The matrix I is a constant skew symmetric matrix. The matrix C is a constant matrix.

A nonlinear state observer for the Generalized Hamiltonian form (3) is given by

$$\dot{\hat{x}} = \mathcal{J}(y)\frac{\partial H}{\partial \hat{x}} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial \hat{x}} + \mathcal{F}(y) + K(y-\eta), \quad \hat{x} \in \Re^n, \qquad (4)$$
$$\eta = C\frac{\partial H}{\partial \hat{x}}, \quad \eta \in \Re^m$$

K is the observer gain.

The state estimation error, defined as $e(t) = x(t) - \hat{x}(t)$ and the output estimation error, defined as $e_y(t) = y(t) - \eta(t)$ are governed by

$$\dot{e} = \mathcal{J}(y)\frac{\partial H}{\partial e} + (\mathcal{I} + \mathcal{S} - KC)\frac{\partial H}{\partial e}, \quad e \in \Re^n, \qquad (5)$$
$$e_y = C\frac{\partial H}{\partial e}, \quad e_y \in \Re^m,$$

where the vector $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified energy function*, $\partial H(e)/\partial e = \partial H/\partial x - \partial H/\partial \hat{x} = \mathcal{M}\, e(x-\hat{x}) = \mathcal{M}\, e$. We set, when needed, $(\mathcal{I} + \mathcal{S}) = \mathcal{W}$

Definition 1 (Chaotic synchronization) ([22]) *The slave system (4) (nonlinear state observer) synchronizes with the chaotic master system in the special class of Generalized Hamiltonian form (3), if*

$$\lim_{t\to\infty} \left\| x(t) - \hat{x}(t) \right\| = 0 \qquad (6)$$

no matter which initial conditions $x(0)$ *and* $\hat{x}(0)$ *have.*

A necessary and sufficient condition for global asymptotic stability to zero of the estimation error (5) is given by the following theorem.

Theorem 1 *([6]) The state* $x(t)$ *of the nonlinear system (3) can be globally, exponentially, asymptotically estimated, by the state* $\hat{x}(t)$ *of the observer (4) if and only if, there exists a constant matrix K such that the symmetric matrix*

$$[\mathcal{W} - KC] + [\mathcal{W} - KC]^T = [\mathcal{S} - KC] + [\mathcal{S} - KC]^T$$
$$= 2\left[S - \tfrac{1}{2}(KC + C^T K^T)\right]$$

is negative definite.

III. Hyperchaotic Chua Circuit (sixth-order)

Consider the modified sixth-order Chua's circuit described by [Suykens et al., 1997]:

$$\begin{aligned}
\dot{x}_1 &= \alpha[x_2 - h(x_1)] \\
\dot{x}_2 &= x_1 - x_2 + x_3, \\
\dot{x}_3 &= -\beta x_2, \\
\dot{x}_4 &= \alpha[x_5 - h(x_4)] + K_p(x_4 - x_1) \\
\dot{x}_5 &= x_4 - x_5 + x_6, \\
\dot{x}_6 &= -\beta x_6
\end{aligned} \qquad (7)$$

with nonlinear function given by

$$h(x_1) = m_{2n-1}x_1 + \tfrac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x_1 + c_i| - |x_1 - c_i|) \qquad (8)$$

where n = 3, K_p = 0.01, α = 9, β = 14.28, m = [0.9/7, -3/7, 3.5/7, 2.7/7, 4/7, -2.4/7], and c = [1, 2.15, 3.6, 6.2, 9], the modified sixth-order Chua's circuit (7)-(8) exhibits hyperchaotic behavior, with two positive Lyapunov exponents. Figure 1 shows the attractors x_1 *vs* x_2, x_1 *vs* x_3, x_1 *vs* x_5, and x_1 *vs* x_6.

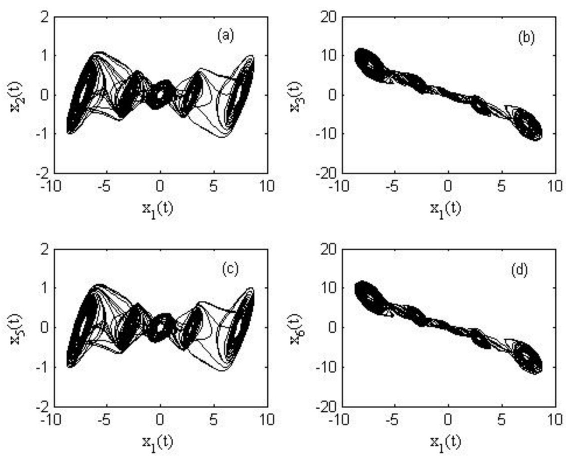

Fig.1. Different hyperchaotic attractors: (a) x_1 vs x_2, (b) x_1 vs x_3, (c) x_1 vs x_5 (c), and (d) x_1 vs x_6.

IV. SYNCHRONIZATION OF TWO HYPERCHAOTIC CHUA'S CIRCUITS

Taking as Hamiltonian energy function to

$$H(x) = \tfrac{1}{2}\left[\tfrac{1}{\alpha}x_1^2 \quad x_2^2 \quad \tfrac{1}{\beta}x_3^2 \quad \tfrac{1}{\alpha}x_4^2 \quad x_5^2 \quad \tfrac{1}{\beta}x_6^2\right], \tag{9}$$

The modified sixth-order Chua's circuit (7)-(8) in Hamiltonian form (master circuit according to Eq. (3)) is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \frac{K_P}{2} & 0 & 0\\ 0 & 0 & \beta & 0 & 0 & 0\\ 0 & -\beta & 0 & 0 & 0 & 0\\ -\frac{K_P}{2} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \beta\\ 0 & 0 & 0 & 0 & -\beta & 0\end{bmatrix}\frac{\partial H}{\partial x} + \tag{10}$$

$$\begin{bmatrix} 0 & \alpha & 0 & -\frac{K_P}{2} & 0 & 0\\ -\alpha & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -\frac{K_P\alpha}{2} & 0 & 0 & K_P & \alpha & 0\\ 0 & 0 & 0 & \alpha & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\frac{\partial H}{\partial x} + \begin{bmatrix}\alpha h(x_1)\\ 0\\ 0\\ \alpha h(x_4)\\ 0\\ 0\end{bmatrix}.$$

The destabilizing vector field calls for x_1 (t) and x_4 (t) to be used as the outputs of the master circuit (10). The matrices C, S, and I are given by

$$C = \begin{bmatrix}\alpha & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha & 0 & 0\end{bmatrix},$$

$$S = \begin{bmatrix} 0 & \alpha & 0 & -\frac{K_P}{2} & 0 & 0\\ -\alpha & -1 & 0 & 0 & 0 & 0\\ -\frac{K_P}{2} & 0 & 0 & 0 & 0 & 0\\ -\frac{K_P}{2} & 0 & 0 & K_P & \alpha & 0\\ 0 & 0 & 0 & \alpha & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ -\alpha & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -\beta\\ 0 & 0 & 0 & 0 & -\beta & 0\end{bmatrix}.$$

The nonlinear state observer (slave circuit) for (10) is designed as

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\\ \dot{\hat{x}}_3\\ \dot{\hat{x}}_4\\ \dot{\hat{x}}_5\\ \dot{\hat{x}}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \frac{K_P}{2} & 0 & 0\\ 0 & 0 & \beta & 0 & 0 & 0\\ 0 & -\beta & 0 & 0 & 0 & 0\\ -\frac{K_P}{2} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \beta\\ 0 & 0 & 0 & 0 & -\beta & 0\end{bmatrix}\frac{\partial H}{\partial x} + \tag{11}$$

$$\begin{bmatrix} 0 & \alpha & 0 & -\frac{K_P}{2} & 0 & 0\\ -\alpha & -1 & 0 & 0 & 0 & 0\\ -\frac{K_P}{2} & 0 & 0 & 0 & 0 & 0\\ -\frac{K_P\alpha}{2} & 0 & 0 & K_P & \alpha & 0\\ 0 & 0 & 0 & \alpha & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\frac{\partial H}{\partial x} + \begin{bmatrix}\alpha h(x_1)\\ 0\\ 0\\ \alpha h(x_4)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix} k_1 & 0\\ k_1 & 0\\ k_3 & 0\\ 0 & k_4\\ 0 & k_5\\ 0 & k_6\end{bmatrix} e_y,$$

Where the error is $e_y = \begin{bmatrix} x_1 - \hat{x}_1\\ x_4 - \hat{x}_4\end{bmatrix}$.

From (10) and (11) we have that the synchronization error dynamics is governed by

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\\ \dot{e}_5\\ \dot{e}_6\end{bmatrix} = \begin{bmatrix} 0 & \frac{K_2\alpha}{2} & \frac{K_3\alpha\beta}{2} & \frac{K_P\alpha}{2} & 0 & 0\\ -\frac{K_2\alpha}{2} & 0 & \beta & 0 & 0 & 0\\ -\frac{K_3\alpha\beta}{2} & -\beta & 0 & 0 & 0 & 0\\ -\frac{K_P\alpha}{2} & 0 & 0 & 0 & \frac{K_5\alpha}{2} & \frac{K_6\alpha\beta}{2}\\ 0 & 0 & 0 & -\frac{K_5\alpha}{2} & 0 & \beta\\ 0 & 0 & 0 & \frac{K_6\alpha}{2} & -\beta & 0\end{bmatrix}\frac{\partial H}{\partial e} +$$

$$\begin{bmatrix} K_1\alpha & \alpha\left(1-\frac{K_2}{2}\right) & \frac{K_3\alpha\beta}{2} & -\frac{K_P\alpha}{2} & 0 & 0\\ -\alpha\left(1-\frac{K_2}{2}\right) & -1 & 0 & 0 & 0 & 0\\ -\frac{K_3\alpha\beta}{2} & 0 & 0 & 0 & 0 & 0\\ -\frac{K_P\alpha}{2} & 0 & 0 & 0 & \alpha\left(1-\frac{K_5}{2}\right) & -\frac{K_6\alpha\beta}{2}\\ 0 & 0 & 0 & -\frac{K_5\alpha}{2} & -1 & 0\\ 0 & 0 & 0 & -\frac{K_6\alpha}{2} & 0 & 0\end{bmatrix}\frac{\partial H}{\partial e}. \tag{12}$$

One may now choose the observer gain $K = (k_1, k_2, k_3, k_4, k_5, k_6)^T$ in order to guarantee asymptotic exponential stability to zero of the synchronization error $e(t) = x(t) - \hat{x}(t)$, as will be shown in next section.

V. STABILITY CONDITIONS

In this section, we examine the stability of the synchronization error (12) between the master (10) in Hamiltonian form and slave (11) nonlinear state observer. Invoking to Theorem 1 and applying the Sylvester's criterion - which provides a test for negative definite of a matrix- thus, we have that the matrix 2[S-(1/2)(KC+C ^{T} K ^{T})] will be negative definite matrix, if we choose k_1, k_2, k_3, k_4, k_5, and k_6 such that the following condition are satisfied:

$$
\begin{array}{lcl}
-2K_1\alpha & \leq & 0 \\
4K_2 - {K_2}^2 & \leq & 4.44 \\
2{K_3}^2\alpha^2 & \leq & 0 \\
-4{K_3}^2 K_P\alpha - 4{K_3}^2 K_4\alpha^2 & \leq & 0 \\
4K_5 - {K_5}^2 & \leq & 4.00049 \\
K_6 & \equiv & 0
\end{array}
\qquad (6)
$$

For next numerical simulations, we have used the gains k_3, k_4 and k_6 are equal to zero, $k_1 = 1$, $k_2 = 2$, and $k_4 = 1.82$. The initial conditions: $x(0) = (0.1, 0.1, 0, 0.1, 0.1, 0)$ and $x(0) = (0, 0, 0, 0, 0, 0)$. Fig. 2 shows the synchronization between master (10) and slave (11) with sixth-order Chua's circuits.

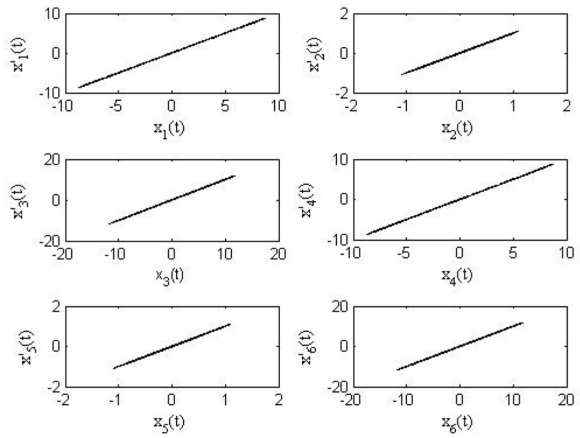

Fig. 2. Synchronization between hyperchaotic Chua's circuits (10) and (11).

VI. Application to encoding, transmission, and decoding

Synchronization of two six-order Chua's circuits allows us to design secret communication systems, where the confidential information is hidden into the transmitted hyperchaotic signal. In this paper, we present two cases, encoding, transmission, and decoding of analog and binary signals.

A. Trasmission of analog message

Two channels are used to synchronize master and slave modified Chua's circuits (10) and (11) via coupling hyperchaotic signals $x_1(t)$ and $x_4(t)$. Meanwhile, the other channel is used to transmit hidden message m(t) = 0.01 sin(t) (see Fig. 4), which is added to signal $x_3(t)$ of the transmitter *Tx,* the transmitted signal to receiver Rx is $s(t)$. At the receiver end, the recovered message m'(t) is given.

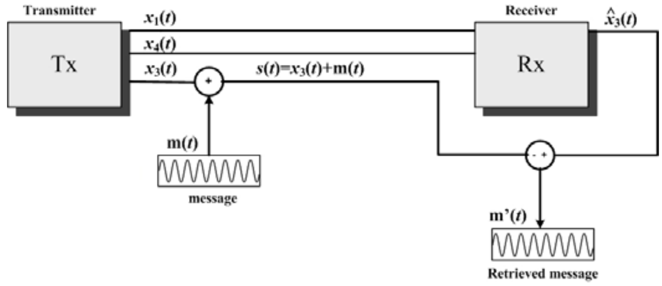

Fig. 4. Secret communication scheme to transmit analog messages.

The information signal m(t) is added to channel $x_3(t)$ of the transmitter *Tx* (Fig.5a). The transmitted signal s(t) (Fig. 5b)) is received the receiver end *Rx*. The signal $s(t)$ is subtracted to the output $\hat{x}_3(t)$ generates in *Rx*, then we recovery the information $m'(t)$ (see Figure 5c). Finally, Fig. 5d shows the error between original and recovery messages.

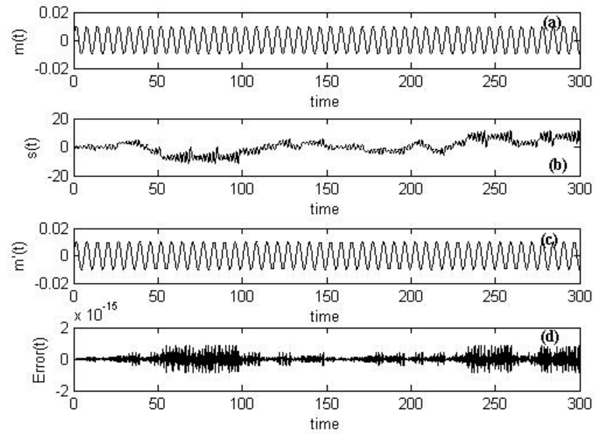

Fig. 5. (a) Confidential message, (b) transmitted hyperchaotic signal, (c) recovered message, and (d) error between original and recovered messages.

B. Transmission of binary message

The binary message $m(t)$ (see Fig. 6) is modulated by using the parameter α. By commutating from α = 9 (for encoding a "0" bit) to α' = 13 (for encoding a "1" bit). The state signals x_1(t) and x_4(t) are used to synchronize transmitter and receiver, moreover is possible to recovery the message using parametric commutation. This technique is based on if there exists synchrony or not.

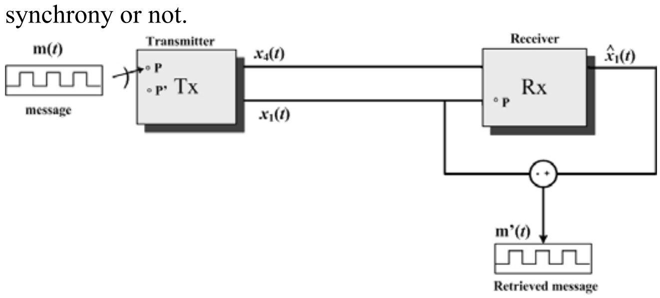

Fig. 6 Secret communication scheme to transmit binary messages.

Figure 7 shows the encoding, transmission, and decoding of binary signal. Fig. 7(a) shows the original binary message. Fig. 7(b) shows the hyperchaotic transmitted signal, and Fig. 7 (c) shows the recovery binary message.

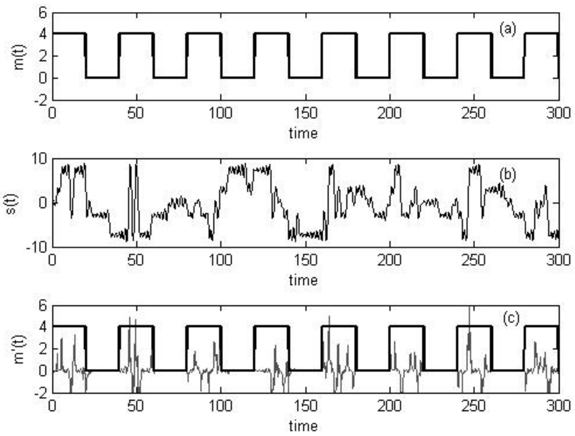

Fig. 7. (a) Confidential message, (b) transmitted hyperchaotic signal, and (c) recovered message.

VII. Conclusion

In this work, we have synchronized hyperchaotic dynamics in a modified sixth-order Chua's circuit through the Generalized Hamiltonian forms and observer approach. Based on this synchronization property, it is achieved secret transmission of confidential information. In addition, has been shown the quality of the recovered information, and at the same time, we have increased the encryption security by using extremely complex dynamics.

We overcame the low security objections against low dimensional chaos-based communication schemes, we confronted two problems: make the transmitted signal more complex, and reduce the redundancy in the transmitted signal. To achieve the first goal, it was necessary to use hyperchaos to generate very complex transmitted signals by using a modified sixth-order Chua's circuit.

To achieve the second goal, Generalized Hamiltonian forms and observer methodology for hyperchaos synchronization offers a very promising approach. The approach can be implemented on experimental setup, and shows great potential for actual communication systems in which the encoding is required to be secure.

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