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A New Version of Unscented Kalman Filter

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***Abstract*—**This paper presents a new algorithm which yields a nonlinear state estimator called iterated unscented Kalman filter. This state estimator makes use of both statistical and analytical linearization techniques in different parts of the filtering process. It outperforms the other three nonlinear state estimators: unscented Kalman filter (UKF), extended Kalman filter (EKF) and iterated extended Kalman filter (IEKF) when there is severe nonlinearity in system equation and less nonlinearity in measurement equation. The algorithm performance has been verified by illustrating some simulation results.

***Keywords*—**Extended Kalman Filter, Iterated EKF, Nonlinear state estimator, Unscented Kalman Filter.

I. Introduction

ACCURATE estimation of state variables of systems is important for fault detection and control applications. However, estimation in nonlinear systems is not easy to deal with. The optimal (Bayesian) solution to the problem requires propagation of description of full probability density function (pdf) [1]. This solution is general and includes factors such as multimodality, asymmetries, and discontinuities. However, since the form of pdf is not restricted, it cannot, in general, be represented using finite number of parameters. Therefore, any practical estimator must use an approximation of some kinds. Many different types of approximations have been developed; unfortunately, most are either computationally unmanageable or require special assumptions about the form of the process and observation models that cannot be satisfied in practice. For these and other reasons, the KF remains the most widely used estimation algorithm.

The most common application of the KF to nonlinear systems is in the form of extended KF (EKF) [2, 3]. EKF was first used by Wu et al. to find the 3D location. Exploiting the assumption that all transformations are quasi-linear, the EKF simply linearizes all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in the KF equations. In addition, EKF is very convenient and fast for real-time processing and quite straightforward to implement if a priori information of the measurement and process noise covariance matrices are available. Linearization in EKF introduces errors in the corresponding state estimations. The linearization is performed usually by Taylor expansion and the error is due to neglecting higher order terms. For reducing this error, using second-order EKF is proposed [2]. However, this method has a burden computation in problems with large measurement dimensions. Another approach to reduce this error is the measurement (and/or system) relinearization about a reference trajectory in the state space [4, 5]. This method is called Iterated Extended Kalman Filter (IEKF) and suits the cases in which the nonlinearity is only in the measurement equation [5]. Although the EKF maintains the elegant and computationally efficient recursive update form of the KF, it suffers a number of serious limitations: 1) Linearized transformations are only reliable if the error propagation can be well approximated by a linear function. If this condition does not hold, the linearized approximation can be extremely poor. At best, this undermines the performance of the filter. At worst, it causes its estimates to diverge altogether. However, determining the validity of this assumption is extremely difficult because it depends on the transformation, the current state estimate, and the magnitude of the covariance. 2) It only works well if the various random vectors are approximately Gaussian distributed. For complicated densities, the expectation covariance representation does not suffice. 3) Linearization can be applied only if the Jacobian matrix exists. However, this is not always the case. Some systems contain discontinuities, others have singularities and in others the states themselves are inherently discrete. 4) In some applications, it is too difficult to find the Jacobian matrix analytically. In these cases, numerical approximations of the Jacobian matrix are needed. However, this introduces other types of problems because now the influence of having approximations rather than the true values is involved. 5) The convergence of the standard EKF is very dependent on the choice of initial state estimate and tuning of filter parameters is crucial to success of the estimates. 6) In the EKF, the Kalman gain matrix depends on the data. With that, the stability of the filter is not assumed anymore. Moreover, it is very hard to analyze the behavior of the filter. 7) The EKF does not guarantee unbiased estimates. In addition, the calculated error covariance matrices do not necessarily represent the true error covariance. The analysis of these effects is also hard to deal with.

Recently, the Unscented Transform has been used in the EKF framework [6, 7] and the resulting filter is referred to as the Unscented Kalman Filter (UKF). It uses the intuition that it is easier to approximate a probability distribution function than to approximate an arbitrary nonlinear function or

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transformation. Thus the state distributions are approximated by the Gaussian density, which is represented by a set of deterministically chosen sample points. The idea of nonlinear filtering using the Gaussian representation of the posterior density via a set of deterministically chosen sample points has been proposed by several authors [8, 9]. The whole class of nonlinear filters (including UKF) is referred to as the linear regression Kalman filter, because they are all based on statistical linearization rather than analytical linearization (as the EKF). The statistical linearization is performed via the linear regression through the regression (sample) points. These sample points (Sigma points) completely capture the true mean and covariance of the Gaussian density. When propagated through nonlinear systems, they capture the true mean and covariance accurately to the second order (Taylor series expansion) of any nonlinearity [6].

UKF yields performance equivalent to the Kalman Filter for linear systems, yet generalizes elegantly to nonlinear systems without the linearization steps required for the EKF. The UKF consistently achieves a better level of accuracy than the EKF in systems with severe nonlinearities. Remarkably, the computational complexity of the UKF is in the same order as that of the EKF. Lefebvre et al. [10] have studied several modifications of Kalman Filters for nonlinear systems. They categorize all the different versions of Kalman Filters such as Central Difference Filter (CDF), Unscented Kalman Filter (UKF), and Divided Difference Filter (DDI) as linear regression Kalman Filters (LRKF) and compare them to EKF and IEKF [4, 11]. They mention that EKF and IEKF are generally better than LRKF for systems with no severe nonlinearities, yet they require a careful tuning. An interesting result of their study is that IEKF outperforms EKF, because it uses the measurements to linearize the measurement function, whereas in EKF and LRKF the measurement is not used [10].

In this paper, an iterated unscented Kalman filter is developed which is new in its kind. It achieves better performance than the other three estimators (EKF, IEKF and UKF) when there is severe nonlinearity in system equation and softer nonlinearity in measurement equation. In this method, we have used the statistical linearization for the prediction step and the analytical linearization for the update step of filtering. We demonstrate the performance benefits in two examples and argue that the ease of implementation and more accurate estimation features of the new filter recommend its use over the EKF and IEKF in almost all applications.

II. Backgrounds

The Unscented Kalman Filter (UKF) is a straightforward extension of the Unscented Transform (UT) to the recursive state estimation. The UT is a method for calculating the statistics of a random variable that undergoes a nonlinear transformation. The UKF uses the intuition that it is easier to approximate a probability distribution function rather than to approximate an arbitrary nonlinear function or transformation. Following this intuition, a set of points, called sigma points, are generated whose sample mean and sample covariance are: $\hat{\mathrm{x}}(k|k)$, the state estimation at kth iteration, and $\mathrm{P}(k|k)$, the state covariance matrix, respectively. The nonlinear function is applied to each of these points in turn to yield a transformed sample, and the predicted mean and covariance are calculated from the transformed sample.

Although this superficially resembles a Monte Carlo method, the samples are not drawn at random. Rather, the samples are deterministically chosen so that they capture specific information about the distribution. In general, this intuition can be applied to capture many kinds of information about many types of distributions. Here, we consider the special case of capturing the mean and covariance of an assumed Gaussian distribution.

The n-dimensional random variable $\mathrm{x}(k)$ with mean $\hat{\mathrm{x}}(k|k)$ and covariance matrix $\mathrm{P}(k|k)$ is approximated by $2n+1$ weighted samples or sigma points selected by the algorithm [6]:

$$\begin{aligned}
&\chi_0(k|k) = \hat{\mathrm{x}}(k|k) && i = 0\\
&W_0 = \frac{\kappa}{(n+\kappa)}\\
&\chi_i(k|k) = \hat{\mathrm{x}}(k|k) + \left(\sqrt{(n+\kappa)\mathrm{P}(k|k)}\right)_i && i = 1,\ldots,n\\
&W_i = \frac{1}{2(n+\kappa)}\\
&\chi_i(k|k) = \hat{\mathrm{x}}(k|k) - \left(\sqrt{(n+\kappa)\mathrm{P}(k|k)}\right)_i && i = n+1,\ldots,2n\\
&W_i = \frac{1}{2(n+\kappa)}
\end{aligned} \tag{1}$$

where κ is a scaling parameter such that $\kappa + n \neq 0$ and $\kappa \in \Re$, $\left(\sqrt{(n+\kappa)\mathrm{P}(k|k)}\right)_i$ is the ith row or column of the matrix square root of $(n+\kappa)\mathrm{P}(k|k)$, and W_i is the weight that is associated with the ith point. The weights are normalized; that is, satisfy $\Sigma_{i=0}^{2n} W_i = 1$.

III. The New Filter: Iterated Uncented Kalman Filter

The set of samples chosen by (1) have the same sample mean, covariance, and all higher odd-ordered central moments as the distribution of $\mathrm{x}(k)$. The matrix square root and κ affect the forth and higher order sample moments of the sigma points [7].

Consider the following nonlinear filtering problem defined by

$$\begin{aligned}
\mathrm{x}(k+1) &= f_k\left(\mathrm{x}(k), \mathrm{w}(k)\right)\\
\mathrm{z}(k) &= h_{k+1}(\mathrm{x}(k)) + \mathrm{v}(k)
\end{aligned} \tag{2}$$

where $\mathrm{x}(k)$ is the vector state of the system at time step k, $\mathrm{w}(k)$ is the vector noise process caused by disturbances and modeling errors, $\mathrm{z}(k)$ is the observation vector, and $\mathrm{v}(k)$ is the additive measurement noise. It is assumed that the noise vector $\mathrm{w}(k)$ and $\mathrm{v}(k)$ are zero mean and

$$E\left[\mathrm{w}(i)\mathrm{w}^T(j)\right] = \delta_{ij}\mathrm{C}_\mathrm{w}(i)$$
$$E\left[\mathrm{v}(i)\mathrm{v}^T(j)\right] = \delta_{ij}\mathrm{C}_\mathrm{v}(i) \qquad \forall i,j$$
$$E\left[\mathrm{v}(i)\mathrm{w}^T(j)\right] = 0$$

The assumption is that the posterior density at time k is Gaussian, i.e. $p(\mathrm{x}_k \left| \mathrm{z}_k \right.) = N(\mathrm{x}_k; \hat{\mathrm{x}}_k, \mathrm{P}_{k|k})$. The first step is to represent this density function by a set of $2n$ sample points $\chi_i(k|k)$ and their weights W_k^i, $i = 0,...,2n$. Each sigma point is instantiated through the process model to yield a set of transformed sample

$$\chi_{k+1|k}^i = f_k\left(\chi_i(k|k)\right) \tag{3}$$

The prediction step is then performed as follows:

$$\hat{\mathbf{x}}(k+1\left|k\right.) = \sum_{i=0}^{2n} W_k^i \chi_{k+1|k}^i \tag{4}$$

and the predicted covariance matrix is computed as

$$\mathrm{P}(k+1\left|k\right.) = \sum_{i=0}^{2n} W_k^i [\chi_{k+1|k}^i - \hat{\mathrm{x}}(k+1\left|k\right.)][\chi_{k+1|k}^i - \hat{\mathrm{x}}(k+1|k)]^T \tag{5}$$

The mean vector and covariance matrix are calculated using standard vector and matrix operations which means the algorithm is suitable for any choice of process model. The implementation is convenient because it is not required to evaluate the Jacobians, which are needed in an EKF. The method has a further advantage that it yields more accurate predictions than those determined through analytical linearization [6, 7]

The second step is an update step. The knowledge gained from observing the measurement $\mathrm{z}(k)$ is used to refine the density function. Using the Bayes theorem on the conditional density for memoryless sensory systems yields

$$\begin{aligned} p\left(\mathrm{x}(k)\left|\mathrm{Z}(k)\right.\right) &= p\left(\mathrm{x}(k)\left|\mathrm{Z}(k-1), \mathrm{z}(k)\right.\right) \\ &= \frac{1}{c} p\left(\mathrm{z}(k)\left|\mathrm{x}(k), \mathrm{Z}(k-1)\right.\right) p\left(\mathrm{x}(k)\left|\mathrm{Z}(k-1)\right.\right) \\ &= \frac{1}{c} p\left(\mathrm{z}(k)\left|\mathrm{x}(k)\right.\right) p\left(\mathrm{x}(k)\left|\mathrm{Z}(k-1)\right.\right) \end{aligned} \tag{6}$$

where $\mathrm{z}(k)$ is the set of received observations from $\mathrm{z}(1)$ up to $\mathrm{z}(k)$ and c is a normalization constant

$$c = \oint_{\mathrm{x}(k)\in X} p\left(\mathrm{z}(k)\left|\mathrm{x}(k)\right.\right) p\left(\mathrm{x}(k)\left|\mathrm{Z}(k-1)\right.\right) d\mathrm{x}(k)$$

The approximation is made such that both predicted state and the measurement noise are considered to be normally distributed. Thus, the posterior probability density $p(\mathrm{x}(k)\left|\mathrm{Z}(k)\right.)$, which is the product of two Gaussian, is also a Gaussian. Therefore, the MMSE estimate coincides with the MAP estimate and the task is now to find the maximum of $p(\mathrm{x}(k)\left|\mathrm{Z}(k)\right.)$. Equivalently, we can maximize its logarithm. After the elimination of the irrelevant constants and factors, it all boils down to minimization the following function

$$\begin{aligned} f(\mathrm{x}) = &\underbrace{\frac{1}{2}(\mathrm{x}-\bar{\mathrm{x}}_p)^T \mathrm{C}_p^{-1}(\mathrm{x}-\bar{\mathrm{x}}_p)}_{\text{due to } p(\mathrm{x}(k)\left|\mathrm{Z}(k-1)\right.)} \\ &+ \underbrace{\frac{1}{2}(\mathrm{z}-\mathrm{h}(\mathrm{x}))^T \mathrm{C}_\mathrm{v}^{-1}(\mathrm{z}-\mathrm{h}(\mathrm{x}))}_{\text{due to } p(\mathrm{z}(k)\left|\mathrm{x}(k)\right.)} \end{aligned} \tag{7}$$

For brevity, the following notations have been used:

$$\begin{aligned} \bar{\mathrm{x}}_p &= \hat{\mathrm{x}}(k+1\left|k\right.) \\ \mathrm{C}_p &= \mathrm{P}(k+1\left|k\right.) \\ \mathrm{z} &= \mathrm{z}(k) \\ \mathrm{C}_\mathrm{v} &= \mathrm{C}_\mathrm{v}(k) \end{aligned}$$

The strategy of finding minimum is to use Newton-Raphson iteration starting from $\bar{\mathrm{x}}_0 = \hat{\mathrm{x}}(k+1\left|k\right.)$. In the l-th iteration step, we have already an estimate $\bar{\mathrm{x}}_{l-1}$ obtained from the previous step. We expand $f(\mathrm{x})$ to a second order Taylor series approximation:

$$\begin{aligned} f(\mathbf{x}) \cong &f(\bar{\mathbf{x}}_{l-1}) + (\mathbf{x}-\bar{\mathbf{x}}_{l-1})^T \frac{\partial f(\bar{\mathbf{x}}_{l-1})}{\partial \mathbf{x}} \\ &+ \frac{1}{2}(\mathbf{x}-\bar{\mathbf{x}}_{l-1})^T \frac{\partial^2 f(\bar{\mathbf{x}}_{l-1})}{\partial \mathbf{x}^2}(\mathbf{x}-\bar{\mathbf{x}}_{l-1}) \end{aligned} \tag{8}$$

where $\partial f / \partial \mathrm{x}$ is the gradient and $\partial^2 / \partial \mathrm{x}^2$ is the Hessian of $f(\mathrm{x})$. The estimate $\bar{\mathrm{x}}_l$ is the minimum of the approximation. It is found by equating the gradient of the approximation to zero. Differentiation of (8) gives

$$\begin{aligned} &\frac{\partial f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}} + \frac{\partial^2 f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}^2}(\mathrm{x}-\bar{\mathrm{x}}_{l-1}) = 0 \\ &\Downarrow \\ &\bar{\mathrm{x}}_l = \bar{\mathrm{x}}_{l-1} - \left(\frac{\partial^2 f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}^2}\right)^{-1} \frac{\partial f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}} \end{aligned} \tag{9}$$

The Jacobian and Hessian of $f(\mathrm{x})$, in explicit form, are obtained from (7) as

$$\begin{aligned} \frac{\partial f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}} &= \mathrm{C}_p^{-1}(\bar{\mathrm{x}}_{l-1} - \mathrm{x}_p) - \mathrm{H}_l^T \mathrm{C}_\mathrm{v}^{-1}(\mathrm{z} - \mathrm{h}(\bar{\mathrm{x}}_{l-1})) \\ \frac{\partial^2 f(\bar{\mathrm{x}}_{l-1})}{\partial \mathrm{x}^2} &= \mathrm{C}_p^{-1} + \mathrm{H}_l^T \mathrm{C}_\mathrm{v}^{-1} \mathrm{H}_l \end{aligned} \tag{10}$$

where $H_l = H(\bar{x}_{l-1})$ is the Jacobian matrix of $h(x)$ evaluated at $\bar{x}_{l-1}$.

Substitution of (10) in (9) yields the following iteration scheme:

$$\bar{x}_l = \bar{x}_{l-1} - \left(C_p^{-1} + H_l^T C_v^{-1} H_l\right)^{-1} \left[C_p^{-1}(\bar{x}_{l-1} - x_p) - H_l^T C_v^{-1}(z - h(\bar{x}_{l-1}))\right] \quad (11)$$

The required number of further iterations depends on how fast $\bar{\mathbf{x}}_l(k)$ converges. It is common practice to fix the number of iterations to some practical number L. The final result is set to the last iteration, i,e, $\hat{\mathbf{x}}(k|k) = \bar{\mathbf{x}}_L$.

The factor $\left(C_p^{-1} + H_l^T C_v^{-1} H_l\right)^{-1}$ in (11) can be regarded as the error covariance matrix associated with $\hat{\mathbf{x}}(k|k)$

$$P(k|k) = \left(C_p^{-1} + H_l^T C_v^{-1} H_l\right)^{-1} \quad (12)$$

This insight gives another connection to the last term in (11) because, in fact, the term $P(k|k)H_l^T C_v^{-1}$ can be regarded as the Kalman gain matrix $\mathbf{K}_l$ during the l-th iteration.

IV. Simulation Results

This section applies the proposed new algorithm to two tracking problems and compares their performance against those of the UKF, IEKF, and EKF algorithms presented as what follows:

Example 1: Consider the state estimation problem that can be assumed as a target which is tracked through linear dynamic system equations and there is some nonlinearity in its measurement equation as follows

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1}}_{\mathbf{x}(k+1)} = \underbrace{\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_1 \end{bmatrix}_k}_{\mathbf{F}.\mathbf{x}(k)} + \underbrace{\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_k}_{\mathbf{w}(k)}$$

$$z(k) = x_1(k).x_2^2(k) + v(k) \quad (13)$$

where the sampling time is considered to be $T = 0.05\ s$. $w(k)$ is the Gaussian vector noise process due to disturbances and modeling errors with covariance matrix $C_W = 0.01I$, $z(k)$ is the received observation data at time instant k, and $v(k)$ is additive measurement Gaussian noise with variance 100. It is assumed that the noise vectors $w(k)$ and $v(k)$ are zero mean independent noises. From the linear state equations, one may assume that x_1 and x_2 can be interpreted as the target position and the target velocity respectively.

100 Monte Carlo runs are carried out and the average is represented by the root mean square error (RMSE) criterion as a measure of the performance in this simulation:

$$RMSE(k) = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\mathbf{x}_k - \hat{\mathbf{x}}_k^i)^2}\ , \qquad k = 1,2,...,10\ ;\ m = 100$$

where $\hat{x}_k^i$ denotes the state estimate vector of the ith Monte Carlo run for the kth sample. It is also obvious that the target is tracked for 100 data samples.

Figs. 1 and 2 show the corresponding state estimation error performances for four tracking methods, the new filter, UKF, IEKF, and EKF.

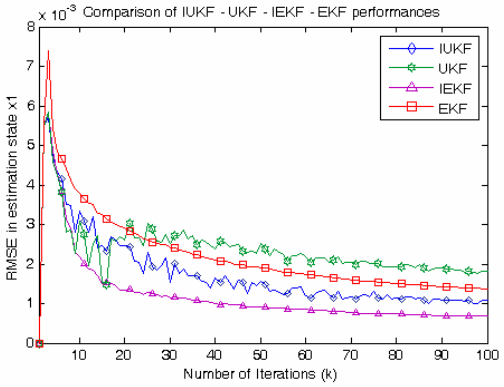

Fig. 1 Position estimation RMSE (example 1)

Because of the linearity behavior of the state dynamic equations, the error performances are all around 10^{-3} and actually, these values are very low in comparison with the ones that would be obtained from other problems with some nonlinearity in state dynamics. It is because the conditions presented here are near to the optimality conditions of the Kalman filter, i.e. linearity of system equations with existence of additive Gaussian noises.

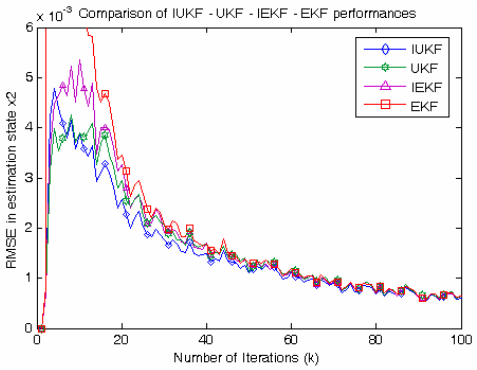

Fig. 2 Velocity estimation RMSE (example 1)

As it is seen from Fig. 1, although IEKF has gained the best error performance of all the other filters, the difference between the IEKF error performances and the one achieved from the new proposed filter is of negligible value and in brief, we may argue that they both have gained almost the same error performances. This argument seems to be true as it is illustrated in Fig. 2. Here, the new proposed filter has gained the best error performance of all the others if we are to

review the results in detail.

Example 2: In this example, we are to examine the performance of the new proposed filter in dealing with the existence of some nonlinearity in both state dynamic equations and measurement equations. Consider the following state estimation problem:

$$\begin{cases} x_1(k+1) = -k.x_2(k) + w_1(k) \\ x_2(k+1) = -\exp\left[10^{-3} x_2(k)\right] + w_2(k) \\ z(k) = x_1(k).x_2(k) + v(k) \end{cases} \quad (14)$$

where $w_1(k)$, $w_2(k)$ and $v(k)$ are zero mean independent additive Gaussian noise processes with

$$E\left[\mathrm{w}(i)\mathrm{w}^T(j)\right] = \delta_{ij}\mathrm{C}_\mathrm{w}(i) = \begin{bmatrix} 300 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$E\left[v(i)v^T(j)\right] = \delta_{ij}\mathrm{C}_\mathrm{v}(i) = 3 \qquad \forall i, j$$

Using the RMSE criterion as a measure of the performance, the superiority of the new iterated unscented Kalman filter is clearly represented in Fig. 3.

As it is illustrated in Figs. 3 and 4, IEKF and EKF have achieved the worse error performances compared to the two other filters and the difference in results is significant.

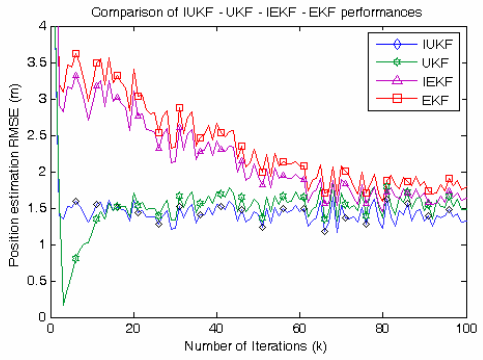

Fig. 3 RMSE in estimating x_1 (example 2)

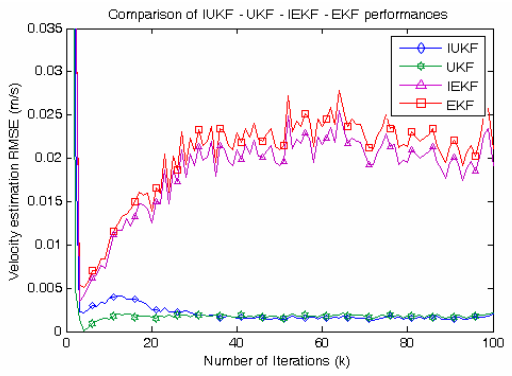

Fig. 4 RMSE in estimating x_2 (example 2)

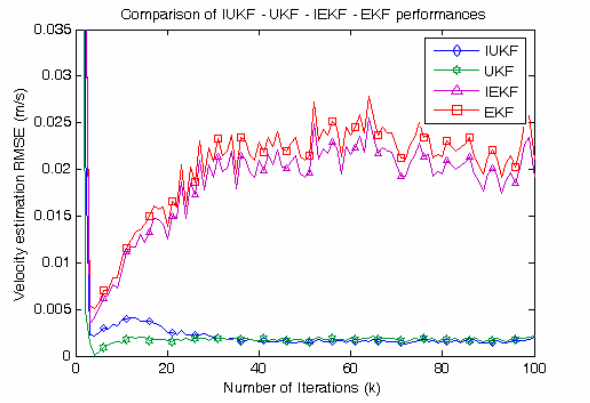

Fig. 4 RMSE in estimating x_2 (example 2)

In general, the more severe the nonlinearity of the state dynamics is, the more error occurs in states estimation obtained by IEKF and EKF. The new iterated unscented Kalman filter (IUKF), and the UKF have shown better performances in presence of the severe nonlinearity in state equations. Considering Fig. 4, it is obvious that although IUKF converges more slowly than UKF, it achieves better steady state error performance.

V. Conclusion

A new iterated unscented Kalman filter (IUKF) has been proposed. The algorithm yields a filter which is more accurate than the unscented Kalman filter (UKF), iterated extended Kalman filter (IEKF), and extended Kalman filter (EKF) when there is severe nonlinearity in state dynamics and softer nonlinearity in measurement equations. Its performance and effectiveness is demonstrated by two numerical examples. The ease of implementation and more accurate estimation features of the new filter recommend its use over the EKF and IEKF in almost all nonlinear applications.

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