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GPI observer-based tracking control and synchronization of chaotic systems

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Abstract—Based on general proportional integral (GPI) observers and sliding mode control technique, a robust control method is proposed for the master-slave synchronization of chaotic systems in the presence of parameter uncertainty and with partially measurable output signal. By using GPI observer, the master dynamics are reconstructed by the observations from a measurable output under the differential algebraic framework. Driven by the signals provided by GPI observer, a sliding mode control technique is used for the tracking control and synchronization of the master-slave dynamics. The convincing numerical results reveal the proposed method is effective, and successfully accommodate the system uncertainties, disturbances, and noisy corruptions.

Keywords—GPI observer, sliding mode control, master-slave synchronization, chaotic systems

I. Introduction

CHAOTIC system is of a kind of complex nonlinear dynamics, which are extremely sensitive to initial conditions, this means that although the mathematical description of chaotic system is deterministic, its behavior is still unpredictable[1]. This feature makes chaos very fantastic and valuable in certain scientific and engineering areas.

The problem of master-slave synchronization of chaotic systems was originated by the outstanding work of Pecora and Carroll[2] in 1990, thereafter, there have a great number of research activities (see [2] - [9] and references therein) motivated by possible applications of chaotic system in various scientific realms, ranging from physics, secure communication, control theory, signal processing, to mention a few. Several methods to chaos control and synchronization have been proposed, such as LMI based robust control[3], adaptive control[5], [7], sliding mode control[6], H_∞ control[8] and improving PD controller in[9], etc.. Most of these methods are based on the state feedback control, however, in many practical realms the states are not completely measurable, further more, the measurement noise is unavoidable, hence observer based output-feedback control is more practical and necessary for the engineering implementations. In literature [4], an improved Luenberger-type observer with the persistency of excitation was proposed for the states reconstruction, on the base of which, an adaptive control scheme was proposed for a class of chaotic systems. However this method required prior knowledge of the system, which is hardly obtained while the presence of system uncertainties, disturbances and parametric perturbations. Hence we turn to another ad hoc technique, model-free control or data-driven control, emerging in recent years. Model-free control was benefited from the differential algebraic framework, which was popular in 1990s' since of the work of Fliess[10], recently, there have a great number of studies on the model-free control(see [11] and the references therein). The main idea of model-free control is to locally model system online via the estimation of the input-output data, then a simple controller for the local model can be designed well, such as so-called i-PID method.

This paper focuses on establishing output feedback controller for the tracking control and synchronization of the chaotic systems. An anther improved Luenberge-type observer - GPI observer[12] was used to reconstruct the states and to estimate the high order derivatives or the generalized disturbance. Driven by the GPI observer the sliding mode control technique is equipped in our design.

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II. Problem formulation

In this paper, we consider a class of chaotic systems as the following form

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad 1 \le i \le n-1 \\ \dot{x}_n = f(x,t) + \Delta f(x,t) + d(t) + bu(t) + \Delta bu \\ y = x_1 + w(t) = Cx + w(t), \ C = [1, 0, ..., 0] \end{cases} \tag{1}$$

where $x = [x_1, x_2, ..., x_n]^T = \left[x_1, \dot{x}_1, ..., x_1^{(n-1)}\right]^T \in R^n$ is the state vector, the notation $(\cdot)^{(n)}$ represents the n^{th} order derivative of $(\cdot)$, $f := R \times R^n \to R$ is the well known function of x and t, $\Delta f(x,t)$ is the uncertain part of chaotic dynamics, $u(t) \in R$ is the control input, constant number b and varying $\Delta b(t)$ are the input coefficients, $d(t) \in R$ and $w(t)$, respectively, are the external disturbance and the measurement noise.

The goal of this paper is to design an output-feedback controller for the system above such that $y(t)$ perfectly track a given reference signal $y_r(t)$ in the presence of system uncertainties, external disturbances and measurement noise. If $y_r(t)$ is generated by another chaotic system, i.e., master system, then the slave system (system (1)) is driven to achieve synchronization with the master system. Here the master system and the slave system are not necessarily with identical structures.

In general, we made some assumptions as follows.

Assumption 1. $f(x,t)$ and $\Delta f(x,t)$ are bounded for all $t > 0$ with known upper bounds $f_M > 0$ and $\Delta f_M > 0$, respectively, meanwhile, the input coefficient perturbation $\Delta b(t)$

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is bounded for all $t > 0$ and its upper bound denoted by b_M is known .

Assumption 2. $d(t)$ and $w(t)$ are bandlimited and bounded for all $t > 0$ with known upper bounds $d_M > 0$ and w_M, respectively.

Assumption 3. The control input $u(t)$ belongs to the extended L_2 space, i.e., any truncation of $u(t)$ on a finite time interval is essentially bounded.

Assumption 4. The reference signal $y_r(t)$ is bounded and its finite derivatives exist.

III. Main results

A. GPI observer

Rewrite (1) as

$$\begin{cases} x_1^{(n)} = \varphi(t) + bu \\ y = x_1 + w(t) \end{cases} \tag{2}$$

where we define the time function $\varphi :\mapsto \varphi(t, x(t), d(t))$, i.e., denote by $\varphi(t) = f(x,t) + \Delta f(x,t) + \Delta bu + d(t)$. Two further assumptions are made based on the assumptions 1-4.

Assumption 5. The values of $\varphi(t)$ are unknown while they are known to be uniformly, absolutely, bounded for every feasible solution of (1).

Assumption 6. For any positive integer p, there exists a small positive, real number φ_M such that $\xi^{(p)}(t)$ is uniformly absolutely bounded, i.e.,

$$\sup_{t\geq 0} \left|\varphi^{(p)}(t)\right| < \varphi_M, \forall p \in Z^+ < \infty \tag{3}$$

Based on the assumptions 4 and 5, the classical GPI observer design for (2) can be found in [13]. However the performance of the classical GPI observer would be undermined dramatically by noisy corruption, hence we borrow the idea of GPI$_+$ from *D. L. Martinez-Vazquez et al.* [14] in this study. Since the integration can annihilate random noisy effects, we construct a new virtual state $x_0(t) = \int_0^t y(\tau)d(\tau)$ henceforth an augmented system

$$\begin{aligned} \dot{x}_0 &= x_1 + w \\ x_1^{(n)} &= \varphi + bu \\ y_0 &= x_0 \\ y &= x_1 + w \end{aligned} \tag{4}$$

The full order GPI observer for the augmented system above is written as

$$\begin{aligned} \dot{\hat{x}}_0 &= \hat{x}_1 + \rho_{p+n}(y_0 - \hat{x}_0) \\ \dot{\hat{x}}_1 &= \hat{x}_2 + \rho_{p+n-1}(y_0 - \hat{x}_0) \\ &\vdots \\ \dot{\hat{x}}_n &= z_1 + bu + \rho_p(y_0 - \hat{x}_0) \\ \dot{z}_1 &= z_2 + \rho_{p-1}(y_0 - \hat{x}_0) \\ &\vdots \\ \dot{z}_{p-1} &= z_p + \rho_1(y_0 - \hat{x}_0) \\ \dot{z}_p &= \rho_0(y_0 - \hat{x}_0) \end{aligned} \tag{5}$$

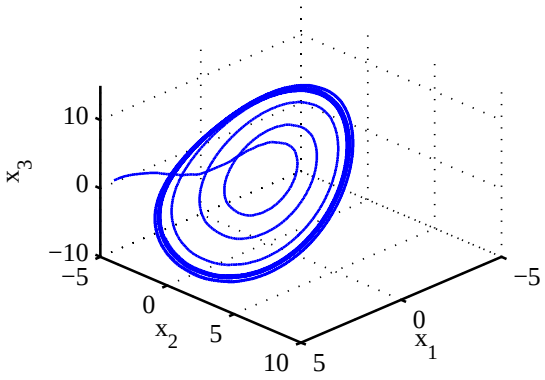


Fig. 1. The chaotic trajectory of Genesio system when $a = 1.5, b = 2.8, c = 6$.

Let the estimation error $\epsilon = y_0 - \hat{x}_0 = x_0 - \hat{x}_0$. Using (5) and (4) we henceforth have

$$\begin{aligned} \epsilon^{(p+n+1)} + \rho_{p+n}\epsilon^{(p+n)} + \ldots + \rho_1\dot{\epsilon} + \rho_0\epsilon \\ = \varphi^{(p)}(t) - w^{(p+n)}(t) \end{aligned} \tag{6}$$

which is a perturbed $(p+n)^{th}$ order linear time invariant system. Given appropriate coefficients vector $\mathrm{P} = [\rho_{\mathrm{p+n}}, ..., \rho_0]$ such that the corresponding characteristic polynomial

$$P(s) = s^{p+n+1} + \rho_{p+n}s^{p+n} + ... + \rho_0$$

is stable, i.e., the roots of $P(s) = 0$ locate deep in the left complex plain, the observer error ϵ asymptotically converges to the region defined by $\frac{\varphi_M + \max|w^{(p+n)}|}{\|\mathrm{P}\|}$, which can be modified by tuning the observer parameters vector P. The details of the proof can be found in [13] and [14].

B. Sliding mode control

In accordance with the GPI observer proposed in the last subsection, the states of (2) are reconstructed and the generalized disturbance $\varphi(t)$ is estimated from the output signal in noisy environment. Let $e^{(i)} = x_1^{(i)} - y_r^{(i)}$, then we can design a sliding manifold as follows

$$s = e^{(n-1)} + k_{n-2}e^{(n-2)} + ... + k_1\dot{e} + k_0 e \tag{7}$$

Chose appropriate parameter vector $\mathcal{K} = [k_0, k_1, ..., k_{n-2}, 1]^T$ such that the roots of the polynomial $P_s = s^{n-1} - k_{n-2}s^{n-2} + ... + k_1 s + k_0$ lie in the left complex plain. In ideal situation, the control law for (4) can be designed as

$$u = \frac{1}{b}[-\varphi(t) + y_r^{(n)} - m\dot{s} - ls + e^{(n)}] \quad (m, l > 0) \tag{8}$$

Differentiate s once respect to time, by using (4) and (8) we therefore have

$$m\dot{s} + ls = 0 \tag{9}$$

Since of $m, l > 0$, there have $\lim_{t\to\infty} s(t) = 0$. Because P_s is stable, then $\lim_{t\to\infty} e(t) = 0$. However the terms of $\varphi(t)$ and e are unknown precisely since of the presence of disturbances, uncertainties, and partially measurable states,

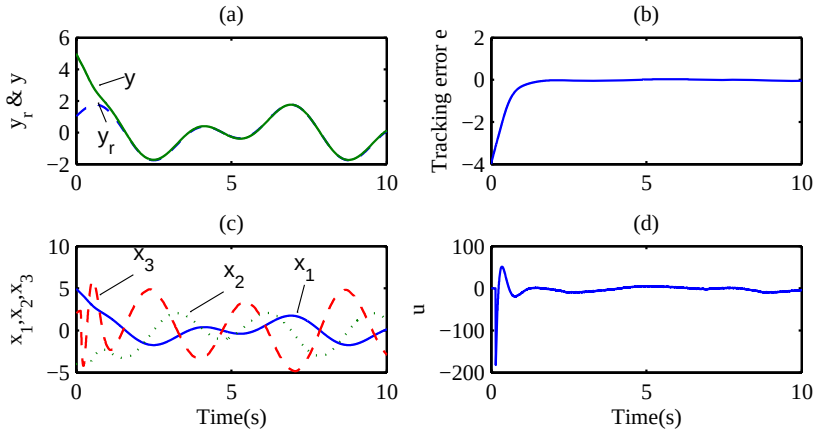


Fig. 2. The time response of system output, states , tracking error e and the control input u for tracking the desired output $y_r = sin(2t) + cos(t)$.

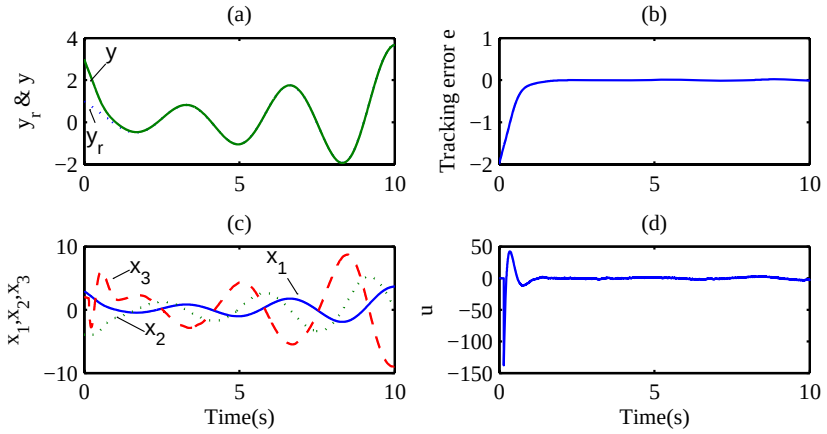


Fig. 3. The time response of system output, states , tracking error e and the control input u for synchronization control.

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hence the controller law (8) is impossible to be implemented. Fortunately, we can obtained $\hat{\varphi}(t)$ and $\hat{x}_1^{(i)}$ can be obtained by the GPI observer (5). Denote $\hat{e}^{(i)} = \hat{x}_1^{(i)} - y_r^{(i)} = x_1^{(i)} - y_r^{(i)} + \epsilon^{(i+1)}, (i = 0, ..., n)$, then rewrite the sliding manifold as

$$\begin{aligned} \hat{s} &= \hat{e}^{(n-1)} + k_{n-2}\hat{e}^{(n-2)} + ... + k_1\dot{\hat{e}} + k_0\hat{e} \\ &= s + \epsilon^n + k_{n-2} + \epsilon^{(n-1)} + ... + k_1\epsilon^{(2)} + k_0\epsilon^{(1)} \end{aligned} \quad (10)$$

Denote $\omega = \epsilon^n + k_{n-2} + \epsilon^{(n-1)} + ... + k_1\epsilon^{(2)} + k_0\epsilon^{(1)}$. Construct the new control law

$$u = \frac{1}{b}[-\hat{\varphi}(t) + y_r^{(n)} - m\dot{\hat{s}} - l\hat{s} + \hat{e}^{(n)}] \quad (l > 0) \quad (11)$$

which rendering a closed-loop system

$$\begin{aligned} & e^{(n)} = \varphi(t) - \hat{\varphi}(t) - m\dot{\hat{s}} - l\hat{s} + \hat{e}^{(n)} \\ & \rightarrow m\dot{\hat{s}} + l\hat{s} = -\epsilon^{(n+1)} - \epsilon^{(n)} \\ & \rightarrow m\dot{s} + ls = -\dot{\omega} - \omega - \epsilon^{(n+1)} - \epsilon^{(n)} \end{aligned} \quad (12)$$

The estimation error ϵ^i is bounded and small, hence the righthand of the last equation of (12 denoted as $\Omega = m\dot{\omega} + l\omega$ is bounded, then the sliding manifold s will converge into a bounded region in finite time, i.e., the tracking error will be confined into a small region, which can be minimized by well choice of the observer parameters.

IV. Numerical Simulation

In order to validate and demonstrate the effectiveness of the proposed control method, two illustrative examples are presented. All simulating experiments are conducted with Matlab environment, and the 4^{th} order Runge-Kutta numerical algorithm with a fixed time step 0.001 second is used to solve the differential equations. We consider the Genesio chaos model as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + x_1^2 \\ y = x_1 \end{cases} \quad (13)$$

where x_1, x_2, x_3 are state variables, and a, b and c are the positive real constants satisfying $ab < c$. When $a = 1.5, b = 2.8, c = 6$ the system (10) is chaotic. The Fig.1 presents the chaotic trajectory of Genesio system with the initial state vector $x = [5, -4, 2]^T$.

Assume the system (15) is with the presence of bounded uncertainty term

$$\Delta f(x,t) = 0.2\sin(\pi x_1)\sin(2\pi x_2)$$

, $\Delta b(t) = 0.1\sin(0.2t)$, and bounded disturbance term $d(t) = 0.3Sig(\cos(3t))$ ($Sig(\cdot)$ is a sign function), and bounded measurement noise $w(t) : \mathcal{N}(0, 0.002)$, therefore an uncertain Genesio system as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + x_1^2 + \Delta f(x,t) + d(t) \\ \qquad + bu(t) + \Delta b(t)u \\ y = x_1 + w(t) \end{cases} \quad (14)$$

In order to make the system (14) tracking a given reference signal y_r, we choose the sliding manifold as

$$\hat{s} = \hat{e}^{(1)} + 5\hat{e}$$

design the controller based on (11) as follows

$$u = \frac{1}{b}[-\hat{\varphi}(t) + y_r^{(2)} - 10\dot{\hat{s}} - 20\hat{s} + \hat{e}^{(n)}] \quad (15)$$

with $b = 1$, where estimation signals are provided by the GPI observer (5), whose parameters are set as $P_o(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2)^4$ with $\zeta = 1$ and $\omega_n = 40$.

The simulation results shown in Fig.2 are under the proposed controller (14) for tracking a given reference signal $y_r(t) = \sin(2t)$. The tracking errors (Fig.2 (b)) are rapidly converge to the small region around zero.

Another application of the proposed method is to the synchronization control of master-slave system. We take the unperturbed system (15 as the master system and the perturbed system (14) as the slave system. The initial states are set as $x_m = [1, -1, 0]^T$ and $x_s = [3, -4, 2]^T$. The results of synchronization control under the identical controller are presented in Fig.3 and Fig.4.

V. Conclusions

In this paper we present a novel method for the tracking control and synchronization of a class of chaotic systems. Based on the GPI observer, the states and the generalized disturbance are estimated from the output signal in noisy environment, then a classical sliding mode control technique is used to form the feedback controller. The proposed method treats the system uncertainties, external disturbances, and parametric perturbation as a generalized disturbance, which are estimated by GPI observer. The additional measurement noise is minimized by the integral effects of GPI observer. Finally, the convincing simulation results validate the effectiveness of the proposed method, and demonstrated the proposed method successfully dealing with the system uncertainties, external disturbances, parametric perturbation, and the measurement noisy.

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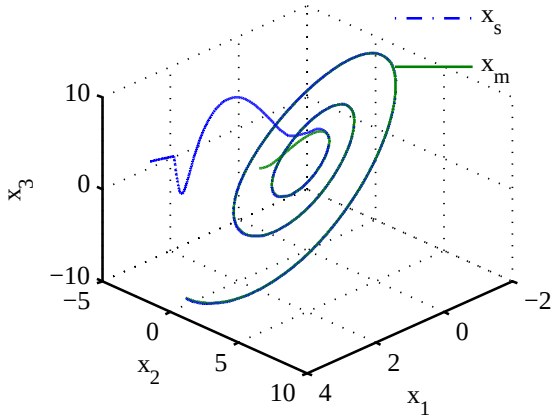


Fig. 4. The trajectories of master and slave chaotic system.

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