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Efficient Realization of an ADFE with a New Adaptive Algorithm

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Abstract— Decision feedback equalizers are commonly employed to reduce the error caused by intersymbol interference. Here, an adaptive decision feedback equalizer is presented with a new adaptation algorithm. The algorithm follows a block-based approach of normalized least mean square (NLMS) algorithm with set-membership filtering and achieves a significantly less computational complexity over its conventional NLMS counterpart with set-membership filtering. It is shown in the results that the proposed algorithm yields similar type of bit error rate performance over a reasonable signal to noise ratio in comparison with the latter one.

Keywords— Decision feedback equalizer, Adaptive algorithm, Block based computation, Set membership filtering.

I. Introduction

IN modern digital communication systems, digital signals are transmitted at a high speed through band-limited time dispersive channels which causes multipath fading and signal distortion, resulting in intersymbol interference (ISI). Channel equalization is an effective approach to remove ISI from the received signal [1].

The decision feedback equalizer (DFE) is an important component in many digital communication receivers and is used to suppress intersymbol interference (ISI) caused by time dispersive channels [2]-[5]. DFE provides better performance in ISI cancellation than linear equalizer, especially if the channel has spectral nulls. DFE incorporates a feedforward filter that operates on the received signal to suppress precursor ISI, with a feedback filter that operates on previously detected channel symbols to suppress postcursor ISI.

Since the channel is time-varying the coefficients of the DFE are usually trained by some adaptive algorithm leading towards an adaptive DFE (ADFE) structure. Two well known adaptive algorithms of two different classes are least mean square (LMS) and recursive least squares (RLS) [6]. Among these two, the RLS algorithm faster than its counterpart at the expense of more computational complexity. Therefore, from implementation view point, we need a fast algorithm with somewhat less computational complexity than RLS algorithm. The normalized least mean square (NLMS) algorithm [7] can be viewed as special case of LMS algorithm which takes into account the variations in the signal level at the filter output and selects the normalized step size parameter, resulting in a stable as well as fast converging adaptive algorithm. For fast convergence properties, NLMS algorithm has found many applications where primarily static input processes are unknown or changing with time that include adaptive equalization, adaptive noise cancellation, adaptive line enhancing, adaptive array processing etc [8]. Further, set-membership NLMS (SM-NLMS) algorithm [9] reduces the computational complexity when compared with NLMS algorithm. In this paper, we propose a new adaptive algorithm for ADFE which performs satisfactorily in comparison with SM-NLMS. This new algorithm can be perceived as a block-based NLMS algorithm with set-membership filtering which gives significantly reduced computational complexity when compared with SM-NLMS. It is shown in the results that the proposed algorithm yields similar type of bit error rate performance over a reasonable signal to noise ratio in comparison with the latter one.

This paper is organized as follows. Section II describes general ADFE structure. In Section III, we briefly deal with BBNLMS algorithm for linear equalizer and discuss about weight update equations along with step size value for convergence. In Section IV, we introduce the proposed scheme for ADFE and finally, Section V presents the results and also briefs about effectiveness of a proposed scheme.

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II. Adaptive Decision Feedback Equalizer

A simple nonlinear equalizer, which is particularly useful for channels with severe amplitude distortion, use decision feedback to cancel the interference from symbol which has already been detected. Fig. 1 shows such an ADFE structure. The equalized signal is given by equation (4), is the sum of outputs of the feedforward and feedback parts of the equalizer. The tap input vector of feedforward filter (FFF) $\mathbf{v}_k$, the tap input vector of feedback filter (FBF) $\mathbf{u}_k$ and filter coefficients vector of both FFF and FBF and $\mathbf{w}_k$ with time index is k given by

$$\mathbf{v}_k = [x_{k+L-1}\ x_{k+L-2}\ \cdots\ x_k]^T \tag{1}$$

$$\mathbf{u}_k = [\hat{y}_{k-1}\ \hat{y}_{k-2}\ \cdots\ \hat{y}_{k-M}]^T \tag{2}$$

$$\mathbf{w}_k = [w_{f,-L+1}\ w_{f,-L+2}\ \cdots w_{f,0}\ \ w_{b,1}\ \cdots\ w_{b,M}]^T \tag{3}$$

where the number of feedforward filter (FFF) taps and decision feedback filter taps (FBF) are L and M respectively, x_k is the input signal and $\hat{y}_k$ is the decision value of the filter output. Then the output of the ADFE can be expressed as

$$y_k = \sum_{l=-L+1}^{0} w_{f,l} x_{k-l} + \sum_{m=1}^{M} w_{b,l} \hat{y}_{k-l} \tag{4}$$

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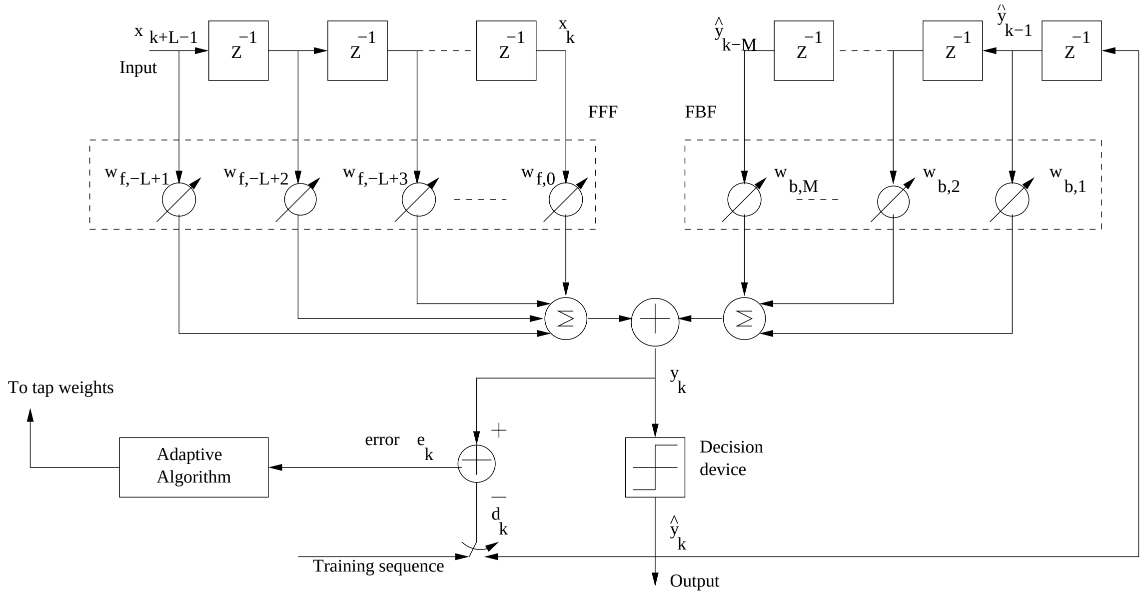

Fig. 1. A generic adaptive decision feedback equalizer structure.

and if $\mathbf{x}_k$ is

$$\mathbf{x}_k = [x_{k+L-1} \ \cdots \ x_{k-1} \ x_k \ \ \hat{y}_{k-1} \ \cdots \ \hat{y}_{k-M+1} \ \hat{y}_{k-M}]^T \quad (5)$$

then

$$y_k = \mathbf{x}_k^T \mathbf{w}. \quad (6)$$

The error between the desired signal d_k and the filter output y_k is defined as

$$e_k = d_k - y_k. \quad (7)$$

The feedforward filter is like the linear transversal equalizer, decision made on the equalized signal are fed back via second transversal filter. The feedforward filter and feedback filter coefficients may be adjusted simultaneously to minimize the mean square error, i.e., $E(e_k^2)$.

III. Block-based NLMS Algorithm

The block-based NLMS algorithm explained in [10], is used for linear equalizer. In this algorithm we find out the maximum magnitude within tap input vector $\mathbf{v}_k$, to consider only that particular value to update the step size of entire block of data. The weight update equation of block-based NLMS recursion takes the following form

$$\mathbf{c}(k+1) = \begin{cases} \mathbf{c}(k) + \frac{\tilde{\mu}}{x_{M_i}^2} e(k)\mathbf{v}(k), & \text{for } x_{M_i} \neq 0 \\ \mathbf{c}(k), & \text{for } x_{M_i} = 0 \end{cases} \quad (8)$$

where x_{M_i} is maximum value of vector $\mathbf{v}_k$ at i^{th} iteration, $\mathbf{v}_k$ is data vector at the input of the filter, $\mathbf{c}_k$ is the vector of filter coefficients at k^{th} iteration, e_k carry their usual meaning as has been described by eq. (7), constant is defined as follows

$$0 < \tilde{\mu} < \frac{2}{L} \quad (9)$$

where L is number of filter coefficients. However, the main advantage of above simpler algorithm is that it reduces the number of MAC operations required for iteration.

IV. Proposed BB-NLMS Algorithm with Set-Membership Filtering

The proposed algorithm is based on principles of the set membership filtering (SMF) which is Explained in [9]. It invites two normalization steps. In SMF, a non zero bound on the magnitude of error signal e_k, is decided. Based on that bound a set $\mathbf{H}_k$ of vectors is defined whose elements are vectors which produce error in that bound, i.e.,

$$\mathbf{H}_k = \{\mathbf{c} : |e_k| \leq \gamma\} \quad (10)$$

where e_k is error defined as in eq. (4), γ is nonzero bound in the error, and $\mathbf{H}_k$ is called the constraint set associated with $\{\mathbf{v}_k, d_k\}$. The boundaries of the $\mathbf{H}_k$ are decided by the two hyper planes,

$$d_k - \mathbf{c}_k^T \mathbf{v}_k = \gamma \quad (11)$$

$$d_k - \mathbf{c}_k^T \mathbf{v}_k = -\gamma. \quad (12)$$

In this algorithm weights are updated if the error e_k, exceeds the bound specified by (7). Otherwise no update is required. the weight update equations are as follows

$$\mathbf{c}(k+1) = \begin{cases} \mathbf{c}(k) + \frac{\alpha_k}{x_{M_i}^2} e(k)\mathbf{v}(k), & \text{for } x_{M_i} \neq 0 \\ \mathbf{c}(k), & \text{for } x_{M_i} = 0 \end{cases} \quad (13)$$

where

$$\alpha_k = \begin{cases} \tilde{\mu}\{1 - \frac{\gamma}{|e_k|}\}, & \text{for } |e_k| \geq \gamma \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

and

$$x_{M_i} = \max \ \{|\mathbf{v}_i(k)|\}. \quad (15)$$

The weights are updated if the error exceeds the bound specified by (7). Otherwise no update is required. There by all data samples are not updated, hence the computational complexity is reduced when compared to BBNLMS.

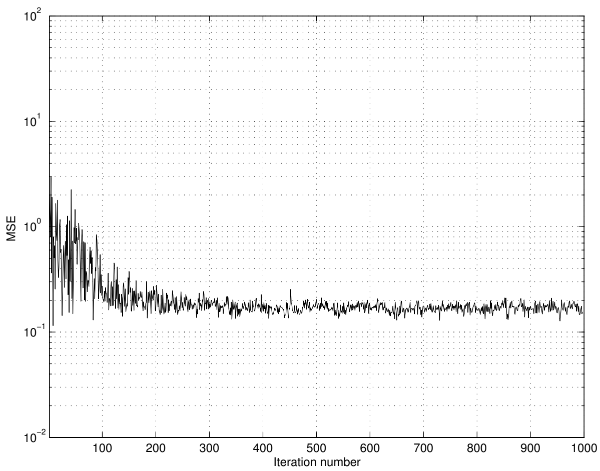

Fig. 2. MSE characteristics of the ADFE with BBNLMS-SM algorithm as the adaptive algorithm with eigenvalue spread = 6. 8 and SNR = 15 dB.

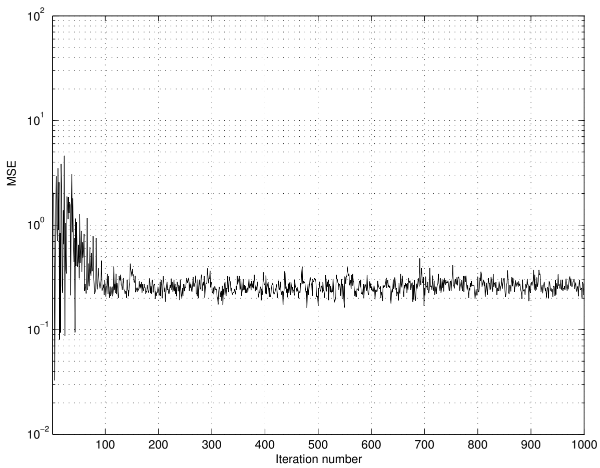

Fig. 3. MSE characteristics of the ADFE with BBNLMS algorithm as the adaptive algorithm with eigenvalue spread = 6. 8 and SNR = 15 dB.

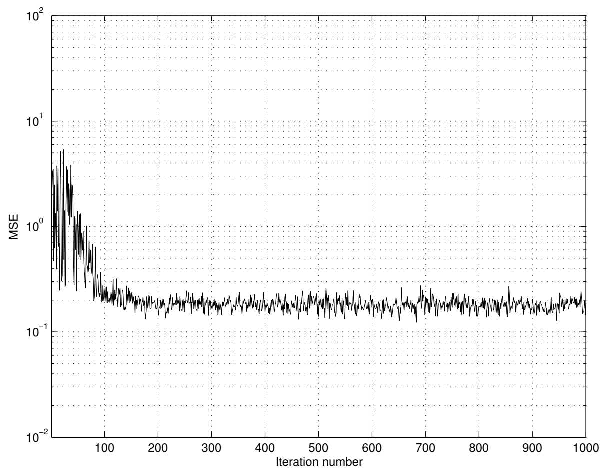

Fig. 4. MSE characteristics of the ADFE with SMNLMS algorithm as the adaptive algorithm with eigenvalue spread = 6. 8 and SNR = 15 dB.

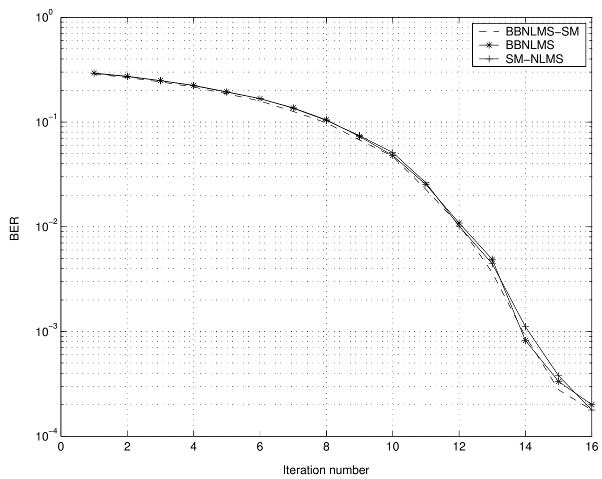

Fig. 5. BER curves for DFE with BBNLMS, SMNLMS and BBNLMS-SM algorithms with an eigenvalue spread = 6.8.

V. Results and Discussions

In this section, the performance of the decision feedback equalizer with proposed algorithm (BBNLMS-SM) is evaluated through computer simulation by comparing with the ADFE with SM-NLMS algorithm. Channel used for simulations is simple ISI channel with additive gaussian noise (AWGN).

The ISI channel model used for our simulation is given by

$$\mathbf{h}_k = \begin{cases} 0.5[1+\cos(\frac{2\pi(k-2)}{K})], & \text{for } k = 1,2,3,4,5 \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

where K represents parameter to adjust the degree of ISI. The received signal x_k, is then given by

$$x_k = d_k * h_k + n_k \quad (17)$$

where n_k is additive gaussian noise, d_k is the QPSK modulated signal and '*' represents the usual convolution operation. The taps are used for feedforward and feedbackward filters of the ADFE are 6 and 4 respectively. In Figs. 2-4, MSE curves of ADFE with adaptive algorithms the proposed BBNLMS with set-membership filtering algorithm, BBNLMS algorithm and SM-NLMS algorithm are shown. In all these simulations, SNR and eigenvalue spread has been kept as 15 dB and 6.8 respectively. Also the error bound γ has been specified within the range [0.3725, 1.0299], where $\gamma = \sigma_v^2\sqrt{2exp(-\sqrt{\sigma^2})}$ with σ_v^2 being the observation noise variance. From the above figures, it is seen that the convergence speed of proposed BBNLMS-SM algorithm is same as the convergence speed of SM-NLMS algorithm.

The bit error rate (BER) performance of BBNLMS-SM, SM-NLMS and BBNLMS algorithms are shown in Fig. 5, by varying SNR from 1 to 16 dB with 6 and 4 taps in the FFF and FBF filters respectively. It is found that the BER performance of proposed algorithm is same as of other two algorithms, namely, SM-NLMS and BBNLMS. Fig. 6 demonstrates the effect of eigenvalue spread on MSE for this new algorithm, where the spread has been taken as 72.3 with SNR being kept as 15 dB as before.

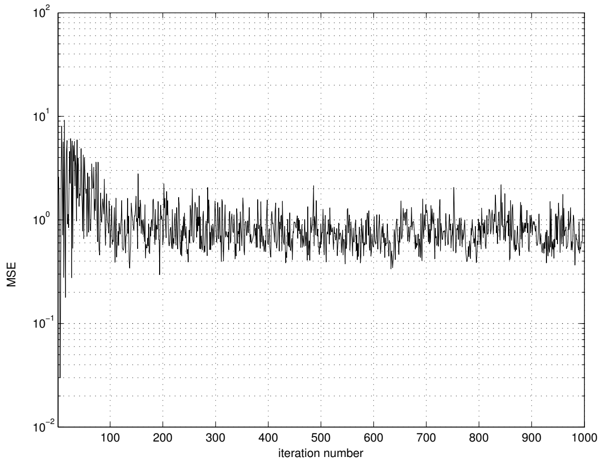

Fig. 6. MSE characteristics of the ADFE with BBNLMS-SM algorithm as the adaptive algorithm with an eigenvalue spread = 72. 3 and SNR = 15 dB.

VI. Conclusions

In this paper, a new adaptive algorithm is proposed for ADFE which gives less computational complexity when compared to a ADFE with SM-NLMS algorithm. The convergence speed and BER performance of the proposed BBNLMS-SM algorithm is similar to the ADFE with other two algorithms, namely, SM-NLMS and BBNLMS. The BBNLMS-SM algorithm requires less MAC operations per iteration when compared to SM-NLMS algorithm. It is also observed that the proposed algorithm saves almost 70% updating computations at high SNR and therefore can serve as a good alternative for high speed decision feedback equalization techniques.

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