Optimal Calculation of Partial Transmission Ratios of Four-Step Helical Gearboxes for Getting Minimal Gearbox Length

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Abstract—This paper presents a new study on the applications of optimization and regression analysis techniques for optimal calculation of partial ratios of four-step helical gearboxes for getting minimal gearbox length. In the paper, basing on the moment equilibrium condition of a mechanic system including four gear units and their regular resistance condition, models for determination of the partial ratios of the gearboxes are proposed. In particular, explicit models for calculation of the partial ratios are proposed by using regression analysis. Using these models, the determination of the partial ratios is accurate and simple.

Keywords—Gearbox design; optimal design; helical gearbox, transmission ratio.

I. INTRODUCTION

IN optimal gearbox design, the optimal determination of partial transmission ratios of a gearbox has a decisive role. This is because the partial ratios are main effected factors on the size, the dimension, the weight, and the cost of the gearboxes. For that reason, optimal determination of the partial ratios has been subjected to many studies.

So far, many researches have been done on the determination of the partial ratios of helical gearboxes. For the gearboxes, the partial ratios can be predicted by several methods:

-By graph method: researches [1], [2], [3] have been carried out for two, three and four-step helical gearboxes (see an example in Fig. 1).
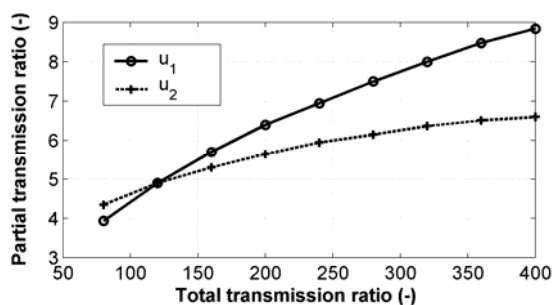

Fig. 1 Determination of partial ratios of three-step helical gearboxes [1]

-By "practical method": this method [4] was based on practical data and it was done for two-step helical gearboxes.

-By models: in this method, based the results of optimization problems, models for prediction of the partial ratios have been found for various objectives, such as for getting minimal cross section dimension of two-step gearboxes [5], for minimal gearbox mass of two and three-step gearboxes [6], or for minimal gear mass of three-step gearboxes [7].

From previous studies, it is apparent that up to now there have been many researches on the determination of the partial ratios for two and three-step helical gearboxes. However, for four-step helical gearboxes, there have not been many studies. In addition, for these gearboxes, there has been only graph method for the prediction of the partial ratios. This paper presents a new result for optimal calculation of partial ratios for four-step helical gearboxes in order to get the minimal gearbox length.

II. DETERMINATION OF GEARBOX LENGTH

In practice, the length of a four-step helical gearbox is decided by the dimension of L which is determined as follows (see Fig. 2):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + a_{w3} + a_{w4} + \frac{d_{w24}}{2} \tag{1}$$

For the first helical gear unit, the center distance can be calculated by the following equation:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right) \tag{2}$$

Or we get

$$a_{w1} = \frac{d_{w21}}{2}\left(\frac{1}{u_1} + 1 \right) \tag{3}$$

Using the same way for the second, the third and the fourth step we have:

$$a_{w2} = \frac{d_{w22}}{2}\left(\frac{1}{u_2} + 1 \right) \tag{4}$$

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$$a_{w3} = \frac{d_{w23}}{2}\left(\frac{1}{u_3}+1\right) \tag{5}$$

$$a_{w4} = \frac{d_{w24}}{2}\left(\frac{1}{u_4}+1\right) \tag{6}$$

With the note that $d_{w11} = d_{w21}/u_1$ and substituting (3), (4), (5) and (6) into (1) we have:

$$L = \frac{d_{w21}}{2}\cdot\left(\frac{2}{u_1}+1\right)+\frac{d_{w22}}{2}\cdot\left(\frac{1}{u_2}+1\right)+$$
$$+\frac{d_{w23}}{2}\cdot\left(\frac{1}{u_3}+1\right)+\frac{d_{w24}}{2}\cdot\left(\frac{1}{u_4}+2\right) \tag{7}$$

In the above equations, u_1, u_2, u_3, u_4 are partial ratios, d_{w11}, d_{w12}, d_{w21}, d_{w22}, d_{w23}, d_{w24} are pitch diameters (mm) and a_{w1}, a_{w2}, a_{w3}, a_{w4} are center distances (mm) of helical gear units 1, 2, 3 and 4, respectively.

The following equation can be used as the design equation for the pitting resistance of the first gear unit [8]:

$$\sigma_{H1} = Z_{M1}Z_{H1}Z_{\varepsilon1}\sqrt{\frac{2T_{11}K_{H1}\sqrt{u_1+1}}{b_{w1}d_{w11}^2 u_1}} \le [\sigma_{H1}] \tag{8}$$

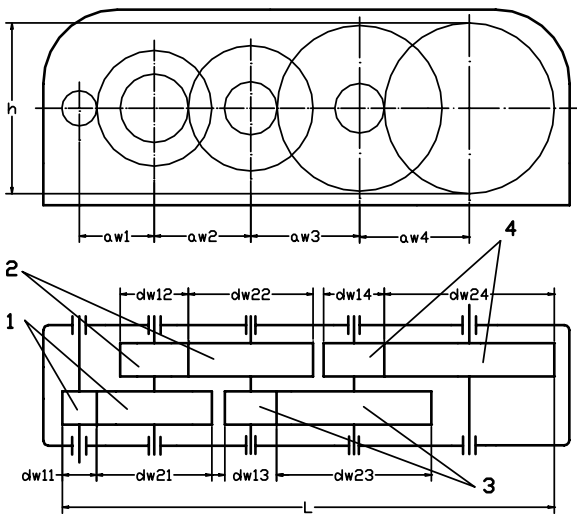Fig. 2 Calculating schema for four-step helical gearbox

From (8) we have:

$$[T_{11}] = \frac{b_{w1}\cdot d_{w11}^2\cdot u_1}{2\cdot(u_1+1)}\frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon1})^2} \tag{9}$$

Where, b_{w1} and d_{w11} are determined by the following equations:

$$b_{w1} = \psi_{ba1}\cdot a_{w1} = \frac{\psi_{ba1}\cdot d_{w11}\cdot(u_1+1)}{2} \tag{10}$$

$$d_{w11} = \frac{d_{w21}}{u_1} \tag{11}$$

Substituting (10) and (11) into (9) we get:

$$[T_{11}] = \frac{\psi_{ba1}\cdot d_{w21}^3\cdot[K_{01}]}{4\cdot u_1^2} \tag{12}$$

In which

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon1})^2} \tag{13}$$

From (12) the pitch diameter d_{w21} can be calculated by:

$$d_{w21} = \left(\frac{4[T_{11}]u_1^2}{\psi_{ba1}[K_{01}]}\right)^{1/3} \tag{14}$$

Calculating in the same way, the following equations can be found:

$$d_{w22} = \left(\frac{4[T_{12}]u_2^2}{\psi_{ba2}[K_{02}]}\right)^{1/3} \tag{15}$$

$$d_{w23} = \left(\frac{4[T_{13}]u_3^2}{\psi_{ba3}[K_{03}]}\right)^{1/3} \tag{16}$$

$$d_{w24} = \left(\frac{4[T_{14}]u_4^2}{\psi_{ba4}[K_{04}]}\right)^{1/3} \tag{17}$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $[\sigma_{H1}]$ is allowable contact stresses of the first helical gear unit; ψ_{ba1}, ψ_{ba2}, ψ_{ba3} and ψ_{ba4} are coefficients of helical gear face width of steps 1, 2, 3 and 4, respectively.

From the condition of moment equilibrium of the mechanic system including four gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{T_{11}} = \frac{[T_r]}{[T_{11}]} = u_1\cdot u_2\cdot u_3\cdot u_4\cdot\eta_{brt}^4\cdot\eta_o^4 \tag{18}$$

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In the above equation, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [8]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [8]).

Choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into (18) we have

$$[T_{11}] = \frac{[T_r]}{0.8573 \cdot u_1 \cdot u_2 \cdot u_3 \cdot u_4} \tag{19}$$

Substituting (19) into (14) with the note that $u_1 = u_h / u_2$ we have

$$d_{w21} = \left(\frac{4.6658 \cdot [T_r] \cdot u_h}{\psi_{ba1} \cdot [K_{01}] \cdot u_2^2 \cdot u_3^2 \cdot u_4^2} \right)^{1/3} \tag{20}$$

For the second helical gear unit we also get:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^3 \cdot \eta_o^3 \tag{21}$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ Equation 21 becomes

$$[T_{12}] = \frac{[T_r]}{0.8909 \cdot u_2 \cdot u_3 \cdot u_4} \tag{22}$$

Substituting (22) into (15) we have:

$$d_{w22} = \left(\frac{4.4898 \cdot [T_r] \cdot u_2}{\psi_{ba2} \cdot [K_{02}] \cdot u_3 \cdot u_4} \right)^{1/3} \tag{23}$$

In exactly similar manner, the following equations can be written for the third and the fourth steps:

$$d_{w23} = \left(\frac{4.3201 \cdot [T_r] \cdot u_3}{\psi_{ba3} \cdot [K_{03}] \cdot u_4} \right)^{1/3} \tag{24}$$

$$d_{w24} = \left(\frac{4.1571 \cdot [T_r] \cdot u_4}{\psi_{ba4} \cdot [K_{04}]} \right)^{1/3} \tag{25}$$

Substituting (20), (23), (24) and (25) into (7), the length of the gearbox can be calculated by:

$$L = \frac{1}{2} \left(\frac{[T_r]}{[K_{01}]} \right)^{1/3} \cdot \left[\left(\frac{4.6658 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2 \cdot u_4^2} \right)^{1/3} \cdot \left(\frac{2}{u_1} + 1 \right) + \right.$$
$$+ \left(\frac{4.4898 \cdot u_2}{\psi_{ba2} \cdot K_{C2} \cdot u_3 \cdot u_4} \right)^{1/3} + \left(\frac{4.3201 \cdot u_3}{\psi_{ba3} \cdot K_{C3} \cdot u_4} \right)^{1/3} +$$
$$\left. + \left(\frac{4.1571 \cdot u_4}{\psi_{ba4} \cdot K_{C2}} \right)^{1/3} \cdot \left(\frac{1}{u_4} + 2 \right) \right] \tag{26}$$

Where, $K_{C2} = [K_{02}]/[K_{01}]$, $K_{C3} = [K_{03}]/[K_{01}]$, and $K_{C4} = [K_{04}]/[K_{01}]$.

III. OPTIMIZATION PROBLEM AND RESULTS

Based on Equation 26, the optimal problem for finding the minimal gearbox length can be expressed as follows:
The objective function is:

$$\min L = f(u_h; u_2; u_3; u_4) \tag{27}$$

With the constraints as follows:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

$$u_{4\min} \le u_4 \le u_{4\max}$$

$$K_{C2\min} \le K_{C2} \le K_{C2\max}$$

$$K_{C3\min} \le K_{C3} \le K_{C3\max} \tag{28}$$

$$K_{C4\min} \le K_{C4} \le K_{C4\max}$$

$$\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}$$

$$\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}$$

$$\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}$$

$$\psi_{ba4\min} \le \psi_{ba4} \le \psi_{ba4\max}$$

For solving the above optimization problem, a computer program was built. The following data were used in the program: K_{C2}, K_{C3}, K_{C4} were from 1 to 1.3, ψ_{ba1}, ψ_{ba2}, ψ_{ba3}, ψ_{ba4} were from 0.25 to 0.4 [8], u_2, u_3, u_4 were from 1 to 9 [1]; u_h was from 50 to 400.

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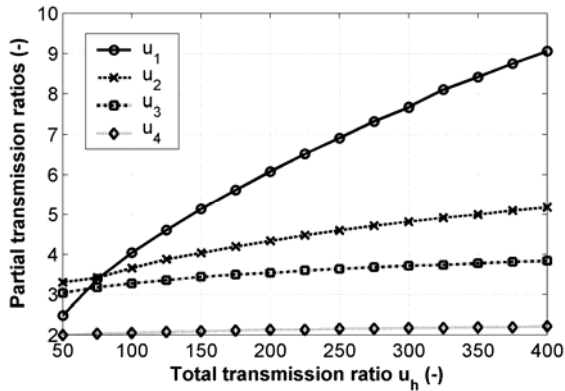International Journal of Mechanical and Mechatronics Engineering
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Fig. 3 Partial transmission ratios versus the total transmission ratio

It is observed that with the increase of the total ratio u_h the partial ratios increase (see Figure 3 - calculated with $K_{c2} = 1.1$, $\psi_{ba1} = 0.3$, $\psi_{ba2} = 0.35$, $\psi_{ba3} = 0.4$ and $\psi_{ba4} = 0.4$). It is observed that with the increase of the total ratio u_h the partial ratios increase. Also, the increase of partial ratio of the first step u_1 when the total ratio increases is much larger than that of the four step u_4. This is because with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driving shaft of the first gear unit T_{11}. As a result, the increase of the partial ratio u_4 should be much smaller than that of u_1 in order to reduce the length of the gearbox.

From the results of the program, models for determining the optimal values of the partial ratios of the second, third and fourth steps were found by regression analysis:

$$u_2 \approx \frac{1.1311 \cdot K_{C2}^{0.4693} \cdot \psi_{ba2}^{0.4741} \cdot u_h^{0.2508}}{K_{C3}^{0.0531} \cdot K_{C4}^{0.1259} \cdot \psi_{ba1}^{0.2948} \cdot \psi_{ba3}^{0.0523} \cdot \psi_{ba4}^{0.1245}} \qquad (29)$$

$$u_3 \approx \frac{1.742 \cdot K_{C3}^{0.4675} \cdot \psi_{ba3}^{0.4722} \cdot u_h^{0.1111}}{K_{C2}^{0.2785} \cdot K_{C4}^{0.0522} \cdot \psi_{ba1}^{0.1289} \cdot \psi_{ba2}^{0.288} \cdot \psi_{ba4}^{0.0579}} \qquad (30)$$

$$u_4 \approx \frac{1.5081 \cdot K_{C4}^{0.4759} \cdot \psi_{ba4}^{0.4756} \cdot u_h^{0.0481}}{K_{C2}^{0.1238} \cdot K_{C3}^{0.2941} \cdot \psi_{ba1}^{0.0563} \cdot \psi_{ba2}^{0.1255} \cdot \psi_{ba3}^{0.2948}} \qquad (31)$$

The above regression models fit quite well with the data. The coefficients of determination were $R^2 = 0.9997$, $R^2 = 0.9985$, and $R^2 = 0.9998$ for Equations 29, 30 and 31, respectively.

Equations 29, 30 and 31 are used to determine the transmission ratio u_2, u_3, and u_4 of steps 2, 3 and 4. After calculating u_2, u_3, and u_4 the partial ratio of the first step u_1 can be predicted as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3 \cdot u_4} \qquad (32)$$

IV. CONCLUSION

It can be concluded that the minimal length of a four-step helical gearbox can be obtained by optimal splitting the total transmission ratio of the gearboxes.

Models for determination of the optimal partial ratios of four-step helical gearboxes in order to get the minimal gearbox length have been suggested.

The partial ratios of the gearboxes can be determined accurately and simply by using explicit models.

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