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Minimum Energy of a Prismatic Joint without: Actuator: Application on RRP Robot

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***Abstract*—This research proposes the state of art on how to control or find the trajectory paths of the RRP robot when the prismatic joint is malfunction. According to this situation, the minimum energy of the dynamic optimization is applied. The RRP robot or similar systems have been used in many areas such as fire fighter truck, laboratory equipment and military truck for example a rocket launcher. In order to keep on task that assigned, the trajectory paths must be computed. Here, the open loop control is applied and the result of an example show the reasonable solution which can be applied to the controllable system.

***Keywords*—** RRP robot, Optimal Control, Minimum Energy and Under Actuator.

I. INTRODUCTION

TODAY, a robot is designed to be versatile so that it can carry out efficiently a large number of motions within its workspace envelop. The robot may perform quite well, in average sense, over the work envelop but may do quite poorly when executing a specific path. A large number of robots end up assembly lines where they perform repetitive motion sequences. These robots must be able to quickly adapt to change in assembly lines where new motion sequences must be executed periodically. A poor performance in one cycle multiples as the cycle repeats. In summary, the current philosophy of design of a robot could be modified to meet the need of flexible manufacturing where a sequence of motion must be performed optimally and the system must be able to quickly adapt to changes in the motion sequence.

The idea proposed in this paper is to design a robot such that some of its control input is malfunction; however, the system could be changed on-line to perform the task without stopping to repair the control actuator. The system could be adapted in an optimal manner consistent with the task. In this paper, the prismatic joint of an RRP robot is supposed to be malfunction such that the one performance index of dynamic optimization called minimum energy could be assigned and solved for the solution of the actuator left in the system. The paper presents how to achieve such a goal and can be applied to a similar system. The paper provides the mathematical formalism, the numerical implementation, and illustration of the method with one example.

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II. OPTIMIZATION PROBLEM

The equation of motion of an n degree of freedom mechanical system are n second-order differential equations of the form

$$\sum_{s=1}^{n} A_{is}(\gamma)\ddot{q}_s + \sum_{s=1}^{n}\sum_{p=1}^{n} h_{ips}(\gamma)\,\dot{q}_p\dot{q}_s + \frac{\partial V(\gamma)}{\partial q_i} = u_i \qquad i = 1,...,n \qquad (1)$$

where $q_{1,...,}q_n$ are generalized coordinates, γ is a set of geometric and inertial parameters characterizing the system, A_{is} are inertial matrix terms, are $h_{ips}\dot{q}_p\dot{q}_s$ Coriolis and centripetal terms, $\partial V/\partial q_i$ are gravity terms, and u_i actuator inputs. It is assumed that the system has an open-chain structure and is properly actuated, i.e. it has as many controlled inputs as the number of degree-of-freedom.

The optimization problem can be stated as finding the optimal parameter γ, the trajectories of state variables and control inputs that take the dynamic system from an inertial position $q(t_0)$ and joint rate $\dot{q}(t_0)$ to a desired final position $q(t_f)$ and joint rate $\dot{q}(t_f)$ while minimizing the cost functional

$$J = \int_0^{t_f} L(q,\dot{q},u,\gamma)dt \qquad (2)$$

Where $q,\dot{q},u,\gamma$ are column vectors with individual elements respectively as $q_i,\dot{q}_i,u_i$ and γ_i. The dynamic equations (1) can be thought of as constraints on the trajectory $q(t)$ while minimizing the cost functional of (2)

The classical solution of this problem with Largrange multipliers is well described in ([3], [4]). These procedures result in the optimality conditions which are $4n$ first-order differential equation referred to as the state and costate equations, n control optimality equations, and n_γ integral equations. The variables appearing in these conditions are

$q(t), u(t), \lambda(t)$, and γ. The solution requires $4n$ boundary equation which are the given $q(t)$ and $\dot{q}(t)$ are the two end points. This two-end point boundary value problem can be solved with multiple shooting techniques which are well-known to be highly computation intensive and sensitive to initial guess of the solution [1].

In order to provide at feel for the computations, the multiple shooting method is briefly described. In time domain of interest, N nodes are created. At each node, the unknowns are the states, Largrange variables, and control inputs. This results in $5nN + n_\gamma$ unknowns. Using the notion of continuity of variables across the nodes, the problem is changed to solution of a set of $5nN + n_\gamma$ nonlinear equation in the same number of variables. It is well known that solution of such nonlinear equations require repeated inversion of $\lfloor 5nN + n_\gamma \times 5nN + n_\gamma \rfloor$ matrics and the solution is quite sensitive to initial guess. For example, if n=6, N=20, n_γ=2, the computations are over (602 x 602) matrices.

In a previous work [2], They have proposed a new method for solving the dynamic optimization problem for classes of linear system without Largrange multipliers. In their work, they have demonstrated that the dynamics equations can be embedded into the cost functional by suitable state transformations that change constrained dynamic optimization problems into unconstrained dynamic optimization problems. As the result, the need for Largrange multipliers is eliminated. The new optimality equations are no longer first-order differential equations but are high-order differential equations. They have shown that the resulting high-order differential equations. They have shown that the resulting high-order differential equations can be solved in a very computationally efficient way using weighted-residual methods. This paper extends these results to design the robot systems.

The organization of this paper is as follows: Section III describes how a constrained optimization problem for a properly actuated mechanical system can be converted to an unconstrained optimization problem, therefore, eliminating the need for Largrange multipliers. The application of this method is discussed in Section IV.

III. Procedure of Dynamic Optimization

A. Unconstrained Optimization Problem

Mechanical systems often have the property that control inputs appear linearly in the dynamic equations, as evident from the structure of (1). One can exploit this property by substituting the expression of u_i from (1) in (2) in terms of higher derivatives of the joint variables. With this substitution, the expression of the new cost functional become

$$J = \int_0^{t_f} L\left[q, \dot{q}, u\left(q, \dot{q}, \ddot{q}, \gamma\right), \gamma\right] dt \tag{3}$$

where $q, \dot{q}, \ddot{q} \in R^n$.

This new expression of the cost functional has the following features.

1) The dynamic equations of (1) have been embedded in the cost functional. The constrained optimization problem, as a result, is reduced to an unconstrained optimization problem and the need for Lagrange multipliers is eliminated.
2) The cost functional of (3) involves second derivatives of the generalized coordinates in contrast to the cost functional of (2) which only involves up to the first derivative of q.

B. Variational Statement

The cost functional of (3) is used here to derive the optimality conditions. The following result from calculus of variations is useful. For a cost functional J of the form

$$J = \int_0^{t_f} F(X, \dot{X}, \ddot{X}, \gamma) dt \tag{4}$$

Where $X \in R^n$ and F is a scalar function. The variation of this definite integral is

$$\begin{aligned}\delta J = & \int_0^{t_f} h^T \left[\frac{\partial F}{\partial X} - \frac{d}{dt}\frac{\partial F}{\partial \dot{X}} + \frac{d^2}{dt^2}\frac{\partial F}{\partial \ddot{X}} \right] dt \\ & + \left[h^T \left(\frac{\partial F}{\partial \dot{X}} - \frac{d}{dt}\frac{\partial F}{\partial \ddot{X}} \right) \right]_{t_0}^{t_f} + \left[\dot{h}^T \left(\frac{\partial F}{\partial \ddot{X}} \right) \right]_{t_0}^{t_f} \\ & + \int_0^{t_f} \delta\gamma^T \frac{\partial F}{\partial \gamma} dt \end{aligned} \tag{5}$$

Where $h = \delta X$ and $\dot{h} = \delta\dot{X}$ are respectively variations of X and $\dot{X}$. If the boundary values of X and $\dot{X}$ are specified at t_0 and t_f, h and $\dot{h}$ are zero at the two end points. Hence, the necessary conditions for optimality become

$$\frac{\partial F}{\partial X} - \frac{d}{dt}\frac{\partial F}{\partial \dot{X}} + \frac{d^2}{dt^2}\frac{\partial F}{\partial \ddot{X}} = 0$$

$$\int_0^{t_f} \frac{\partial F}{\partial \gamma_i} dt = 0, \qquad i = 1,....,n_\gamma . \tag{6}$$

On applying this result to (3), it can be shown that the optimal trajectory of a properly actuated open-chain mechanical system must satisfy

$$\begin{aligned}&\sum_{i=1}^{n} \left[\frac{\partial L}{\partial u_i}\frac{\partial u_i}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial L}{\partial u_i}\frac{\partial u_i}{\partial \dot{q}_j} \right) + \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial u_i}\frac{\partial u_i}{\partial \ddot{q}_j} \right) \right] \\ & + \frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}_j} = 0, \quad (j = 1,...,n)\end{aligned} \tag{7}$$

$$\int_0^{t_f} \left(\frac{\partial L}{\partial \gamma_i} + \sum_{i=1}^{n} \frac{\partial L}{\partial u_i}\frac{\partial u_i}{\partial \gamma_j} \right) dt = 0 \qquad (j = 1,...,n_\gamma) \tag{8}$$

In summary, for an n degree-of-freedom proper-actuated system, the optimality equations are n fourth-order nonlinear differential equations of (15) and n_γ integral equations (17). These equations can be explicitly written in terms of partial derivatives of the elements of inertia matrix and expression of the potential energy of the system and do not have any Lagrange multipliers in them.

IV. Example

To illustrate the theory above with minimum energy, The RRP mechanism is shown in Fig. 1 along with its dimension in MKS unit.

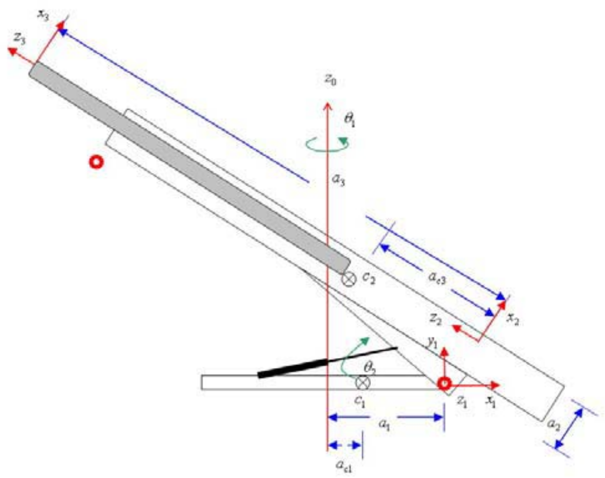

Fig. 1 RRP Robot

From robotic theory [5], the Denevit-Hartenberg parameters are shown in Table I

TABLE I
DENEVIT-HARTENBERG PARAMETERS

i	a_i	α_i	d_i	θ_i
1	a_1	90°	0	$q_1(t)$
2	a_2	-90°	0	$90^\circ - q_2(t)$
3	0	0	$q_3(t)$	0

where

a_i is link length of link i

α_i is link twist of link i

d_i is link offset of link i

θ_i is link angle of link i.

From the mechanism, assigning $a_1 = 2$, $a_2 = 0.6$, $a_{c3} = 2.4$, $m_1 = 1$, $m_2 = 2$, $m_3 = 0.5$, $I_1 = \frac{1}{2} m_1 a_1^2$, $I_2 = \frac{1}{3} m_2 a_3^2$ and $I_3 = 0$. Then the dynamic equations of motion can be computed [5]. The symbolic form of these equations are not shown in this paper since they are quite long expression. However, one can follow step in [5] in order to have them. The boundary conditions are set as initial conditions, $x(t_0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$ and final conditions, $x(t_f) = \begin{pmatrix} 90^\circ & 0 & 0 & 0 & 2 & 0 \end{pmatrix}^T$, where $t_0 = 0$ and $t_f = 1.0$ The cost function of minimum energy is defined as

$$J = \int_0^1 u_1^2 \, dt \,. \tag{18}$$

In order for the cost function in (18) to be minimized, the Calculus of Variations as stated in previous section has been used.

By using software developed by Tawiwat Veeraklaew, [6], the problems of minimum energy can be solved to obtain the optimal solutions. The idea behind this software is to transform the necessary conditions of the dynamic optimization to static optimization. Then one kind of the well known methods called nonlinear programming or linear programming has been used to solve for all parameters that are parameterized through collocation technique. The comparison for each variable such as state and control variables of the dynamic systems in this example are shown in figure below as Fig. 2 to Fig. 6.

V. Conclusion

The above results can be concluded that applying minimum energy to the under actuator system like RRP robot when the prismatic joint is malfunction, the trajectory paths can be obtained easily. This result makes more flexible in order to design some dynamic system that has similar situation as under actuator dynamic system.

The results in this paper show that the minimum energy can be used; however, the other objective function called minimum jerk is also quite challenge to be used for comparison for the future work.

Acknowledgement

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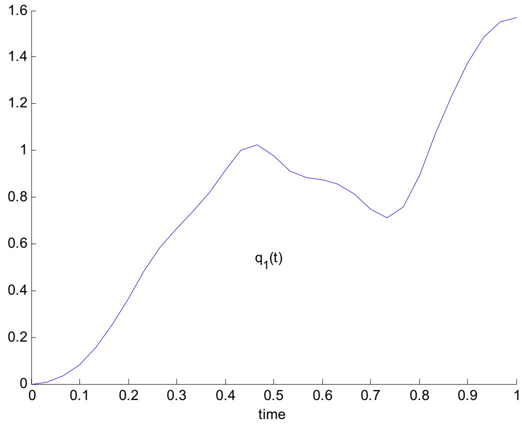

Fig. 2 Solutions of the first state variable from minimum energy

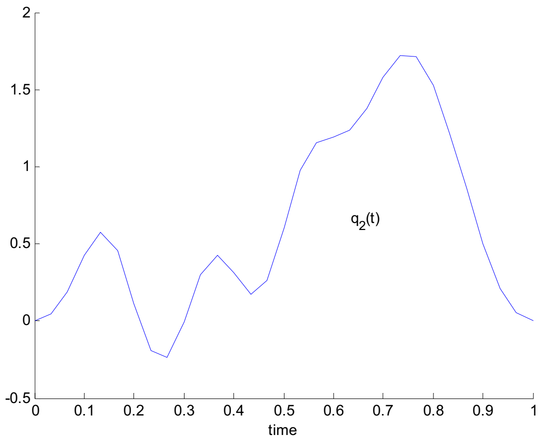

Fig. 3 Solutions of the second state variable from minimum energy

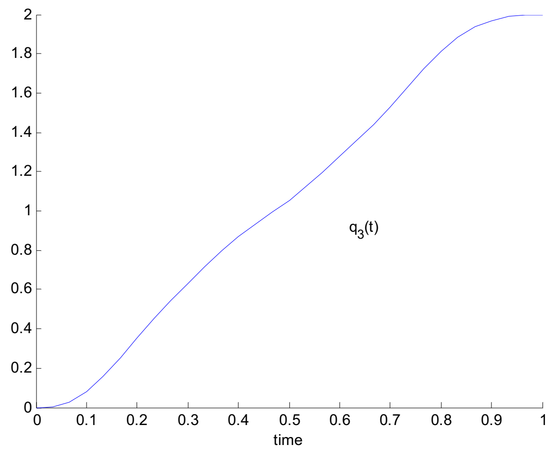

Fig. 4 Solutions of the third state variable from minimum energy

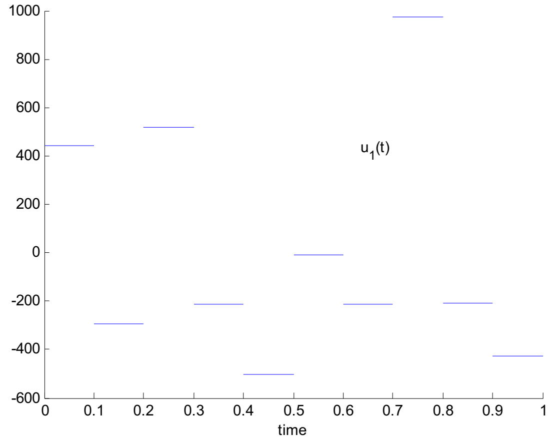

Fig. 5 Solutions of the first control variable from minimum energy

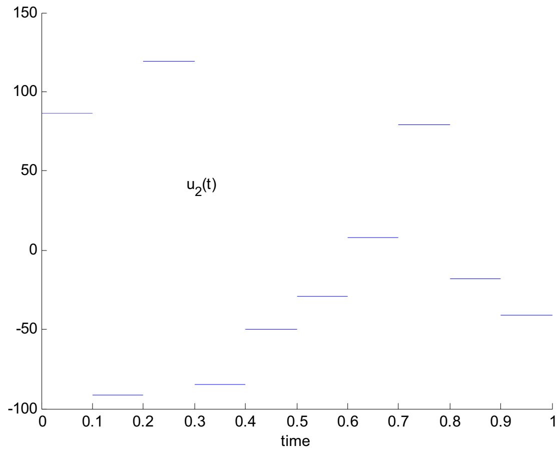

Fig. 6 Solutions of the second control variable from minimum energy

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