Sensitivity computations of Time Relaxation Model with an application in Cavity Computation

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Abstract—We present a numerical study of the sensitivity of the so called time relaxation family of models of fluid motion with respect to the time relaxation parameter χ on the two dimensional cavity problem. The goal of the study is to compute and compare the sensitivity of the model using finite difference method (FFD) and sensitivity equation method (SEM).

Keywords—sensitivity, time relaxation, deconvolution, Navier-Stokes equations.

I. INTRODUCTION

THE study of sensitivity has become an important tool in the understanding of fluid behavior. To obtain a meaningful solution for the Navier Stokes equations one needs to work with a fine mesh, which becomes expensive as well as time consuming. Fluid models are developed in order to avoid these obstacles. Even when a fluid flow model has performed well in practice, the reliability of the model is often not addressed [2]. The reliability can be affected if the model displays sensitivity to certain parameters. Sensitivity analysis eliminates the arising uncertainties and provides a reliable interval for the parameters to be chosen from. During the years, there have been investigations on the sensitivity topics, [1]–[3], [8], [13], [14], [16]. Two methods could be used in order to calculate the sensitivities: Forward Finite Difference method (FFD) or the Sensitivity Equation Method (SEM). SEM can be classified into two methods: Continuous Sensitivity Equation Method (CSEM) and Automatic Differentiation Method (ADM). The difference between ADM and CSEM is in the order of operations of discretization and differentiation. CSEM implements differentiation first and then implements discretization, whereas ADM implements discretization first and then uses differentiation, [10]. When using a flow solver code, finite difference quotient is easy to use. Nevertheless, it might not be a reliable way to compute sensitivites of the model, see [6]. When solving sensitivity for the flow using CSEM, when the flow is obtained, only a linear equation needs to be solved in order to compute the sensitivity [1]. CSEM has been extensively used to compute the sensitivities with respect to different regularization parameters, see [5]–[7], [9], [11]. This paper explores the sensitivity of a time relaxation type model with respect to a regularization parameter defined below.

The governing equations of fluid motion are the Navier Stokes equations, which are defined as follows:

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \nu \triangle \mathbf{u} = \mathbf{f}, \text{ in } \Omega \times [0, T]$$
$$\nabla \cdot \mathbf{u} = 0, \text{ in } \Omega \times [0, T]$$

Here $\mathbf{u}$ and p represent velocity vector and pressure respectively, ν represents the viscosity and $\mathbf{f}$ represents the body force. Time Relaxation model (TRM) was introduced by Stolz, Adams and Kleiser and was developed from regularized Chapman-Enskog expansion of conservation laws [21]. The model was computationally tested on compressible flows with shocks and on turbulent flows [17], [18], [21], i.e. on the aerodynamic noise [19]. A continous finite element analysis for the model along with numerical results can be found in [21]. TRM consists of the Navier-Stokes equations with an addition of a stabilization term:

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \nu \triangle \mathbf{u}$$
$$+ \chi(\mathbf{u} - G_N \overline{\mathbf{u}}) = \mathbf{f}, \text{ in } \Omega \times [0, T]$$
$$\nabla \cdot \mathbf{u} = 0, \text{ on } \Omega \times [0, T] \qquad (1)$$

Here, $\bar{\mathbf{u}}$ represents an averaged function of $\mathbf{u}$ satisfying:

$$-\delta^2 \triangle \bar{\mathbf{u}} + \bar{\mathbf{u}} = \mathbf{u}, \text{in } \Omega$$
$$\bar{\mathbf{u}} = 0, \text{on } \partial\Omega \qquad (2)$$

G_N represents the continuous van Cittert deconvolution operator and is defined as follows:

$$G_N \mathbf{u} := \sum_{n=0}^{N} (I - G)^n \mathbf{u}$$

For order of deconvolution $N = 0$ and $N = 1$, and $\mathbf{u} \in \mathbf{X}_h$ we have:

$$G_0 \mathbf{v} = \mathbf{v},$$
$$G_1 \mathbf{v} = 2\mathbf{v} - \overline{\mathbf{v}},$$
$$G_2 \mathbf{v} = 3\mathbf{v} - 3\overline{\mathbf{v}} + \overline{\overline{\mathbf{v}}},$$
$$G_3 \mathbf{v} = 4\mathbf{v} - 6\overline{\mathbf{v}} + 4\overline{\overline{\mathbf{v}}} - \overline{\overline{\overline{\mathbf{v}}}}.$$

Higher order of deconvolution increases accuracy, however it also requires significant computational time [15]. Herein, computations are carried out for the fundamental case, i.e. order of deconvolution $N = 0$. Here, χ represents the time relaxation coefficient and has units 1/time. Since different values of χ will cause different responses of the flow, it is natural to explore how the change of the flow will be affected by altering this parameter. In this paper we obtain sensitivity computations using both FFD and SEM. The sensitivity using FFD is obtained by the formula:

$$\frac{\mathbf{u}(\chi + \triangle \chi) - \mathbf{u}(\chi)}{\triangle \chi} \qquad (3)$$

Sensitivity of the solution $(\mathbf{u}, p)$ with respect to χ for the SEM is obtained by differentiating (1) (with $N = 0$) with respect

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to χ:

$$\mathbf{s}_t + \mathbf{u} \cdot \nabla\mathbf{s} + \mathbf{s} \cdot \nabla\mathbf{u} + \nabla r - \nu \triangle\mathbf{s}$$
$$+ (\mathbf{u} - \bar{\mathbf{u}}) + \chi(\mathbf{s} - \mathbf{w}) = \mathbf{f}, \text{ in } \times [0, T]$$
$$\nabla \cdot \mathbf{s} = 0, \text{ in } \Omega \times [0, T]$$
$$s = 0, \text{ on } \partial\Omega \times [0, T] \qquad (4)$$

where $s = \dfrac{\partial \mathbf{u}}{\partial \chi}$, $r = \dfrac{\partial p}{\partial \chi}$ and $\mathbf{w} = \dfrac{\partial \bar{u}}{\partial \chi}$. Here w satisfies the following filtering equation:

$$-\delta^2 \triangle\mathbf{w} + \mathbf{w} = \mathbf{s}, \text{ in } \Omega$$
$$\mathbf{w} = 0, \text{ on } \partial\Omega \qquad (5)$$

As we see in (4), $\mathbf{u}$ appears in the sensitivity equation. Hence, in order to obtain the solution for (4) we need to couple (1) with (4).

II. NUMERICAL SCHEME

This section presents the algorithm in order to numerically solve (1) and (4). The finite element spaces (X^h, Q^h) are defined respectively below:

$$X^h \subset X = H_0^1(\Omega) := \{v \in H^1(\Omega) : v|_{\partial\Omega} = 0\},$$
$$Q^h \subset Q = L_0^2(\Omega) := \{q \in L^2(\Omega)| \int_\Omega q = 0\}.$$

Also, bilinear $a(\cdot, \cdot) : X \times X \to \mathbb{R}$ and trilinear $b^*(\cdot, \cdot, \cdot) : X \times X \times X \to \mathbb{R}$ forms are defined respectively as follows:

$$a(\mathbf{u}, \mathbf{v}) := \nu(\nabla\mathbf{u}, \nabla\mathbf{v}),$$
$$b^*(\mathbf{u}, \mathbf{v}, \mathbf{w}) := \frac{1}{2}(\mathbf{u} \cdot \nabla\mathbf{v}, \mathbf{w}) - \frac{1}{2}(\mathbf{u} \cdot \nabla\mathbf{w}, \mathbf{v})$$

The Crank-Nicolson method, which is second order approximation in time, is used for the discretization of the time derivative.

Given $(\mathbf{X}_h, Q_h)$, end-time $T > 0$, the time step is chosen $\triangle t < T = M\triangle t$, find the TRM solution $(\mathbf{u}_h^{n+1}, p_h^{n+1}) \in (X_h, Q_h)$, for n=0,1,2....M-1 satisfying:

$$\frac{1}{\triangle t}(\mathbf{u}_h^{n+1} - \mathbf{u}_h^n, \mathbf{v}_h) + \nu a(\mathbf{u}_h^{n+1/2}, \mathbf{v}_h)$$
$$+ b^*(\mathbf{u}_h^{n+1/2}, \mathbf{u}_h^{n+1/2}, \mathbf{v}_h) - (p_h^{n+1}, \nabla \cdot \mathbf{v}_h)$$
$$+ \chi(\mathbf{u}_h^{n+1/2} - \bar{\mathbf{u}}_h^{n+1/2}, \mathbf{v}_h) = (\mathbf{f}^{n+1/2}, \mathbf{v}_h),$$
$$(\nabla \cdot \mathbf{u}_h^{n+1}, q_h) = 0, \qquad \forall q_h \in Q_h$$
$$\delta^2(\nabla\bar{\mathbf{u}}_h^{h\,n+1}, \nabla\mathbf{v}_h) + (\bar{\mathbf{u}}_h^{h\,n+1}, \mathbf{v}_h) = (\bar{\mathbf{u}}_h^{n+1}, \mathbf{v}_h),$$
$$\forall v_h \in X_h$$

and the sensitivity solution $(\mathbf{s}_h^{n+1}, v^{n+1}) \in (X_h, Q_h)$, for n=0,1,2....M-1 satisfying:

$$\frac{1}{\triangle t}(\mathbf{s}_h^{n+1} - \mathbf{s}_h^n, \mathbf{v}_h) + \nu a(\mathbf{s}_h^{n+1/2}, \mathbf{v}_h)$$
$$+ b^*(\mathbf{s}_h^{n+1/2}, \mathbf{u}_h^{n+1/2}, \mathbf{v}_h) + b^*(\mathbf{u}_h^{n+1/2}, \mathbf{s}_h^{n+1/2}, \mathbf{v}_h)$$
$$- (r_h^{n+1}, \nabla \cdot \mathbf{v}_h) + (\mathbf{u}_h^{n+1/2} - \bar{\mathbf{u}}_h^{n+1/2}, \mathbf{v}_h)$$
$$+ \chi(\mathbf{s}_h^{n+1/2} - \mathbf{w}_h^{n+1/2}, \mathbf{v}_h) = (\mathbf{f}^{n+1/2}, \mathbf{v}_h),$$
$$(\nabla \cdot \mathbf{s}_h^{n+1}, q_h) = 0, \qquad \forall q_h \in Q_h$$
$$\delta^2(\nabla\bar{\mathbf{w}}_h^{h\,n+1}, \nabla v_h) + (\bar{\mathbf{w}}_h^{h\,n+1}, \mathbf{v}_h) = (\bar{\mathbf{s}}_h^{n+1}, \mathbf{v}_h),$$
$$\forall \mathbf{v}_h \in X_h$$

III. SENSITIVITY COMPUTATIONS

χ	Method	T=0.1	T=1
$\chi = 0.005$	FFD	0.0174288	3.71334
	SEM	0.0173	0.146137
$\chi = 0.05$	FFD	0.01738	4.69683
	SEM	0.0172	0.144227
$\chi = 0.5$	FFD	0.0169013	5.06646
	SEM	0.0169	0.127838
$\chi = 1$	FFD	0.016389	5.42801
	SEM	0.01.658	0.112962
$\chi = 10$	FFD	0.00987	0.0232246
	SEM	0.019	0.0384867

TABLE I
SENSITIVITY OF TRM ON THE FINE MESH $h = 1/36$ AND $Re = 10^4$

χ	Method	T=0.1	T=1
$\chi = 0.005$	FFD	$1.73219 \cdot 10^{-5}$	$1.1232 \cdot 10^{-5}$
	SEM	$2.11071 \cdot 10^{-5}$	$1.12607 \cdot 10^{-5}$
$\chi = 0.05$	FFD	$1.73213 \cdot 10^{-5}$	$1.12315 \cdot 10^{-5}$
	SEM	$2.11134 \cdot 10^{-5}$	$1.12542 \cdot 10^{-5}$
$\chi = 0.5$	FFD	$1.73151 \cdot 10^{-5}$	$1.12257 \cdot 10^{-5}$
	SEM	$2.12031 \cdot 10^{-5}$	$1.1247 \cdot 10^{-5}$
$\chi = 1$	FFD	$1.73082 \cdot 10^{-5}$	$1.12194 \cdot 10^{-5}$
	SEM	$2.13586 \cdot 10^{-5}$	$1.13585 \cdot 10^{-5}$
$\chi = 10$	FFD	$1.71862 \cdot 10^{-5}$	$1.1106 \cdot 10^{-5}$
	SEM	$3.07014 \cdot 10^{-5}$	$2.41182 \cdot 10^{-5}$

TABLE II
SENSITIVITY OF TRM ON THE FINE MESH $h = 1/36$ AND RE=1

The experiment is done on the two-dimensional cavity problem with the flow domain $\Omega = [0, 1] \times [0, 1]$. The boundary and initial conditions for the problem are $u(t, x, y) = (16x^2(1 - x^2), 0)^t$ and $u(0, x, y) = (3y^2 - y, 0)^t$ respectively. Since initial and boundary conditions have no dependence on χ, they are set to zero for the sensitivity $\mathbf{s}$. In the computations for the cavity problem we use $\nu = 0.0001$ and $\nu = 1$, implying $Re = 10^4$ and $Re = 1$ respectively. The time step is chosen to be $\triangle t = 0.01$ and we use Taylor-Hood finite elements. To carry out the computations we use finite element method software Freefem++ [20]. The FFD computations are done with (3) and $\triangle\chi = 0.01$. We let s_{FFD} and s_{SEM} represent the sensitivity using FEM and SEM respectively. The following tables represent $\|s_{FFD}(t)\|_{L^2(\Omega)}$ and $\|s_{SEM}(t)\|_{L^2(\Omega)}$ for different parameter settings.

According to the results obtained from tables I-II and tables III-IV, the sensitivity is greatly decreased when a lower Reynolds number is used. Also, at the final time, the difference between FFD and SEM becomes more noticable.

Tables V and VI computed the maximum sensitivity via SEM. It shows that this model is not strongly sensitive to the parameter χ for this benchmark problem. Table VII shows the execution time for FFD and SEM for different values of χ. The table suggests that SEM is computed significantly faster that the FFD and we believe that it is more accurate [6]. Figures 1 and 2 below show the similarity/difference of computations via SEM and FFD.

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χ	Method	T=0.1	T=1
$\chi = 0.005$	FFD	0.023147	5.43861
	SEM	0.0231474	0.255481
$\chi = 0.5$	FFD	0.0223875	2.54603
	SEM	0.0223875	0.214977
$\chi = 1$	FFD	0.220898	0.81
	SEM	0.216504	0.1804
$\chi = 10$	FFD	0.0213324	0.01951
	SEM	0.0156569	0.043

TABLE III
SENSITIVITY OF TRM ON A COARSE MESH $h = 1/16$

χ	Method	T=0.1	T=1
$\chi = 0.005$	FFD	$1.04284 \cdot 10^{-4}$	$9.80413 \cdot 10^{-5}$
	SEM	$1.01806 \cdot 10^{-4}$	$9.77097 \cdot 10^{-5}$
$\chi = 0.5$	FFD	$1.01786 \cdot 10^{-4}$	$9.80238 \cdot 10^{-5}$
	SEM	$1.04208 \cdot 10^{-4}$	$9.75814 \cdot 10^{-5}$
$\chi = 1$	FFD	$1.01584 \cdot 10^{-4}$	$9.78493 \cdot 10^{-5}$
	SEM	$1.0354 \cdot 10^{-4}$	$9.64125 \cdot 10^{-5}$
$\chi = 10$	FFD	$1.01362 \cdot 10^{-4}$	$9.7656 \cdot 10^{-5}$
	SEM	$1.03001 \cdot 10^{-4}$	$9.53558 \cdot 10^{-5}$

TABLE IV
SENSITIVITY OF TRM ON A COARSE MESH $h = 1/16$

χ	T=0.1	T=1
0.005	0.000116971	0.000116971
0.05	0.000116965	0.000116965
0.5	0.000116906	0.000116906
1	0.000116841	0.000116841
10	0.000115714	0.000115714

TABLE V
MAX. SENSITIVITY VIA SEM WITH $h = 1/36$ AND $Re = 10^4$

χ	T=0.1	T=1
0.005	0.0173037	0.146137
0.05	0.0172701	0.144227
0.5	0.0169409	0.127838
1	0.0165878	0.0940275
10	0.0119636	0.0384867

TABLE VI
MAX. SENSITIVITY VIA SEM WITH $h = 1/36$ AND $Re = 1$

χ	0.005	0.05	0.5	1	10
FFD	353.94	354.92	431.89	357.02	366.72
SEM	177.84	205.61	187.05	179.41	179.23

TABLE VII
COMPUTATIONAL TIME USING FFD AND SEM FOR WITH $T = 0.1$,
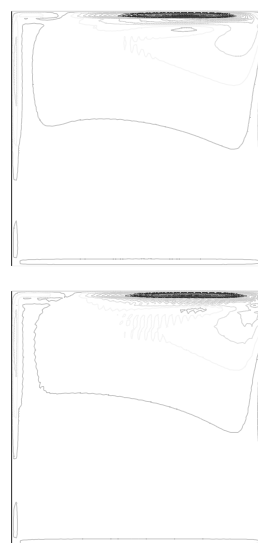$Re = 10^4$ AND $h = 1/36$

Fig. 1. Similarity of SEM (top) and FFD (bottom) at $T = 0.1$ with $\chi = 0.5$, $h = 1/36$
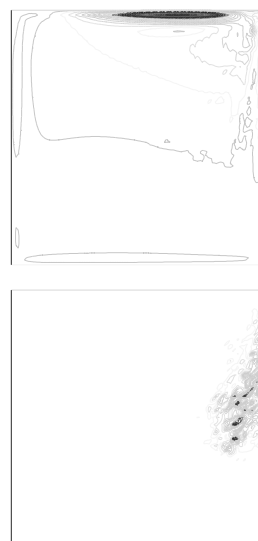
Fig. 2. Difference of SEM (top) and FFD (bottom) at $T = 1$ with $\chi = 0.5$, $h = 1/36$

IV. CONCLUSIONS AND FUTURE DIRECTIONS

Sensitivity of the time relaxation model to the time relaxation coefficient χ on a two dimensional cavity problem was computed via SEM and FFD methods. The numerical comparison between the methods shows that the sensitivity is the highest about $\chi = 1$. Also, on a coarser mesh we can see that the sensitivity becomes larger than on the finer mesh, which agrees with the assumption that we need finer mesh for sensitivity computations [2]. Also, sensitivity is higher for larger Re based on our obtained results. Overall, based on the maximum sensitivity results, the TRM does not seem to have strong sensitivity to the parameter χ for this benchmark problem. The further studies will include stochastic finite element discretization, which should give more insights into the parameter sensitivity. Other benchmark problems will be investigated as well.

REFERENCES

[1] F. Pahlevani, Sensitivity Computations of Eddy Viscosity Models with an Application in Drag Computation, *International Journal for Numerical Methods in Fluids.*, 52-4:381-392, 2006.
[2] M. Anitescu and W. J. Layton, Sensitivities in Large Eddy Simulation and Improved Estimates of Turbulent Flow Functionals *J.C.P*, 178: 391-426, 2001.

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:5, No:7, 2011

[3] M. Anitescu and W. J. Layton, Uncertainties in large eddy simulation and improved of turbulent flow functionals, 2002.

[4] M. Anitescu, F. Pahlevani and W. J. Layton, Implicit for local effects and explicit for nonlocal effects is unconditionally stable *Electronic Trnasactions of Numerical Analysis*, 18: 174-187, 2004.

[5] J. Borggaard and J. Burns, A sensitivity equation approach to shape optimization in fluid flows *Flow control*, 49-78, 1995.

[6] J. Borggaard, D. Pelletier and E. Turgeon, A continuous sensitivity equation method for flows with temperature, *SIAM*, 14-24, 1993.

[7] J. Borggaard, D. Pelletier and E. Turgeon, A continuous sensitivity equation method for flows with temperature dependent properties, *in Proceedings of the 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplanary Analysis and Design*, 2481 1993.

[8] J. Borggaard, D. Pelletier and E. Turgeon, Sensitivity and uncertainty analysis for variable property flows, *in Proceedings of the 39th AIAA Aerospace Sciences Meeting and Exhibit*, 0140, 1993.

[9] J. Borggaard and J. Burns, A PDE sensitivity equation method for optimal aerodynamic design, *Journal of Computational Physics*, 136:366-384, 1997.

[10] J. Borggaard and A. Verma, Solutions to continuous sensitivity equations using Automatic Differentiation, *SIAM Journal of Scientific Computing*, 22:39-62, 2001.

[11] A. Godfrey and E. Cliff, Direct calculation of aerodynamic force derivatives: A sensitivity equation approach, *in proceedings of the 36th AIAA Aerospace Sciences Meeting and Exhibit*, 0393, 1998.

[12] J. Guermond, Stabilization of Galerkin approximations of transport equations by subgrid modeling, *M2AN*, 33: 1293-1316, 1999.

[13] M. Gunzburger, Sensitivities, adjoints and flow optimization, *Int. Jour. Num. Meth. Fluids*, 31:53-78, 1999.

[14] P. Sagaut and T.Lê, Some investigations of the sensitivity of large eddy simulation, *Tech. Rep.*, 1997-12, ONERA.

[15] A. Dunca and Y. Epshteyn, On the Stolz-Adams deconvolution LES model, *SIAM J. Math Analysis*, 2006

[16] L. Stanley and D. Stewart, Design sensitivity analysis: Computational issues of sensitivity equation methods, *Frontiers in Mathematics, SIAM*, Philadelphia, 2002.

[17] N. A. Adams and S. Stolz, Deconvolution methods for subgrid-scale approximation in large eddy simulation *Modern Simulation Strategies for Turbulent Flow*, 2001.

[18] N. A. Adams and S. Stolz, A subgrid-scale deconvolution approach for shock capturing *J.C.P*, 178: 391-426, 2001.

[19] R. Guenanff, Non-stationary coupling of Navier-Stokes/Euler for the generation and radiation of aerodynamic noises, PhD thesis, *Universite Rennes*, Rennes, France, 2004.

[20] F. Hecht, O. Pironneau and K. Oshtsuka, *Software Freefem++*, http://www.freefem++.org, 2003.

[21] V. J. Ervin, W. J. Layton and Monika Neda, Numerical Analysis of a Higher Order Time Relaxation Model of Fluids, *International Journal of Numerical Analysis and Modeling*, 4(3-4): 648-670.

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2) M. Neda, *Discontinuous Time Relaxation Method for the Time Dependent Navier-Stokes Equations*, Advances in Numerical Analysis, Volume 2010, 419021, 1-21, 2010.

3) W. J. Layton, C. C. Manica, M. Neda and L. G. Rebholz, *Numerical analysis and computational comparisons of the NS-alpha and NS-omega regularizations*, Computer Methods in Applied Mechanics and Engineering, Volume 199, Issues 13-16, 916-931, 2010.

4) A. Labovsky, W. Layton, C. Manica, M. Neda and L. Rebholz, *The stabilized, extrapolated trapezoidal finite element method for the Navier-Stokes equations*, Computer Methods in Applied Mechanics and Engineering, Volume 198, Issues 9-12, pp. 958-974, 2009.

5) W. Layton, C. Manica, M. Neda, M. Olshanskii and L. Rebholz, *On the accuracy of the rotation form in simulations of the Navier-Stokes equations*, Journal of Computational Physics, Volume 228, Issue 9, 3433-3447, 2009.

6) W. Layton, C. Manica, M. Neda and L. Rebholz, *Numerical Analysis and Computational Testing of a high-order Leray-deconvolution turbulence model*, Numerical Methods for Partial Differential Equations, Volume 24, Issue 2, pp. 555-582, 2008.

7) M. Anitescu, G. Palmiotti, W. Yang and M. Neda, *Stochastic finite-element approximation of the parametric dependence of eigenvalue problem solution*, Proceedings of the Mathematics, Computation and Supercomputing in Nuclear Applications, Monterey, 2007.

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