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Acceptance Single Sampling Plan with fuzzy parameter with The Using of Poisson Distribution

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Abstract--This purpose of this paper is to present the acceptance single sampling plan when the fraction of nonconforming items is a fuzzy number and being modeled based on the fuzzy Poisson distribution. We have shown that the operating characteristic (oc) curves of the plan is like a band having a high and low bounds whose width depends on the ambiguity proportion parameter in the lot when that sample size and acceptance numbers is fixed. Finally we completed discuss opinion by a numerical example. And then we compared the oc bands of using of binomial with the oc bands of using of Poisson distribution.

Keywords--Statistical quality control, acceptance single sampling, fuzzy number.

I. INTRODUCTION

STATISTICAL quality control (SQC) is an efficient method of improving a firm s process quality of production. Sampling for acceptance or rejection a lot is an important field in SQC.

Acceptance single sampling is one of the sampling methods for acceptance or rejection which is long with classical attribute quality characteristic. In different acceptance sampling plans the fraction of defective items, is considered as a crisp value, but in practice the fraction of defective items value must be know exactly. Many times these values are estimated or it is provided by experiment. The vagueness present in the value of p from personal judgment, experiment or estimation may be treated formally with the help of fuzzy set theory. As known, fuzzy set theory is powerful mathematical tool for modeling uncertain resulting.

In this basis defining the imprecise proportion parameter is as a fuzzy number. With this definition, the number of non-conforming items in the sample has a binomial distribution with fuzzy parameter. However if fuzzy number p is small we can use the fuzzy poison distribution to approximate values of the fuzzy binomial. Classical acceptance sampling plans have been studied by many researchers. They are thoroughly elaborated by Schilling (1982). Single sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988) kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991),and Grzegorzewski(1998,2001b). Grzegrozewski (2000b,2002) also considered sampling plan by variables with fuzzy requirements. Sampling plan by attributes for vague data were considered by Hrniewicz (1992, 1994).

We provide some definition and preliminaries of fuzzy sets theory and fuzzy probability in section 2. In section 3 the fuzzy probability of acceptance of the lot, was considered broadly, and its values in special case was computed. In section 4, we deal with oc band of such a plan, with a example.

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II. PRELIMINARIES AND DEFINITIONS

Parameter p (probability of a success in each experiment) of the crisp binomial distribution is known exactly, but sometimes we are not able to obtain exact some uncertainty in the value p and is to be estimated from a random sample or from expert opinion. The crisp poison distribution has one parameter, which we also assume is not known exactly.

Definition1: the fuzzy subset $\tilde{N}$ of real line IR, with the membership function $\mu_N : IR \to [0,1]$ is a fuzzy number if and only if (a) $\tilde{N}$ is normal (b) $\tilde{N}$ is fuzzy convex (c) μ_N is upper semi continuous (d) supp ($\tilde{N}$) is bounded [3].

Definition2: A triangular fuzzy number $\tilde{N}$ is fuzzy number that membership function defined by three number $a_1 < a_2 < a_3$ where the base of the triangle is the interval [a_1, a_3] and vertex is at x=a_2 [3].

Definitoin3: The α -cut of a fuzzy number $\tilde{N}$ is a non-fuzzy set defined as $N[\alpha] = \{x \in IR; \mu_N(x) \geq \alpha\}$. Hence we have $N[\alpha] = [N_\alpha^L, N_\alpha^U]$ where

$$N_\alpha^L = \inf\{x \in IR; \mu_N(x) \geq \alpha\}$$
$$N_\alpha^U = \sup\{x \in IR; \mu_N(x) \geq \alpha\}$$

Definition4: Due to the uncertainty in the k_i's values we substitute $\widetilde{k}_i$, a fuzzy number, for each k_i and assume that 0< $\widetilde{k}_i$ <1 all i. Then X together with the $\widetilde{k}_i$ value is a discrete fuzzy probability distribution. We write $\widetilde{P}$ for fuzzy P and we have $\widetilde{P}(\{x_i\}) = \widetilde{k}_i$. Let $A = \{x_1, ..., x_l\}$ be subset of X. Then define:

$$\widetilde{P}(A)[\alpha] = \{\sum_{i=1}^{l} k_i \mid s\} \qquad (1)$$

For $0 \leqq \alpha \leqq 1$, where stands for the statement "$k_i \in \widetilde{k}_i[\alpha], 1 \leqq i \leqq n, \sum_{i=1}^{n} k_i = 1$" this is our restricted fuzzy arithmetic[1].

Definition5: In m independent Bernoulli experiment let us assume that p, probability of a "success" in each experiment is not known precisely and needs to be estimated, or obtained from expert opinion.

So that p value is uncertain and we substitute $\widetilde{p}$ for p and $\widetilde{q}$ for q so that there is a $p \in p[1]$ and a $q \in q[1]$ with $p + q = 1$.

Now let $\widetilde{P}(r)$ be the fuzzy probability of r successes in m independent trials of the experiment. Under our restricted fuzzy algebra we obtain

$$\widetilde{P}(r)[\alpha] = \{C_m^r \, p^r \, q^{m-r} \mid s\} \qquad (2)$$

For $0 \leqq \alpha \leqq 1$, where now S is the statement, "$p \in \widetilde{p}[\alpha], q \in \widetilde{q}[\alpha], p + q = 1$".

If $\widetilde{P}(r)[\alpha] = [P_{r_1}(\alpha), P_{r_2}(\alpha)]$ then

$P_{r_1}(\alpha) = \min\{C_m^r \, p^r \, q^{m-r} \mid s\}$ and

$P_{r_2}(\alpha) = \max\{C_m^r \, p^r \, q^{m-r} \mid s\}$ and if $\widetilde{P}[a,b]$ be the fuzzy probability of x successes so that fuzzy a $\leqq x \leqq$ b, then

$$\widetilde{P}([a,b])[\alpha] = \{\sum_{x=a}^{b} C_m^x \, p^x \, q^{m-x} \mid s\} \qquad (3)$$

if $\widetilde{P}([a,b])[\alpha] = [P_1([a,b])[\alpha], P_2([a,b])[\alpha]]$ then:

$$P_1([a,b])[\alpha] = \min\left\{\sum_{x=a}^{b} C_m^x \, p^x \, q^{m-x} \mid s\right\} \text{ and}$$

$$P_2([a,b])[\alpha] = \max\left\{\sum_{x=a}^{b} C_m^x \, p^x \, q^{m-x} \mid s\right\}$$

Where S is the same with past case[1].

Definition 6: let x be a random variable having the poison mass function. If $P(x)$ stands for the probability that $X = x$, then

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x} \qquad (4)$$

For $x = 0,1,2,...$ and parameter $\lambda > 0$.

Now substitute fuzzy number $\widetilde{\lambda} > 0$ for λ to produce the fuzzy poison probability mass function. Let $\widetilde{P}(x)$ to be the fuzzy probability that $X = x$. Than we find α-cut of this fuzzy number as

$$\widetilde{P}(x)[\alpha] = \left\{\frac{e^{-\lambda} \lambda^x}{x} \middle| \lambda \in \lambda[\alpha]\right\} \qquad (5)$$

For all $\alpha \in [0,1]$. Let X be a random variable having the fuzzy binomial distribution and $\widetilde{p}$ in the definition 4 be small. Which means that all $p \in \widetilde{p}[\alpha]$ are sufficiently small. Then $\widetilde{P}[a,b][\alpha]$ using the fuzzy poison approximation[2].

$$\widetilde{P}[a,b][\alpha] \approx \left\{\sum_{x=a}^{b} \frac{e^{-\lambda} \lambda^x}{x!} \middle| \lambda \in n\widetilde{p}[\alpha]\right\} \qquad (6)$$

III. ACCEPTANCE SINGLE SAMPLING PLAN WITH FUZZY PARAMETER

Suppose that we want to inspect a lot with a large size of N. First take a randomized sample of size n from the lot, then inspect all items in the sample, and the number of defective items (d) will be count down. If the number of observation defective items is less than or equal to acceptance number, then the lot will be accepted, otherwise the lot rejection [10]. If the size of lot be large, the random variable d has a binomial distribution with parameter n and p in which p indicates the lot s defective items. However if the size of sample be large and p is small then the random variable d has a Poisson approximation distribution with $\lambda = np$. So, the probability for the number of defective items to be exactly equal to d is:

$$P(d) = \frac{e^{-np} (np)^d}{d!} \qquad (7)$$

and the probability for acceptance of the lot (p_a) is:

$$p_a = P(d \le c) = \sum_{d=0}^{c} \frac{e^{-np}(np)^d}{d!} \qquad (8)$$

Suppose that we want to inspect a lot with the large size of N, such that the proportion of damaged items is not known precisely. So we represent this parameter with a fuzzy number $\widetilde{p}$ as follows: $\widetilde{p} = (a_1, a_2, a_3), p \in \widetilde{p}[1], q \in \widetilde{q}[1], p + q = 1$.

A single sampling plan with a fuzzy parameter if defined by the sample size n ,and acceptance number c ,and if the number of observation defective product is less than or equal to c ,the lot will be acceptance. If N is a large number, then the number of defective items in this sample (d) has a fuzzy binomial distribution, and if $\widetilde{p}$ is a small, then random variable d has a fuzzy Poisson distribution with parameter $\widetilde{\lambda} = n\widetilde{p}$ [1]. So the fuzzy probability for the number of defective items in a sample size that is exactly equal to d is:

$$\widetilde{P}(d - defective)[\alpha] = \left[P^L[\alpha], P^U[\alpha]\right] \qquad (9)$$

$$P^L[\alpha] = \min\left\{\frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in n\widetilde{p}[\alpha]\right\},$$

$$P^U[\alpha] = \max\left\{\frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in n\widetilde{p}[\alpha]\right\}$$

and fuzzy acceptance probability is as follows:

$$\widetilde{p}_a = \left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\} \qquad (10)$$

$$= \left[P^L[\alpha], P^U[\alpha]\right]$$

$$P^L[\alpha] = \min\left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\},$$

$$P^U[\alpha] = \max\left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\}$$

Example1: The experience of Mazandaran Kesht Va Sanat Shomal Complex management shows that half percent are ill-packed. Major customers choose and inspect 60 items of this product available in a large store to buy them. If the number of nonconforming items in this sample equals zero or one, the customer will buy all products in the store. If the number of nonconforming increases, the customer will not buy them. Because of the proportion of defective products has explained linguistically, we can consider that as fuzzy number $\widetilde{p} = (0, 0.005, 0.01)$. Therefore, the probability purchasing will be described in the following:

$$n = 60, c = 1, \widetilde{p} = [0, 0.005, 0.01],$$

$$\widetilde{\lambda} = [0, 0.3, 0.6], \widetilde{\lambda}[\alpha] = [0.3\alpha, 0.6 - 0.3\alpha]$$

$$\widetilde{p}_a[\alpha] = \left\{(1+\lambda)e^{-\lambda} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\}$$

According to that the $(1+\lambda)e^{-\lambda}$ decreasing, then:

$$\widetilde{p}_a[\alpha] = \left[(1.6 - 0.3\alpha)e^{-(0.6-0.3\alpha)}, (1+0.3\alpha)e^{-0.3\alpha}\right]$$

under $\alpha = 0$ we obtain $\widetilde{p}_a[0] = [0.8781, 1]$

that is, it is exected that for very 100 lots in such a process, 88 to 100 lots will be accepted.

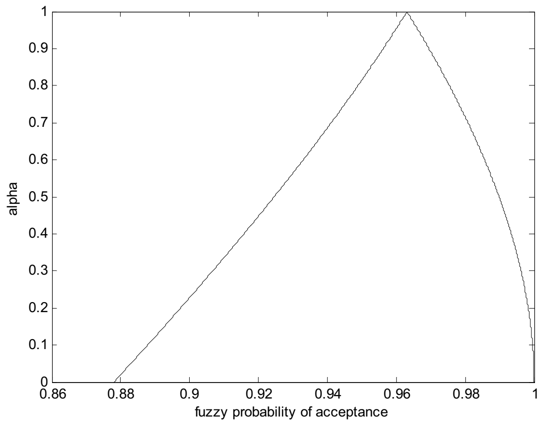

Fig.1 fuzzy probability of acceptance with $\widetilde{p} = (0, 0.005, 0.01)$

IV. OC-BAND WITH FUZZY PARAMETER

Operating characteristic curve is one of the important criteria in the sampling plan. By this curve, one could be determined the probability of acceptance or rejection of a lot having some specific defective items [10]. the oc curve represents the performance of the acceptance sampling plans by plotting the probability of acceptance a lot versus its production quality, which is expressed by the proportion of nonconforming items in the lot [4]. Oc curve aids in selection of plans that are effective in reducing risk and indicates discriminating power of the plan.

Suppose that the event A is the event of acceptance of a lot. Then the fuzzy probability of acceptance a lot in terms of fuzzy fraction of defective items would be as a band with upper and lower bounds. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on that. The less uncertainty value results in less bandwidth, and if proportion parameter gets a crisp value, lower and upper bounds will become equal, which that oc curve is in classic state. Knowing the uncertainty degree of proportion parameter (given a_1, a_2, a_3) and variation of its position on horizontal axis, we have different fuzzy number

($\widetilde{p}$) and hence we will have different proportion (p) which the oc bands are plotted in terms of it. To achieve this aim we consider the structure of $\widetilde{p}$ as follows:

$\widetilde{p}$ = (k , a_2+k , a_3+k), $p \in \widetilde{p}[1], q \in \widetilde{q}[1], p+q=1$ and

$$\widetilde{\lambda} = n\widetilde{p} = (nk, na_2 + nk, na_3 + nk)$$

Which with variation of k in the domain of [0,1-a_3], the oc band, is plotted according to the calculation of follow fuzzy probability:

$$\widetilde{p}[\alpha] = [p_1(\alpha), p_2(\alpha)]$$
$$= [k + a_2\alpha \ , a_3 + k - (a_3 - a_2)\alpha]$$
$$\widetilde{\lambda}[\alpha] = [\lambda_1[\alpha], \lambda_2[\alpha]] =$$
$$[nk + na_2\alpha, nk + na_3\alpha - n(a_3 - a_2)k]$$
$$\widetilde{p}_a = \widetilde{P}_k(A)[\alpha] = [P_k^L[\alpha], P_k^U[\alpha]] \qquad (11)$$

$$P_k^{\,L}[\alpha] = \min\left\{\sum_A \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\}$$
$$P_k^{\,U}[\alpha] = \max\left\{\sum_A \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha]\right\}$$

In Kesht Va Sanat Complex company related to example 1 we had $c = 1, a_2 = 0.005, a_3 = 0.01$ then we have

$$\widetilde{\lambda}[0] = [nk, nk + 0.01n], 0 \leq k \leq 0.99$$
$$\widetilde{p}_a = \widetilde{P}(1)[0] = [(1 + \lambda_2[\alpha])e^{-\lambda_2[\alpha]}, (1 + \lambda_1[\alpha]e^{-\lambda_1[\alpha]}]$$
$$= [(1 + nk + 0.01n)e^{-(nk+0.01n)}, (1 + nk)e^{-nk}]$$

Table1: fuzzy probability of acceptance

$c = 1, n = 60$

k	$\widetilde{p}$	$\widetilde{p}_a$
0	[0,0.01]	[0.8781,1]
0.01	[0.01,0.02]	[0.6626,0.8781]
0.02	[0.02,0.03]	[0.4628,0.6626]
0.03	[0.03,0.04]	[0.3084,0.4628]
0.04	[0.04,0.05]	[0.1991,0.3084]
0.05	[0.05,0.06]	[0.1257,0.1991]

Figure 2 shows the oc band of the example 1. This figure represents that when the process quality decrease from a very good state to a moderate state, then the oc band will be wider.

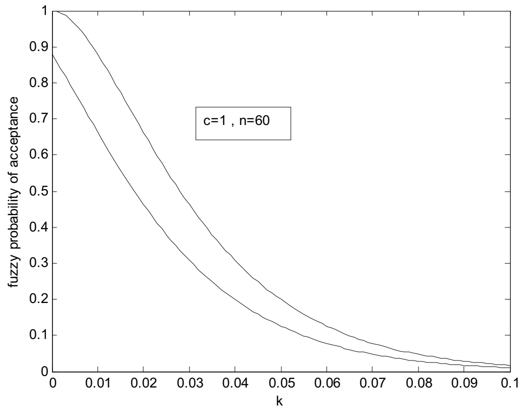

Fig. 2 oc band for a single sampling plan with fuzzy parameter of $c = 1, n = 60$

Example 2: suppose that $c = 0, n = 20$ in example 1 then we have $a_2 = 0.005, a_3 = 0.01, \widetilde{p}[0] = [k, k + 0.01]$ and $\widetilde{\lambda}[0] = [20k, 20k + 0.2], 0 \leq k \leq 0.99$ therefore oc curve in terms fuzzy Binomial distribution and fuzzy Poisson distribution is as follows:

$$\widetilde{p}_{ab} = \{(1-p)^{20} | p \in \widetilde{p}[0]\} = [(0.99-k)^{20}, (1-k)^{20}]$$

$$\widetilde{p}_{ap} = \{e^{-\lambda} | \lambda \in \widetilde{\lambda}[0]\} = [e^{-(0.2+20k)}, e^{-20k}]$$

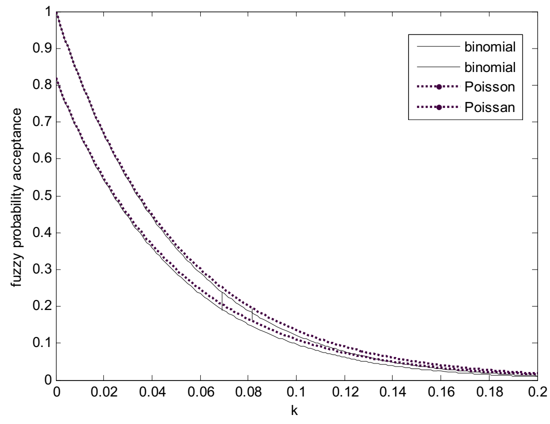

Fig .3 oc band for a single sampling plan with fuzzy parameter of $c = 0, n = 20$

Figure 3 and table 2 show that there two bands approximate. Finally it can be said that oc band with using from fuzzy Poisson distribution optimal approximant for oc band with using from fuzzy binomial distribution. With regard this, such plan can be designed based on oc fuzzy Poisson distribution.

Table2: fuzzy probability of acceptance
$c = 0, n = 20$

k	$\widetilde{p}_{ab}$	$\widetilde{p}_{ap}$
0	$[0.8179,1]$	$[0.8187,1]$
0.01	$[0.6676,0.8179]$	$[0.6703,0.8187]$
0.02	$[0.5437,0.6676]$	$[0.5488,0.6703]$
0.03	$[0.4420,0.5437]$	$[0.4493,0.5488]$

Figure 4 shows two oc bands for n=20, n=40. Indicating that oc bands are convex with zero acceptance number and this leads to a quick reduction of fuzzy probability of acceptance for proportion of defective items with small fuzzy numbers, and it will be more the increase of n.

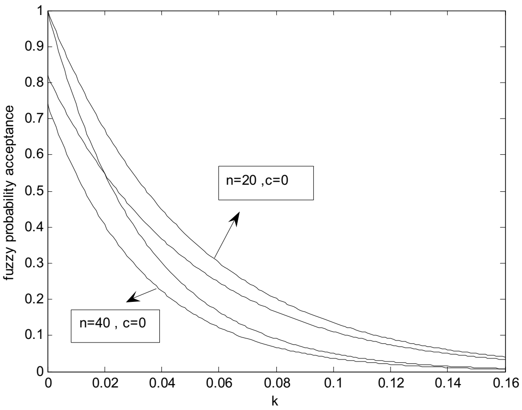

Fig .4 oc band for a single sampling plan with fuzzy parameter of $n = 20, c = 0$; $n = 40, c = 0$

V. CONCLUSION

In the present paper we have proposed a method for designing acceptance single sampling plans with fuzzy quality characteristic with using fuzzy Poisson distribution. These plans are well defined since if the fraction of defective items is crisp they reduce to classical plans. As it was shown that oc curves of the plan is like a band having a high and low bounds. We had shown that in this plan oc bands are convex with zero acceptance number.

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