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Mechanical Buckling of Engesser-Timoshenko Beams with a Pair of Piezoelectric Layers

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***Abstract*—**This paper presents the elastic buckling of homogeneous beams with a pair of piezoelectric layers surface bonded on both sides of the beams. The displacement field of beam is assumed based on the Engesser-Timoshenko beam theory. Applying the Hamilton's principle, the equilibrium equation is established. The influences of applied voltage, dimensionless geometrical parameter and piezoelectric thickness on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

***Keywords*—**Mechanical Buckling, Engesser-Timoshenko beam theory - Piezoelectric layer.

I. INTRODUCTION

THE applications of the smart materials have drawn attention in aerospace engineering, civil engineering, mechanical and even bio-engineering. The analysis of a coupled piezoelectric structure has recently been keenly researched because piezoelectric materials are more extensively used either as actuators or sensors. Examples include the analytical modelling and behaviour of a beam with surface-bonded or embedded piezoelectric sensors and actuators [1–3], and the use of piezoelectric materials in composite laminates and for vibration control [4]. The use of finite element method in the analysis of piezoelectric coupled structures has been studied [5–8]. Crawley and de Luis [9] developed the analytical model for the static and dynamic response of a beam structure with segmented piezoelectric actuators either bonded or embedded in a laminated composite. Owing to their good characteristics of lightweight and electromechanical coupling effects, piezoelectric materials have been studied in other application fields, such as the shape control of structures, acoustic wave excitation, health monitoring of structures, etc. [10–12].

Loughlan et al. [13] carried out experimental tests which illustrate the feasibility of buckling control in composite structural elements using induced strain actuation by using shape memory actuators. Shen [14] presented a post-buckling analysis for cross-ply laminated cylindrical shells with piezoelectric actuators subjected to the combined action of external pressure and heating and under different electric voltage situations. LaPeter and Cudney [15] proposed an analytic model for the segmented piezoelectric actuators bonded on a beam or a plate, and found the equivalent forcing functions of the actuators. The piezoelectric bimorph column structures were used as sensing elements.

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Dobrucki and Pruchnicki [16] presented an analysis theory of an axisymmetric piezoelectric bimorph. They also described a sensing theory for using the axisymmetric piezoelectric bimorph. Chandrashekhara and Bhatia [17] developed a finite element model for the active buckling control of laminated composite plates with surface bonded or embedded piezoelectric sensors that are either continuous or segmented. The dynamic buckling behavior of the laminated plate subjected to a linearly increasing compression load is investigated in their work. Chase and Bhashyam [18] derived optimal design equations to actively stabilize laminated plates loaded in excess of the critical buckling load using a large number of sensors and actuators. Such work finds application in aircraft wing skins.

In this analysis, the mechanical buckling of a homogeneous Engesser-Timoshenko beam with piezoelectric actuators subjected to axial compressive loads is studied. Appling the Hamilton's principle, the equilibrium equations of beam are derived and solved. The effects of the applied voltage and dimensionless geometrical parameter on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

II. FORMULATION

Displacements of a beam can be written as a function of its mid-plane displacements on the basis of the Engesser-Timoshenko beam theory in the following forms [19]:

$$u(x,z) = z\phi(x) \qquad w(x,z) = w_0(x,z) \tag{1}$$

In view of the displacement field given in Eqs. (1), the strain displacement relations are given by [19]:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = z\frac{d\phi}{dx} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi + \frac{dw}{dx} \tag{2}$$

Consider a homogeneous beam with piezoelectric actuators and rectangular cross-section as shown in Fig. 1. The thickness, length, and width of the beam are denoted, respectively, by $h, L,$ and b. Also, h_T and h_B are the thickness of top and bottom of piezoelectric actuators, respectively. The $x-y$ plane coincides with the midplane of the beam and the $z-$axis located along the thickness direction.

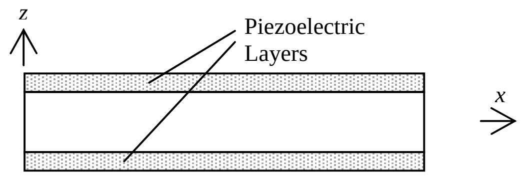

Fig. 1 Schematic of the problem studied.

The Young's modulus E and the Poisson's ratio ν are assumed to be constant. The constitutive relations for homogeneous Engesser-Timoshenko beam with piezoelectric layers are given by [20]:

$$\begin{aligned}\sigma_{xx} &= Q_{11}\varepsilon_{xx} - e_{31}E_z \\ \sigma_{xz} &= Q_{55}\gamma_{xz} - e_{15}E_x\end{aligned} \tag{4}$$

where

$$E_i = \frac{V}{h_i} \tag{5}$$

where σ_{xx}, σ_{xz}, Q_{11} and Q_{55} are the normal ,shear stresses and plane stress-reduced stiffnesses and e_{31}, e_{15} are piezoelectric elastic stiffnesses respectively. Also, u and w are the displacement components in the $x-$ and $z-$directions, respectively.
The potential energy can be expressed as [19]:

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\,\mathrm{d}v \tag{6}$$

Substituting Eqs. (2)-(4) into Eq. (6) and neglecting the higher-order terms, we obtain

$$\begin{aligned}U &= \frac{1}{2}\int_v [(Q_{11}\left(z\frac{d\phi}{dx}\right) - e_{31}E_z)(z\frac{d\phi}{dx}) \\ &+ (Q_{55}(\phi + \frac{dw}{dx}) - e_{15}E_x)(\phi + \frac{dw}{dx})]dv\end{aligned} \tag{7}$$

The width of beam is assumed to be constant, which is obtained by integrating along y over v. Then Eq. (7) becomes

$$\begin{aligned}U &= \frac{b}{2}\int_0^L [D\left(\frac{d\phi}{dx}\right)^2 + \frac{A}{2(1+\nu)}(\phi^2 + (\frac{dw}{dx})^2 \\ &+ 2\phi\frac{dw}{dx})]dx - \frac{b}{2}\int_0^L \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} (ze_{31}E_z\frac{d\phi}{dx} + e_{15}E_x\varphi \\ &+ e_{15}E_x\frac{dw}{dx})dzdx\end{aligned} \tag{8}$$

where

$$\begin{aligned}A &= \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} Q_{55}\mathrm{dz} \\ D &= \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} z^2 Q_{11}\mathrm{dz}\end{aligned} \tag{9}$$

where A and D are the shear rigidity and flexural rigidity respectively. Note that, no residual stresses due to the piezoelectric actuator are considered in the present study and the extensional displacement is neglected. Thus, the potential energy can be written as

$$\begin{aligned}U &= \frac{b}{2}\int_\circ^L [D\ (\frac{d\phi}{dx})^2 + A(\phi^2 + (\frac{dw}{dx})^2 + 2\phi\frac{dw}{dx} \\ &- e_{31}(h_T V_T + h_B V_B)\frac{d\phi}{dx} - e_{15}(V_T + V_B)(\phi + \frac{dw}{dx})]dx\end{aligned} \tag{10}$$

where V_T and V_B are the applied voltages on the top and bottom actuators respectively. The beam is subjected to the axial compressive loads, P as shown in Fig. 2.

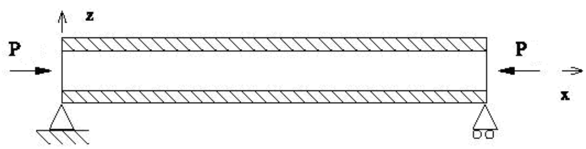

Fig. 2 Simply supported beam under periodic loads.

The work done by the axial compressive load can be expressed as [19]:

$$W = \frac{1}{2}\int_0^L P\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \qquad (11)$$

We apply the Hamilton's principle to derive the equilibrium equations of beam, that is [20]:

$$\delta\int_0^t (T - U + W)\,\mathrm{d}t = 0 \qquad (12)$$

Substitution from Eqs. (10) and (11) into Eq. (12) leads to the following equilibrium equations of the homogeneous Engesser-Timoshenko beam with piezoelectric layers.

$$(P - bA)\frac{d^2 w}{dx^2} + bA(\frac{d\phi}{dx}) = 0$$

$$A(\phi + \frac{dw}{dx}) + 2e_{15}V_T + 2D(\frac{d^2\phi}{dx^2}) = 0 \qquad (13)$$

The boundary conditions for the pin-ended Timoshenko column are given by:

$$w = \frac{d^2 w}{dx^2} = \frac{d\phi}{dx}, \qquad at \quad x = 0 \quad and \quad x = L \qquad (14)$$

Substituting Eq. (14) into (13) and by neglecting the piezoelectric effect, the critical Engesser-Timoshenko buckling load of a homogeneous beam will be derived, that is:

$$p_{cr} = \frac{(\frac{\pi}{L})^2 \frac{bh^3 Q_{11}}{12}}{1 + (\frac{L}{\pi})^2 \frac{12 Q_{55}}{bh^2 Q_{11}}} \qquad (15)$$

The above equation has been reported by Wang and Reddy [19].

III. NUMERICAL RESULTS

This paper presents the mechanical buckling behaviors of simply supported homogeneous Engesser-Timoshenko beams with piezoelectric actuators. It is assumed that both the top and bottom piezoelectric layers have the same thickness; $h_T = h_B$ and the same voltages are applied to both actuators. The material properties of the beam are listed in Table 1. The critical buckling loads for Bernoulli-Euler homogeneous beam and Engesser-Timoshenko homogeneous beam evaluated considering of $h_a / h = 0.1$, $b / h = 1$, $L = 1$, are shown in Fig. 3.

TABLE I MATERIAL PROPERTIES

Property	Piezoelectric layer	Mid layer
Young's modulus E (GPa)	63	223.95
Poisson's ratio ν	0.3	0.3
Length L (m)	0.3	0.3
Thickness h (m)	0.00005	0.01
Density ρ ($\mathrm{Kgm^{-3}}$)	7600	8900
Piezoelectric constant e_{31}, e_{15} ($\mathrm{Cm^{-2}}$)	17.6	-

It is seen that the critical buckling loads for Engesser-Timoshenko beam are generally lower than corresponding values of Bernoulli-Euler[21] beam. Fig. 4. demonstrates the critical buckling loads of homogeneous Engesser-Timoshenko beam for different applied voltage. It is seen that the critical buckling loads for homogeneous Engesser-Timoshenko beam increased with an increase of the applied voltage.

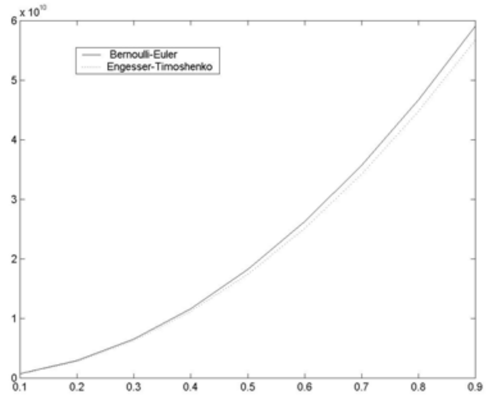

Fig. 3. Comparison of the Critical Buckling Load of Homogeneous Beam with Piezoelectric Actuators Versus h / L .

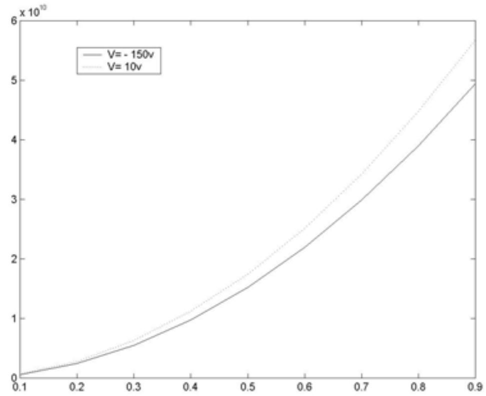

Fig. 5. Effect of Applied Voltage on the Critical Buckling Load of Homogeneous Beam with Piezoelectric Actuators.

IV. CONCLUSION

The mechanical buckling of a homogeneous Engesser-Timoshenko beam with piezoelectric actuators subjected to axial compressive loads is studied. we concluded that the piezoelectric actuators induce tensile piezoelectric force produced by applying negative voltages that significantly affect the stability of the homogeneous Engesser-Timoshenko beam with piezoelectric actuators. The critical buckling loads of homogeneous Engesser-Timoshenko beam under axial compressive load generally increases with the increase of relative thickness h/L.The accuracy of Engesser-Timoshenko beam theory is more than Bernoulli-Euler beam theory.

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