

# Adaptive Nonlinear Backstepping Control

Sun Lim and Bong-Seok Kim

**Abstract**—This paper presents an adaptive nonlinear position controller with velocity constraint, capable of combining the input-output linearization technique and Lyapunov stability theory. Based on the Lyapunov stability theory, the adaptation law of the proposed controller is derived along with the verification of the overall system's stability. Computer simulation results demonstrate that the proposed controller is robust and it can ensure transient stability of BLDCM, under the occurrence of a large sudden fault.

**Keywords**—BLDC Motor Control, Backstepping Control, Adaptive nonlinear control

## I. INTRODUCTION

THE transient stability control of separately excited DC motor (BLDCM) is a typical nonlinear control problem. Although many controller design approaches have been developed, most of the existing controllers are designed by using linear approximation models, which are valid only if the system exhibits small variations around the steady state or pre-fault operating point. Since the BLDCM is inherently nonlinear system, the linear approximation model at a certain equilibrium state is not adequate for large perturbations of state variable since it may cause intolerable errors or even take wrong actions.

In recent years, a lot of interests have been drawn to the applications of nonlinear control theory, e.g., feedback linearization technique, for the control of power electronics to improve system stability and performance [1]-[2].

In addition to the nonlinearities of BLDCM, parameters are not constant and they cannot readily be inferred from available signals. The varying resistances are the result of temperature effects [7].

In this paper, we present a nonlinear feedback linearization and internal backstepping control design to stabilize the system and reduce position tracking error under velocity constraint due to physical limitations

The rest of this paper is organized as follows. In section II a nonlinear model of BLDCM system is given. In the section III feedback linearization applied to BLDCM is obtained. In section IV the adaptive backstepping controller with exact feedback linearization is described. For unknown load, the propped adaptive compensation and estimation are designed and the stability of the closed-loop system is demonstrated in the same section. In section V, the simulation shows the good

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performance and robustness with respect to uncertainty.

Finally, conclusion are drawn in section VII

## II. BLCD MODEL

In this paper, we will consider a simplified dynamic model of a BLDCM that has two winding circuits (armature and filed circuit) [3]. The stator voltage equations, position and speed dynamics of motor are described as follows [4]

$$\frac{dD}{dt} = \frac{1}{g_{ear}} \omega \quad (1)$$

$$\frac{d\omega}{dt} = \frac{1}{J}(T_e - T_L) \quad (2)$$

$$\frac{di_a}{dt} = \frac{1}{L_a}(-R_a i_a - E + u_a) \quad (3)$$

$$\frac{di_f}{dt} = \frac{1}{L_f}(-R_f i_f + u_f) \quad (4)$$

$$T_e = k i_f i_a \quad (5)$$

$$E = k i_f \omega \quad (6)$$

where  $D$ : position,  $\omega$ : speed of motor,  $T_e = k i_f i_a$ : electrical torque,  $i_a$ : the armature current,  $i_f$ : the field current,  $T_L$ : unknown mechanical load torque,  $R_a$ : resistance of the armature winding,  $R_f$ : resistance of the field winding,  $L_a$ : armature inductance,  $L_f$ : field inductance,  $J$ : inertia of motor,  $g_{ear}$ : gear ratio,  $u_a = V_a$ : control input voltage to applied armature winding,  $u_f = V_f$ : control input voltage to applied field winging.

It is clearly shown that the general dynamic model of the motor is highly nonlinear. It can be seen from (1) to (3) that the back EMF term and the electrical torque are the product of state variables.

According to (1) ~ (6), the dynamic model of the BLDCM can be represented as follows:

$$\dot{x} = f(x) + g_a(x)u_a + g_f(x)u_f \quad (7)$$

where

$$f(x) = \begin{bmatrix} \omega & \frac{1}{J}(T_e - T_L) & \frac{1}{L_a}(-R_a i_a - E) & \frac{1}{L_f}(-R_f i_f) \end{bmatrix}^T$$

$$g_a(x) = \begin{bmatrix} 0 & 0 & \frac{1}{L_a} & 0 \end{bmatrix}^T, \quad g_f(x) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{L_f} \end{bmatrix}^T \quad (8)$$

External load and parameter variation are represented as follows:

$$R_a = R_{a0} + \Delta R_a, \quad T_L = T_{L0} + \Delta T_L,$$

$$R_f = R_{f0} + \Delta R_f \quad (9)$$

where the subscript "o" is used to denote the nominal value and "Δ" means the variation w.r.t.(with respect to) nominal value.

As a result,

$$\dot{x} = f(x) + \Delta f(x) + g_a(x)u_a + g_f(x)u_f \quad (10)$$

$$\Delta f(x) = \begin{bmatrix} 0 & -\frac{\Delta T_L}{J} & -\frac{\Delta R_a}{L_a}i_a & -\frac{\Delta R_f}{L_f}i_f \end{bmatrix}^T$$

$$f(x) = \begin{bmatrix} \omega & \frac{1}{J}(T_e - T_{L0}) & \frac{1}{L_a}(-R_{a0}i_a - E) & \frac{1}{L_f}(-R_{f0}i_f) \end{bmatrix}^T$$

In this paper, we assume that thy physical parameters of the BLDCM have modelling error caused by unknown load disturbance, unknown parameter, and high temperature. That is, the parameter  $R_a$  and  $R_f$  in (10) are not exactly known. We consider a nonlinear backstepping controller with adaptive estimator for unknown load and parameters under velocity constraint for the form as follows:

$$u = \begin{bmatrix} u_a & u_f \end{bmatrix}^T = \alpha(z_i, \hat{\theta}_i) \quad \& \quad \dot{\hat{\theta}}_i = \beta(z_i, \hat{\theta}_i) \quad (11)$$

where  $z_i$  are new state variables of BLDCM model,  $\hat{\theta}_i$  are estimation values of unknown uncertainty, i.e., unknown load and unknown parameters.

Control objective is as follows:

$$\lim_{t \rightarrow \infty} \begin{bmatrix} D(t) \\ i_f(t) \end{bmatrix} = \begin{bmatrix} D_{ref} \\ i_{fref} \end{bmatrix}, \quad |\omega(t)| \leq \omega_{\text{constraint}} \quad \text{and}$$

all signals are bounded w.r.t. parameter variation and unknown load.  $\omega_{\text{constraint}}$  is positive specification of user for preventing increment of temperature or increasing system stability in high current operation with long time duration.

### III. APPLICATION OF THE INPUT/OUTPUT LINEARIZATION

The main idea of the input/output feedback linearization approach is to design a nonlinear controller, which transforms the nonlinear system dynamics into a fully or partially decoupled linear one so that linear control technique can be easily applied [1], [2], [4].

Define a new state variable  $z$  as follows by considering output equation:

$$z = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} D(t) \\ \omega(t) \\ \frac{d}{dt}\omega(t) \\ i_f(t) \end{bmatrix} \quad (12)$$

where  $z_0(t)$  and  $z_3(t)$  are output variables, i.e.,

$$h_1 = D(t) \quad \text{and} \quad h_2 = i_f(t).$$

After the coordinate transformation, the dynamic model shown in (10) can be represented as follows:

$$\begin{aligned} \dot{z} &= Az + b \begin{bmatrix} u_a \\ u_f \end{bmatrix} + C \\ &= Az + b \begin{bmatrix} u_a \\ u_f \end{bmatrix} + (C_0 + \Delta C) \end{aligned} \quad (13)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 0 \\ L_{g_a}h_1 & L_{g_f}h_1 \\ 0 & L_{g_f}h_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ L_f h_1 \\ L_f h_2 \end{bmatrix}$$

$$\Delta C = \begin{bmatrix} 0 & -\frac{\Delta T_L}{J} & -\frac{\Delta R_a}{L_a}i_a & -\frac{\Delta R_f}{L_f}i_f \end{bmatrix}^T$$

$$L_f h_1 = \frac{k}{J} \left( \frac{-R_{f0}i_f}{L_f} \right) i_a + \frac{ki_f}{JL_a} (-R_{a0}i_a - E) - \frac{1}{J^2} (T_e - T_{L0})$$

$$L_{g_a} h_1 = \frac{ki_f}{JL_a}, \quad L_{g_f} h_1 = \frac{k}{JL_f} i_a, \quad L_f h_2 = -\frac{R_{f0}}{L_f} i_f, \quad L_{g_f} h_2 = \frac{1}{L_f}$$

in which  $L_f(\cdot)$ ,  $L_g(\cdot)$  denote the Lie derivative w.r.t.  $f$  and  $g$ , [1],[2].  $b$  and  $c$  are also the nominal parameter vectors without parameter variation and external load disturbance.  $\Delta C$  denotes the uncertainty.

Choose the control input as follows:

$$\begin{bmatrix} u_a \\ u_f \end{bmatrix} = g^{-1} \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_f \end{bmatrix} - C \right\} \quad (14)$$

where  $v_a, v_f$  are the newly designed decoupled control inputs, which is introduced in section IV.

*Remark 1*

From the analysis above we can see that the nonlinear compensation law (14) is simple and practically realizable and it is used to make the system (13) linear. Using (14) we can linearize the BLDCM system. The linearized model is independent of the operating point, which is of great importance in electronic power system.

*Remark 2*

By means of input/output linearization, the dynamics of a nonlinear system is decomposed into external and internal dynamic parts. In our application, it is interesting to note that both the relative degree of the linearized system(external dynamic part) and the original nonlinear system have the fourth order. For this reason, internal dynamics dose not have to be analyzed [1,2].

Substituting (14) into (13), the dynamic model can be reorganized as

$$\begin{aligned} \dot{z} &= A_c z + b \begin{bmatrix} v_a \\ v_f \end{bmatrix} + \Delta C \\ &= A_c z + b \left\{ \begin{bmatrix} v_a \\ v_f \end{bmatrix} + E \right\} \end{aligned} \quad (15)$$

where

$$A_c = A, b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, E = b^+ \{\Delta C\}$$

$b^+$  is the left-Penrose pseudo inverse of  $b$ ,  $E$  is lumped uncertainty. The lumped uncertainty is observed by an adaptive estimator and assumed to be a constant during estimation. The above assumption is valid in practical digital signal processing of the estimator because the sampling period of the estimator is short enough compared with the variation of  $E$ .

#### IV. ADAPTIVE NONLINEAR BACKSTEPPING CONTROL

The control problem is to find a control law so that the state  $z$  can track any reference command under velocity constraint.

Equation (15) can be rewritten as

$$\begin{bmatrix} \dot{z}_0 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 - \frac{\Delta T_L}{J} & v_a + \Delta a & v_f + \Delta f \end{bmatrix}^T \quad (16)$$

where  $\Delta a = -\frac{\Delta R_a}{L_a} i_a$ ,  $\Delta f = -\frac{\Delta R_f}{L_f} i_f$  are ease notation of uncertainty of resistance of armature and field.

The armature control loop:

For backstepping control [1], [6],

*Step 1)*

Since  $z_1$  is required to be bounded and the derivative of it is also bounded. Using the same procedure in [8], the Lyapunov candidate function is chosen as

$$V_1(z_0) = k_1 z_0 \tan^{-1}(z_0) \quad (17)$$

where  $k_1$  is positive constant. The position constant speed command is zero. The speed tracking error defines as follows

$$e_1 = z_1 - z_{1ref} \quad (18)$$

where  $z_{1ref} = -c_1 \tan^{-1}(z_0)$  is the speed command,  $c_1$  is the positive constant parameter.

It also shows that

$$\dot{e}_1 = z_2 + \frac{c_1 z_1}{1+z_0^2} \quad (19)$$

Take the derivative of (17) and (18),

$$\begin{aligned} \dot{V}_1(z_0) &= k_1 z_1 \tan^{-1}(z_0) + k_1 z_0 \frac{\dot{z}_0}{1+z_0^2} \\ &= k_1 \{-c_1 \tan^{-1}(z_0) + e_1\} \tan^{-1}(z_0) \\ &\quad + k_1 z_0 \{-c_1 \tan^{-1}(z_0) + e_1\} \frac{1}{1+z_0^2} \\ &= -W_1(z_0) + k_1 e_1 \tan^{-1}(z_0) + \frac{k_1 z_0 e_1}{1+z_0^2} \end{aligned} \quad (20)$$

where  $W_1(z_0) = k_1 c_1 \{\tan^{-1}(z_0)\}^2 + c_1 k_1 \frac{z_0 \tan^{-1}(z_0)}{1+z_0^2}$

are positive definite value.

*Step2)*

Now the second step consists of redefinition of another error to be cancelled:

$$e_2 = z_2 - z_{2ref} \quad (21)$$

where

$$z_{2ref} = -c_1 e_1 - \frac{k_1 z_0}{1+z_0^2} - \frac{k_2^2 - e_1^2}{k_3} \left( k_1 \tan^{-1}(z_0) + \frac{k_1 z_0}{1+z_0^2} \right) + \frac{\hat{T}_L}{J}$$

where  $\hat{T}_L$  is estimated value of unknown external load.

It also shows that

$$\begin{aligned} \dot{e}_2 &= v_a + \Delta a + c_1 \dot{e}_1 + \frac{c_1 z_2}{1+z_0^2} - 2c_1 \frac{z_0 z_1}{(1+z_0^2)^2} \\ &\quad - \frac{2e_1 \dot{e}_1}{k_3} \left\{ k_1 \tan^{-1}(z_0) + \frac{k_1 z_0}{1+z_0^2} \right\} \\ &\quad + \frac{k_2^2 - e_1^2}{k_3} \left\{ \frac{2k_1 z_1}{1+z_0^2} - \frac{2k_1 z_0^2 z_1}{(1+z_0^2)^2} \right\} \end{aligned} \quad (22)$$

For backstepping procedure, another Lyapunov candidate function is chosen as follows:

$$V_2(z_0, e_1) = V_1(z_0) + \frac{1}{2} k_3 \log \left( \frac{k_2^2}{k_2^2 - e_1^2} \right) \quad (23)$$

From (23) we know that if the speed tracking error goes to  $k_2$ , the Lyapunov candidate function  $V_2$  goes to infinite value.

That is,

$$V_2 \rightarrow \infty \text{ as } |e_1| \rightarrow k_2.$$

$k_2$ , i.e., upper bound is designed for increasing system stability or preventing the increment of motor temperature. Since  $z_{2ref}$  is bounded (refer to (3)), from (18) we obtain that

$$|z_1| \leq |e_1| + |z_{1ref}| < k_2 + \frac{\pi}{2} c_1 \quad (24)$$

Thus we have control upper bound automatically by tuning  $c_1$  and  $k_2$ .

The derivative of (23) is

$$\dot{V}_2(z_0, e_1) = -W_1(z_0) + k_1 e_1 \tan^{-1}(z_0) + \frac{k_1 z_0 e_1}{1+z_0^2} + \frac{k_3 e_1 \dot{e}_1}{k_2^2 - e_1^2} \quad (25)$$

Arranging (25) using (19) and (21),

$$\begin{aligned} \dot{V}_2(z_0, e_1) &= -W_1(z_0) + e_1 \left\{ k_1 \tan^{-1}(z_0) \right. \\ &\quad \left. + \frac{k_1 z_0}{1+z_0^2} + \frac{k_3}{k_2^2 - e_1^2} \left( z_2 - \frac{\Delta T_L}{J} + \frac{c_1 z_1}{1+z_0^2} \right) \right\} \\ &= W_1(z_0) - \frac{k_3 x_1 e_1^2}{k_2^2 - e_1^2} + \frac{k_3 e_1}{k_2^2 - e_1^2} e_2 + \frac{k_3 e_1}{k_2^2 - e_1^2} \tilde{T}_L \\ &= -W_2(z_0) + \frac{k_3 e_1}{k_2^2 - e_1^2} e_2 + \frac{k_3 e_1}{k_2^2 - e_1^2} \tilde{T}_L \end{aligned} \quad (26)$$

where  $W_2(z_0, e_1) = W_1(z_0) + \frac{k_3 x_1 e_1^2}{k_2^2 - e_1^2}$  is positive definite value.

Step 3)

Last step, define the Lyapunov candidate function as

$$V_3(z_0, e_1, e_2) = V_2 + \frac{1}{2}e_2^2 + \frac{1}{2\gamma_1}\tilde{T}_L^2 + \frac{1}{2\gamma_2}\tilde{a}^2 \quad (27)$$

where  $\tilde{T}_L = \Delta T_L - \hat{T}_L$ ,  $\tilde{a} = \Delta a - \hat{a}$ .

Its derivative is

$$\begin{aligned} \dot{V}_3(z_0, e_1, e_2) = & -W_2(z_0, e_1) + e_2\dot{e}_2 - \frac{1}{\gamma_1}\tilde{T}_L\dot{\hat{T}}_L \\ & - \frac{1}{\gamma_2}\tilde{a}\dot{\hat{a}} + \frac{k_3e_1}{k_2^2 - e_1^2}e_2 + \frac{k_3e_1}{k_2^2 - e_1^2}\tilde{T}_L \end{aligned} \quad (28)$$

Substituting (22) into (28), it is obtain that

$$\begin{aligned} \dot{V}_3 = & -W_2(z_0, e_1) + e_2 \left\{ \frac{k_3e_1}{k_2^2 - e_1^2} + v_a - \dot{z}_{2ref} \right\} \\ & - \frac{1}{\gamma_1}\tilde{T}_L\dot{\hat{T}}_L + \frac{k_3e_1}{k_2^2 - e_1^2}\tilde{T}_L \end{aligned} \quad (29)$$

The control law is proposed as in the following equation:

$$\begin{aligned} v_a = & -c_2e_2 - c_1\dot{e}_1 - \frac{c_1z_2}{1+z_0^2} + 2c_1\frac{z_0z_1}{1+z_0^2} - \hat{a} \\ & + \frac{2e_1\dot{e}_1}{k_3} \left\{ k_1 \tan^{-1}(z_0) + \frac{k_1z_0}{1+z_0^2} \right\} - \frac{k_3e_1}{k_2^2 - e_1^2} \\ & - \frac{k_2^2 - e_1^2}{k_3} \left\{ \frac{k_1z_1}{1+z_0^2} + \frac{k_1z_1}{1+z_0^2} - \frac{2k_1z_0^2z_1}{(1+z_0^2)^2} \right\} \end{aligned} \quad (30)$$

Substituting (30) into (29), it is obtain that

$$\begin{aligned} \dot{V}_3(z_0, e_1, e_2) = & -W_2(z_0, e_1) - c_2e_2^2 - \frac{1}{\gamma_1}\tilde{T}_L\dot{\hat{T}}_L \\ & - \frac{1}{\gamma_2}\tilde{a}\dot{\hat{a}} + \frac{k_3e_1}{k_2^2 - e_1^2}\tilde{T}_L + e_2\tilde{a} \end{aligned} \quad (31)$$

If the adaptive law for the external disturbance load and parameter is chosen as:

$$\dot{\hat{T}}_L = \gamma_1 \frac{k_3e_1}{J(k_2^2 - e_1^2)}, \quad (32)$$

$$\dot{\hat{a}} = \gamma_2e_2. \quad (33)$$

Using (32) and (33), (31) is simplified as follows:

$$\begin{aligned} \dot{V}_3(z_0, e_1, e_2) = & -W_2(z_0, e_1) - c_2e_2^2 \\ = & -W_3(z_0, e_1, e_2) \leq 0 \end{aligned} \quad (34)$$

The field current control loop:

For the regulation of field current, we use the PI type internal linear controller, which is,

$$v_f = -k_4e_3 - k_5 \int e_3 dt - \hat{f}, \quad (35)$$

$$\dot{\hat{f}} = \gamma_3e_3, \quad (36)$$

where  $e_3 = i_f^* - i_f$ ,  $i_f^*$  is field current command,  $k_4$  and  $k_5$  are positive constants. Constants  $k_4$  and  $k_5$  are adjusted in order to achieve a good compromise between performances in terms of regulation speed and overshoot.

In the following,  $k_5$  is assumed to be zero for simple analysis.

For the stability of field current, define the Lyapunov candidate function as

$$V_4 = \frac{1}{2}e_3^2 + \frac{1}{2\gamma_3}\tilde{f}^2, \quad (37)$$

where  $\tilde{f} = \Delta f - \hat{f}$ .

Its derivative is as:

$$\dot{V}_4 = e_3\dot{e}_3 + \frac{1}{\gamma_3}\tilde{f}\dot{\hat{f}}. \quad (38)$$

Using (35) and (36), (38) result in:

$$\dot{V}_4 = -k_4e_3^2 \leq 0. \quad (39)$$

Theorem

Consider the nonlinear dynamic system expressed by (10) with a velocity constraint, which is defined by

$$\Omega_w = \{z_i \mid |z_i| < U_w\}$$

where  $U_w$  is positive upper bound.

The nonlinear backstepping control law is designed as (30) and (35) with adaptation controller such as (32), (33), (36). Let

$$\frac{\pi}{2}c_1 + k_2 \leq U_w, \quad \frac{\pi}{2}c_1 \leq k_2, \quad c_i > 0 \quad \text{where } i = 1 \sim 3,$$

$k_1$  and  $k_3$  are strictly positive constants. If any initial condition such that  $z_1(0) \in \Omega_w$  is given, the closed-loop system stability is guaranteed, that is, all states are asymptotically stable.

Proof:

Define the total Lyapunov function as

$$V = V_3 + V_4 \quad (40)$$

Differentiating (40) w.r.t. time and using (34) and (39), we get

$$\begin{aligned} \frac{d}{dt}V(z_0, e_1, e_2, e_3) = & -W_3(z_0, e_1, e_2) - k_4e_3^2 \\ = & -W_4(z_0, e_1, e_2, e_3) \end{aligned} \quad (41)$$

where  $W_4 = W_3 + k_4e_3^2$ . From (41),  $V$  is a Lyapunov function of overall system and thus the equilibrium state,  $e = 0$ , of the system is globally asymptotically stable.

Since  $\dot{V}$  is a negative definite function, that is,  $V(z_i, e_i, t) \leq V(z_i(0), e_i(0), 0)$ ,  $z_i(t)$  and  $e_i(t)$  are bounded functions.

Define a function  $W(t) = W_4 \leq -\dot{V}$  and integrate function  $W(t)$  w.r.t. time,

$$\int_0^t W(\tau) d\tau \leq V(z_i(0), e_i(0)) - V(z_i(t), e_i(t)). \quad (42)$$

Because  $V(z_i(0), e_i(0))$  is a bounded function, and

$V(z_i(t), e_i(t))$  is a non-increasing and bounded function, the following result is obtained:

$$\int_0^t W(\tau) d\tau < \infty. \quad (43)$$

Also,  $\dot{W}(t)$  is bounded, so by Barbalats Lemma[6], it can be shown that  $\lim_{t \rightarrow \infty} W(t) = 0$ . This implies,  $e(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

## V. SIMULATION RESULT

The system under study is shown in Fig. 1 and the data of the system are introduced in Table I. In order to investigate the effectiveness and performance, simulation has been carried out using Matlab to evaluate the proposed adaptive nonlinear backstepping control.

TABLE I  
 BLDCM PARAMETERS

Inertia	0.3 kgm <sup>2</sup>
Armature resistance	1.2 Ω
Field resistance	60 Ω
Armature inductance	0.01 mH
Field inductance	60 mH
Gear ratio	1

Now the simulation is implemented. Setting  $c_1 = 50$ ,  $c_2 = 100$ ,  $k_1 = 1$ ,  $k_2 = 50$ ,  $k_3 = 10$ ,  $k_4 = 10$  in the position and field current controller,  $\gamma_i = 0.1$ ,  $i = 1 \sim 3$  in the adaptive law.

Fig. 2 demonstrates that the adaptive nonlinear backstepping controller tracks accurately position well under the velocity constraint.

From the simulation, it can be seen that the proposed adaptive nonlinear backstepping control with feedback linearization can improve performance and the transient stability.

In this example,  $k_i$ ,  $i = 1 \sim 5$ , influences the convergence time to all errors, respectively; however, they also influence the control gain of  $u_a$  and  $u_f$ . The parameter  $\gamma$  is the estimated rate of external load. If  $\gamma$  is chosen to be small, the load convergence can be achieved; however, this results in slow speed. On the other hand, if  $\gamma$  is chosen to be large, the convergence speed is fast; however, the estimated algorithm may become unstable.

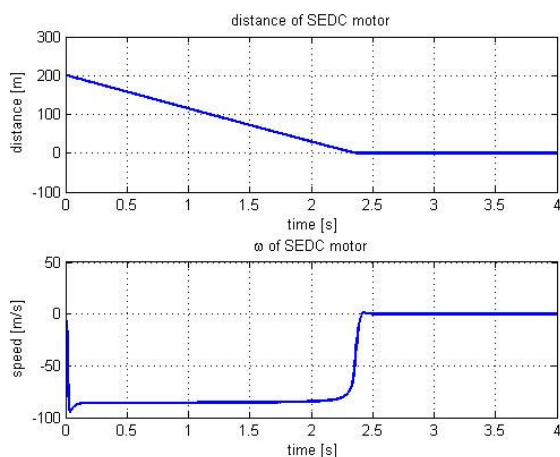


Fig. 1 Distance and Speed of BLDCM with initial condition  $z_1(0) = 200m$ ,  $z_2(0) = z_3(0) = z_4(0) = 0$

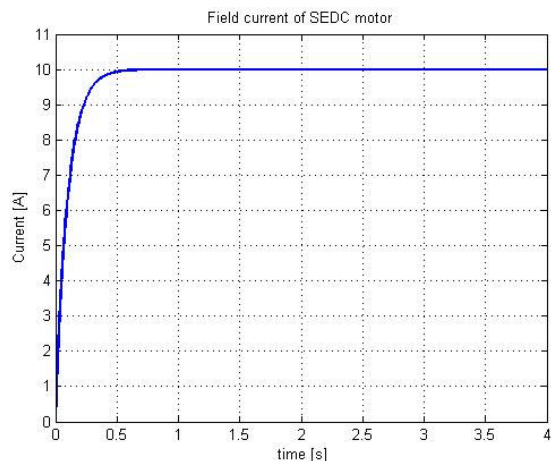


Fig. 2 Field current of BLDCM

## VI. CONCLUSION

An adaptive nonlinear backstepping controller is proposed to reduce the tracking error and to improve the transient performance and stability of BLDCM position control system. Dynamic motion tracking problem of BLDCM with velocity constraint is solved by the proposed controller. The design procedure of the controller proposed in this paper is independent of the operating point. Simulation on this model has shown that the proposed controller can greatly enhance the transient stability of the system regardless of the operating point, parameter variation and load disturbance.

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