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Simulink Approach to Solve Fuzzy Differential Equation under Generalized Differentiability

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Abstract—In this paper, solution of fuzzy differential equation under general differentiability is obtained by simulink. The simulink solution is equivalent or very close to the exact solution of the problem. Accuracy of the simulink solution to this problem is qualitatively better. An illustrative numerical example is presented for the proposed method.

Keywords—Fuzzy differential equation, Generalized differentiability, H-difference and Simulink.

I. Introduction

FUZZY set theory is a powerful tool for modelling uncertainty and for processing vague or subjective information in mathematical models. The main directions of development of this subject have been diverse with applications to variety of real problems like the golden mean [9], quantum optics, gravity [11], synchronize hyperchaotic systems [24], chaotic system, medicine [2], [4], and engineering problems [15]. Particularly, fuzzy differential equation is an important topic from the theoretical point of view (see [1], [12], [17], [18]) as well as its applications like in population models [13], [14], civil engineering and hydraulics.

Differentiable fuzzy valued mappings were initially studied by Puri and Ralescu [19]. They generalized and extended the concept of Hukuhara differentiability (H-derivative) for set valued mappings to the class of fuzzy mappings. Subsequently, using H-derivative, Kaleva [16] started to develop a theory for fuzzy differential equations.

In the last few years, many works have been done by several authors in theoretical and applied fields for fuzzy differential equations with H-derivative (see [20], [21], [22], [23]). Now, in some cases this approach suffers certain disadvantages since the diameter diam(x(t)) of the solution is unbounded as time t increases [10]. This problem demonstrates that in some case this interpretation is not a good generalization of the associated crisp case.

The generalized differentiability was introduced and studied in [5], [6], [7], [8]. This concept allows us to resolve the above mentioned shortcoming. Indeed, the generalized derivative is defined for a larger class of fuzzy number valued functions than Hukuhara derivative. Hence, this differentiability concept is used in the present paper. Under appropriate conditions, the fuzzy initial value problem considered under this interpretation has locally two solutions. In this paper, simulink approach is used to compute the solution of fuzzy differential equation.

Simulink is a MATLAB add-on package that many professional engineers use to model dynamical processes in control systems. Simulink allows to create a block diagram representation of a system and run simulations very easily. Simulink is really translating block diagram into a system of ordinary differential equations. Simulink is the tool of choice for control system design, digital signal processing (DSP) design, communication system design and other simulation applications [3]. This paper focuses upon the implementation of simulink approach for solving fuzzy differential equation.

This paper is organized as follows. In section 2, the basic concepts and fuzzy differential equation are described. In section 3, simulink method is presented. In section 4, numerical example is discussed. The final conclusion section demonstrates the efficiency of the method.

II. Basic concepts and Fuzzy differential equation

Let X be a nonempty set. A fuzzy set u in X is characterized by its membership function $u : X \to$ [0,1]. Then $u(x)$ is interpreted as the degree of membership of a element x in the fuzzy set u for each $x \in X$.

Definition 2.1: Let $\mathcal{F}^n$ be the space of all compact and convex fuzzy sets on $\mathbb{R}^n$. Let $u, v \in \mathcal{F}^n$. If there exists $w \in \mathcal{F}^n$ such that $u = v \oplus w$, then w is called the H-difference of u and v and it is denoted by $u \ominus v$.

Definition 2.2: Let $F : T \to \mathcal{F}^n$ and $t_0 \in T$. The function F is said to be differentiable at t_0 if

(I) an element $F'(t_0) \in \mathcal{F}^n$ exist such that, for all $h > 0$ sufficiently near 0, there are $F(t_0+h) \ominus F(t_0), F(t_0) \ominus F(t_0 - h)$ and the limits

$$\lim_{h\to 0^+} \frac{F(t_0+h)\ominus F(t_0)}{h} = \lim_{h\to 0^+} \frac{F(t_0)\ominus F(t_0-h)}{h}$$

are equal to $F'(t_0)$.

(or)

(II) there is an element $F'(t_0) \in \mathcal{F}^n$ exist such that, for all $h < 0$ sufficiently near 0, there are $F(t_0+h) \ominus F(t_0), F(t_0) \ominus F(t_0 - h)$ and the limits

$$\lim_{h\to 0^-} \frac{F(t_0+h)\ominus F(t_0)}{h} = \lim_{h\to 0^-} \frac{F(t_0)\ominus F(t_0-h)}{h}$$

are equal to $F'(t_0)$

Note that if F is differentiable in the first form (I), then it is not differentiable in the second form (II) and viceversa.

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Given that $F : T \to \mathcal{F}$ is a function and $[F(t)]^\alpha = [f_\alpha(t), g_\alpha(t)]$, for each $\alpha \in [0, 1]$. The following result is the fundamental for solving a fuzzy differential equation.

Theorem 2.3: Let $F : T \to \mathcal{F}$ be a function. Then

(i) If F is differentiable in the first form (I), then f_α and g_α are differentiable functions and

$$[F'(t)]^\alpha = [f'_\alpha(t), g'_\alpha(t)]. \tag{1}$$

(ii) If F is differentiable in the second form (II), then f_α and g_α are differentiable functions and

$$[F'(t)]^\alpha = [g'_\alpha(t), f'_\alpha(t)]. \tag{2}$$

Theorem 2.4: Let $F : T \to \mathcal{F}$ be a continuous function. Then

(i) If F is differentiable in the first form (I), then F' is integrable if and only if $F(a) \prec F(t)$ for all $t \in T$.

(ii) If F is differentiable in the second form (II), then F' is integrable if and only if $F(t) \prec F(a)$ for all $t \in T$.

A. *Fuzzy differential equation*

Consider the fuzzy differential equation

$$x' = F(t, x(t)), \quad x(a) = x_0, \tag{3}$$

where $F : [a, b] \times \mathcal{F} \to \mathcal{F}$ is a continuous fuzzy mapping and x_0 is a fuzzy interval.

The solution of the fuzzy differential equation (3) is dependent of the choice of the derivative: in the first form or in the second form. The equations (1) and (2) in Theorem 1 give us an useful procedure to solve the fuzzy differential equation (3). For this, let

$$[x(t)]^\alpha = [u_\alpha(t), v_\alpha(t)]$$

and

$$[F(t, x(t))]^\alpha = [f_\alpha(t, u_\alpha(t), v_\alpha(t)), g_\alpha(t, u_\alpha(t), v_\alpha(t))].$$

Example 2.1: Let us consider the fuzzy differential equation

$$x'(t) = -\lambda x(t), \quad x(0) = x_0, \tag{4}$$

where $\lambda > 0$ and the initial condition x_0 is a symmetric triangular fuzzy number with support $[-a, a]$. That is,

$$[x_0]^\alpha = [-a(1-\alpha), a(1-\alpha)] = (1-\alpha)[-a, a].$$

If $x'(t)$ is considered in the first form(I), the fuzzy differential system will be as given below:

$$u'_\alpha(t) = -\lambda v_\alpha(t), \quad u_\alpha(0) = -a(1-\alpha)$$
$$v'_\alpha(t) = -\lambda u_\alpha(t), \quad v_\alpha(0) = a(1-\alpha).$$

The solution of this system is $u_\alpha(t) = -a(1-\alpha)e^{\lambda t}$ and $v_\alpha(t) = a(1-\alpha)e^{\lambda t}$. Therefore, the fuzzy function x(t) solving (4) has level sets

$$[x(t)]^\alpha = [-a(1-\alpha)e^{\lambda t}, a(1-\alpha)e^{\lambda t}]$$

for all $t \geq 0$.

If $x'(t)$ is considered in the second form(II), the fuzzy differential system will be as given below:

$$u'_\alpha(t) = -\lambda u_\alpha(t), \quad u_\alpha(0) = -a(1-\alpha)$$
$$v'_\alpha(t) = -\lambda v_\alpha(t), \quad v_\alpha(0) = a(1-\alpha).$$

The solution of this system is $u_\alpha(t) = -a(1-\alpha)e^{-\lambda t}$ and $v_\alpha(t) = a(1-\alpha)e^{-\lambda t}$. Therefore, the fuzzy function x(t) solving (4) has level sets

$$[x(t)]^\alpha = [-a(1-\alpha)e^{-\lambda t}, a(1-\alpha)e^{-\lambda t}]$$

for all $t \geq 0$.

III. Simulink Method

Simulink is an interactive tool for modelling, simulating and analyzing dynamic systems. It enables engineers to build graphical block diagrams, evaluate system performance and refine their designs. Simulink integrates seamlessly with MATLAB and is tightly integrated with state flow for modelling event driven behavior. Simulink is built on top of MATLAB. A Simulink model for the given problem can be constructed using building blocks from the simulink library. The solution curves can be obtained from the model without writing any codes.

A simulink model is constructed for the following system of two differential equations as shown in the Figure 1.

$$\begin{aligned} x'(t) &= -2 * x(t) - 1, \quad x(0) = -1 \\ y'(t) &= -2 * y(t) - 1, \quad y(0) = 1. \end{aligned}$$

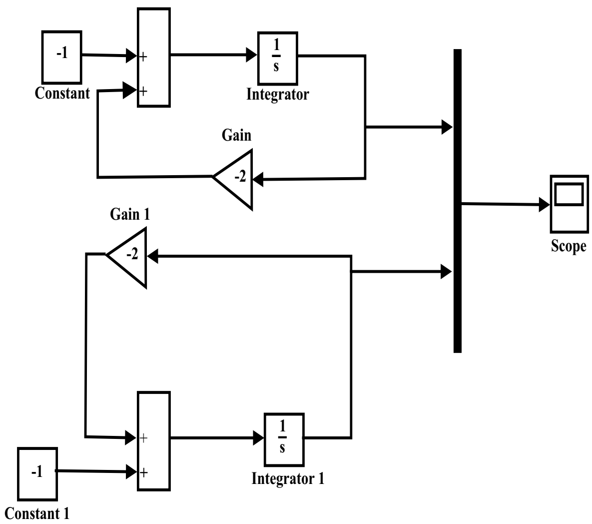

Fig. 1. Simulink model

As soon as the model is constructed, the simulink parameters can be changed according to the problem. The solution of the system of differential equation can be obtained in the display block by running the model.

A. *Procedure for Simulink Solution*

Step 1. Select the required number of blocks from the simulink Library.
Step 2. Connect the appropriate blocks.
Step 3. Make the required changes in the simulation parameters.
Step 4. Run the simulink model to obtain the solution.

IV. Numerical Example

Consider the fuzzy differential equation

$$x'(t) = -x(t) + 1, \quad x(0) = x_0,$$

where $\lambda > 0$ and the initial condition x_0 is a symmetric triangular fuzzy number with support $[-1, 1]$. That is,

$$[x_0]^\alpha = [-(1-\alpha), (1-\alpha)] = (1-\alpha)[-1, 1].$$

If $x'(t)$ is considered in the first form(I), the fuzzy differential system will be as given below:

$$u'_\alpha(t) = -v_\alpha(t) + 1, \quad u_\alpha(0) = -(1-\alpha)$$
$$v'_\alpha(t) = -u_\alpha(t) + 1, \quad v_\alpha(0) = (1-\alpha).$$

If $x'(t)$ is considered in the second form(II), the fuzzy differential system will be as given below:

$$u'_\alpha(t) = -u_\alpha(t) + 1, \quad u_\alpha(0) = -(1-\alpha)$$
$$v'_\alpha(t) = -v_\alpha(t) + 1, \quad v_\alpha(0) = (1-\alpha).$$

A. *Solution obtained using Simulink*

The simulink model is constructed for the above systems of differential equations. The simulink models are shown in Figures 2 and 3. The simulink curves for the systems are shown in Figures 4 and 5 when $\alpha = 0.4$.

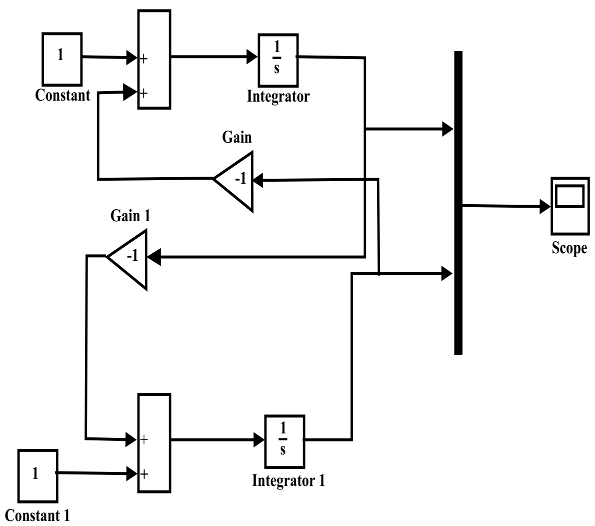

Fig. 2. Simulink model

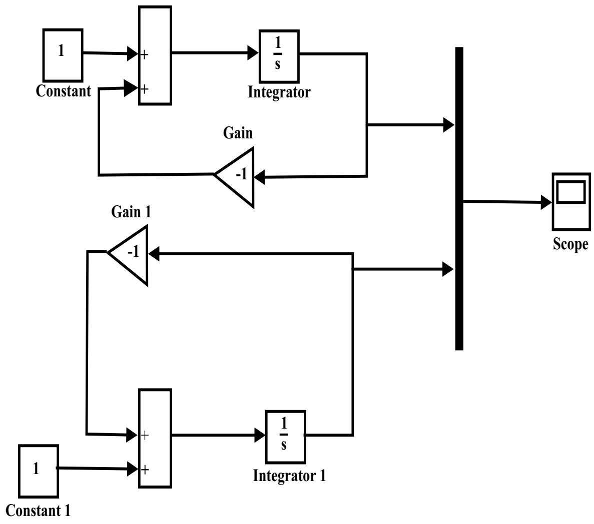

Fig. 3. Simulink model

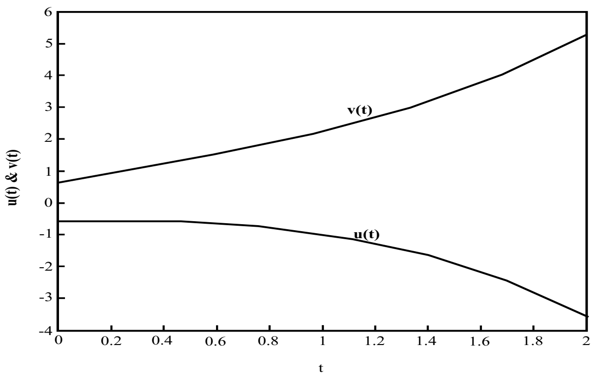

Fig. 4. Simulink curve for the first system

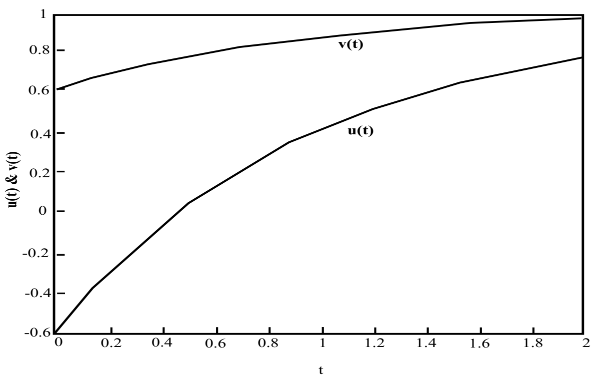

Fig. 5. Simulink curve for the second system

The simulink solution of I type fuzzy differential equation is displayed in Table 1.

The simulink solution of II type fuzzy differential equation is displayed in Table 2.

V. Conclusion

The solution of fuzzy differential equation under generalized differentiability can be computed by using simulink approach.

TABLE I
SOLUTIONS OF I SYSTEM

t	α =0.2		α =0.4	
	u_α	v_α	u_α	v_α
0.0	-0.8000	0.8000	-0.6000	0.6000
0.2	-0.7959	1.1584	-0.5516	0.9141
0.4	-0.8638	1.5231	-0.5654	1.2248
0.6	-1.0065	1.9089	-0.6421	1.5445
0.8	-1.2298	2.3311	-0.7847	1.8860
1.0	-1.5425	2.8067	-0.9988	2.2631
1.2	-1.9573	3.3549	-1.2933	2.6909
1.4	-2.4908	3.9976	-1.6797	3.1865
1.6	-3.1643	4.7605	-2.1737	3.7699
1.8	-4.0050	5.6744	-2.7951	4.4645
2.0	-5.0466	6.7759	-3.5688	5.2981

TABLE II
SOLUTIONS OF II SYSTEM

t	α =0.2		α =0.4	
	u_α	v_α	u_α	v_α
0.0	-0.8000	0.8000	-0.6000	0.6000
0.2	-0.4737	0.8363	-0.3100	0.6725
0.4	-0.2066	0.8659	-0.0725	0.7319
0.6	0.0121	0.8902	0.1219	0.7805
0.8	0.1912	0.9101	0.2811	0.8203
1.0	0.3378	0.9264	0.4114	0.8528
1.2	0.4579	0.9398	0.5181	0.8795
1.4	0.5561	0.9507	0.6054	0.9014
1.6	0.6366	0.9596	0.6770	0.9192
1.8	0.7025	0.9669	0.7355	0.9339
2.0	0.7564	0.9729	0.7835	0.9459

The simulink solution is equivalent or close to the exact solution of the problem. A numerical example is given to illustrate the derived results. In future, simulink approach can be used to solve linear and nonlinear stochastic differential equation in the fuzzy environment.

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REFERENCES

[1] S. Abbasbandy, J. J. Nieto, M. Alavi, Tuning of reachable set in one dimensional fuzzy differential inclusions, *Chaos, Solitons & Fractals*, **26** (2005), 1337–1341.

[2] M. F. Abbod, D. G. Von Keyserlingk, D. A. Linkens, M. Mahfouf, Survey of utilisation of fuzzy technology in medicine and healthcare, *Fuzzy Sets and Systems*, **120** (2001), 331–349.

[3] Albagul, Othman O. Khalifa, Wahyudi, Matlab and Simulink in Mechatranics, *Int. J. engng Ed.*, **21** (2005), 896–905.

[4] S. Barro, R. Marín, *Fuzzy logic in medicine*, Heidelberg: Physica-Verlag, 2002.

[5] B. Bede, S. G. Gal, Almost periodic fuzzy number valued functions, *Fuzzy Sets and Systems*, **147** (2004), 385–403.

[6] B. Bede, S. G. Gal, Generalizations of the differentiability of fuzzy number valued functions with applications to fuzzy differential equations, *Fuzzy Sets and Systems*, **151** (2005), 581–599.

[7] B. Bede, I. J. Rudas, A. L. Bencsik, First order linear fuzzy differential equations under generalized differentiability, *Inform. Sciences*, **177** (2007), 1648–1662.

[8] Y. Chalco-Cano, H. Román-Flores, On new solutions of fuzzy differential equations, *Chaos, Solitons & Fractals*, **38** (2008), 112–119.

[9] D. P. Datta, The Golden mean, scale free extension of real number system, fuzzy sets and 1/f spectrum in physics and biology, *Chaos, Solitons & Fractals*, **17** (2003), 781–788.

[10] P. Diamond, Brief note on the variation of constants formula for fuzzy differential equations, *Fuzzy Sets and Systems*, **129** (2002), 65–71.

[11] M. S. El Naschie, From experimental quantum optics to quantum gravity via a fuzzy Kähler manifold, *Chaos, Solitons & Fractals*, **25** (2005), 969–977.

[12] T. Gnana Bhaskar, V. Lakshmikantham, V. Devi, Revisiting fuzzy differential equations, *Nonlinear Analysis*, **58** (2004), 351–358.

[13] M. Guo, X. Xue, R. Li, The oscillation of delay differential inclusions and fuzzy biodynamics models, *Math. Comput. Model.*, **37** (2003), 651–658.

[14] M. Guo, R. Li, Impulsive functional differential inclusions and fuzzy population models, *Fuzzy Sets and Systems*, **138** (2003), 601–615.

[15] M. Hanss, *Applied fuzzy arithmetic: An introduction with engineering applications*, Springer-Verlag, Berlin, 2005.

[16] O. Kaleva, Fuzzy differential equations, *Fuzzy Sets and Systems*, **24** (1987), 301–317.

[17] O. Kaleva, A note on fuzzy differential equations, *Nonlinear Analysis*, **64** (2006), 895–900.

[18] J. J. Nieto, R. Rodríguez-López, Bounded solutions for fuzzy differential and integral equations, *Chaos, Solitons & Fractals*, **27** (2006), 1376–1386.

[19] M. Puri, D. Ralescu, Differential and fuzzy functions, *J. Math. Anal. Appl.*, **91** (1983), 552–558.

[20] S. Song, Lei Guo, Chumbo Feng, Global existence of solutions to fuzzy differential equations, *Fuzzy Sets and Systems*, **115** (2000), 371–376.

[21] S. Song, C. Wu, Existence and uniqueness of solutions to Cauchy problem of fuzzy differential equations, *Fuzzy Sets and Systems*, **110** (2000), 55–67.

[22] C. X. Wu, S. Song, Stanley Lee, Approximate solutions, existence and uniqueness of the Cauchy problem of fuzzy differential equations, *J. Math. Anal. Appl.*, **202** (1996), 629–644.

[23] C. X. Wu, S. Song, Existence theorem to the Cauchy problem of fuzzy differential equations under compactness-type conditions, *Inform. Sciences*, **108** (1998), 123–134.

[24] H. Zhang, X. Liao, J. Yu, Fuzzy modeling and synchronization of hyperchaotic systems, *Chaos, Solitons & Fractals*, **26** (2005), 835–843.

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