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Soliton Interaction in Birefringent Fibers with Third-Order Dispersion

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***Abstract*—**Propagation of solitons in single-mode birefringent fibers is considered under the presence of third-order dispersion (TOD). The behavior of two neighboring solitons and their interaction is investigated under the presence of third-order dispersion with different group velocity dispersion (GVD) parameters. It is found that third-order dispersion makes the resultant soliton to deviate from its ideal position and increases the interaction between adjacent soliton pulses. It is also observed that this deviation due to third-order dispersion is considerably small when the optical pulse propagates at wavelengths relatively far from the zero-dispersion. Modified coupled nonlinear Schrödinger's equations (CNLSE) representing the propagation of optical pulse in single mode fiber with TOD are solved using split-step Fourier algorithm. The results presented in this paper reveal that the third-order dispersion can substantially increase the interaction between the solitons, but large group velocity dispersion reduces the interaction between neighboring solitons.

***Keywords*—**Birefringence, Group velocity dispersion, Polarization mode dispersion, Soliton interaction, Third order dispersion.

I. Introduction

SINGLE-mode fibers are generally bimodal due to the existence of birefringence[1]. Polarization mode dispersion occurs because of the slight elliptical nature of the core in birefringent fibers. Hence the condition of having input polarization angle $\theta=0^o$ or 90^o is not satisfied in practice. As a result the propagating light is split into two local orthogonally polarized components that travel with different velocities results in group velocity mismatch between them. The resulting difference in propagation times between the two polarization modes is known as differential group delay (DGD) $\Delta\tau$. The dispersion-induced pulse broadening is due to the lowest-order group velocity dispersion term D proportional to group velocity dispersion coefficient β_2. Although the contribution of this term dominates in most cases of practical interest, it is sometimes necessary to include the third-order dispersion term. When optical pulses propagate at relatively far from the zero-dispersion wavelength; the affects of third-order dispersion on solitons are almost negligible. For fundamental soliton, with order N=1, the soliton peak shifts linearly with distance. Physically speaking, the third-order dispersion slows down the soliton and, as a result, the soliton peak is delayed by an amount that increases linearly with distance.

It has been shown [2] that in the case of constant birefringence, the two partial pulses in each of the polarization states are in each other's frequency shift, such that any initial difference in group velocity is eliminated, as a result they are self-trapped [3]. When the pulses propagate at a wavelength in the vicinity of the zero group velocity dispersion, the third-order dispersion plays a crucial role and is potentially a major impairment for soliton transmission system. For a nonlinear pulse, in particular of soliton, its propagation in the zero-dispersion region of the single-mode fiber has been the object of much interest in the past decade.

Solitons have been considered best information carriers for very large bandwidth optical fiber communication systems. However, the mutual interaction between the neighboring solitons increases with polarization mode dispersion [3], so that the useful bandwidth of the optical fibers reduces. It is further found in [4] that, if the fiber is operated near zero-dispersion wavelength, where the second-order dispersion (GVD) vanishes, soliton pulse gets broadened due to third-order dispersion. This further reduces the useful bandwidth.

In this paper, we study numerically the propagation of solitons and their interaction due to third-order dispersion. The remainder of this paper is organized as follows. In Section II the mathematical model of the pulse propagation in fibers with third-order dispersion is formulated. Simulation results and discussions are presented in Section III and finally Section IV concludes the paper.

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II. Mathematical Model of the Fiber with Third-order Dispersion

In this paper we use the slowly varying envelop approximation of fibers by considering third-order dispersion. It is found that the wave envelopes of electric field in each of the polarizations satisfy the following modified coupled equations, which can be derived from Maxwell's equations using the reductive perturbation method.

$$\frac{\partial A_x}{\partial z}+k_1\frac{\partial A_x}{\partial t}+\frac{i}{2}k_2\frac{\partial^2 A_x}{\partial t^2}-\frac{i}{6}k_3\frac{\partial^3 A_x}{\partial t^3}+\frac{\alpha}{2}A_x$$
$$=i\gamma\left(|A_x|^2+\frac{2}{3}|A_y|^2\right)A_x+\frac{i\gamma}{3}A_x^*A_y^2\exp(-2i\Delta\beta z) \quad (1a)$$

$$\frac{\partial A_y}{\partial z}+l_1\frac{\partial A_x}{\partial t}+\frac{i}{2}l_2\frac{\partial^2 A_y}{\partial t^2}-\frac{i}{6}l_3\frac{\partial^3 A_y}{\partial t^3}+\frac{\alpha}{2}A_y$$
$$=i\gamma\left(|A_y|^2+\frac{2}{3}|A_x|^2\right)A_y+\frac{i\gamma}{3}A_y^*A_x^2\exp(2i\Delta\beta z) \quad (1b)$$

where $k_n=\frac{d^n k}{d\omega^n}$, $l_n=\frac{d^n l}{d\omega^n}$ $(n=1,2,3)$ are the derivatives of the dispersion relations in each of the polarization states, $\Delta\beta=k_0(\omega_0)-l_0(\omega_0)$ is the wave-vector mismatch due to linear birefringence of the fiber, γ is the nonlinearity coefficient and α is the attenuation coefficient. A_x and A_y are the wave amplitude envelops in each of the polarization states. The third-terms on the left-hand side of Eqs.(1) represent the second order dispersion terms and fourth terms are responsible for third-order dispersion. The last terms on the right-hand side of Eqs.(1) are due to coherent coupling between two polarization components which lead to degenerate four-wave mixing. If the fiber length L is much larger than the beat length L_B, the sign of the last term in (1) rapidly changes and its contribution averages out to zero. For highly birefringent fibers (L_B~1cm), the four-wave-mixing term can often be neglected for this reason [1].

In anomalous-dispersion regime, by neglecting the last terms in Eqs.(1), the normalized equations can be obtained as follows:

$$\frac{\partial U}{\partial Z}+\delta\frac{\partial U}{\partial T}-\frac{i}{2}\frac{\partial^2 U}{\partial T^2}-\frac{\beta}{6}\frac{\partial^3 U}{\partial T^3}+\Gamma U=i\left(|U|^2+\frac{2}{3}|V|^2\right)U \quad (2a)$$

$$\frac{\partial V}{\partial Z}-\delta\frac{\partial V}{\partial T}-\frac{i}{2}\frac{\partial^2 V}{\partial T^2}-\frac{\beta}{6}\frac{\partial^3 V}{\partial T^3}+\Gamma V=i\left(|V|^2+\frac{2}{3}|U|^2\right)V \quad (2b)$$

where $\delta=\frac{k_1-l_1}{2k_2}\tau=\frac{L_d\Delta n}{2ct_0}=\frac{\Delta n t_0\pi}{\lambda^2 D}$, $Z=\frac{|k_2|z}{\tau^2}$,

$$T=\frac{1}{\tau}\left(t-\frac{z}{v_g}\right),\ A_x=\left|\frac{k_2}{\gamma\tau^2}\right|^{1/2}U,\ A_y=\left|\frac{k_2}{\gamma\tau^2}\right|^{1/2}V,\ \beta=\frac{k_3}{\tau k_2},$$

$$\Gamma=\frac{\alpha\tau^2}{2k_2},\ v_g=\frac{2}{k_1+l_1}.$$

L_d is the dispersion length related to the group velocity dispersion parameter D as $L_d=2\pi ct_0^2/(\lambda^2 D)=t_0^2/\beta_2$, $\beta_2=D\frac{\lambda^2}{2\pi c}$ is called the group velocity dispersion coefficient in ps^2/km, β is the third-order dispersion parameter in ps^3/km, Δn is the constant birefringence, t_0 is the initial pulse width in ps. D is the group velocity dispersion parameter in $ps/km.nm$, τ is the full width at half maximum (FWHM) of pulse intensity and for a hyperbolic secant pulse it is given by $\tau=1.763t_0$.

The usual model of birefringent fiber can be considered as cascade of many short fibers with constant birefringence [5], [6]. We assume that all the fiber pieces have identical length L_c, which is the mode coupling length, and identical magnitude of linear birefringence Δn. With this model, the polarization-mode dispersion parameter in $ps/\sqrt{km}$ is given by $D_p=\sqrt{8L_c/(3\pi)}(\Delta n/c)=\langle\Delta\tau\rangle/\sqrt{z}$ where c is the velocity of light and $\langle\Delta\tau\rangle$ is the average differential group delay (DGD) [7]. The equations under study are the perturbed modified coupled nonlinear Schrödinger equations which are not integrable using the techniques of the inverse scattering theory. However numerical simulations using split-step Fourier algorithm can be performed to study the effects of third-order dispersion on soliton propagation in single-mode fibers. In this algorithm the fiber is subdivided into small sections in which the dispersion and the nonlinearity can be taken into account individually in time domain and frequency domain respectively [8].

III. Simulation Results and Discussions

We did comprehensive simulations using split-step Fourier algorithm to solve the modified coupled nonlinear Schrödinger's equations (2a) and (2b). Numerical results obtained show that, deviation of resultant soliton from its ideal position and interaction between adjacent solitons enhance with increase in third order-dispersion parameter. However if the fibers are operated at GVD parameters other than those at zero-dispersion wavelength the interaction is considerably small.

A. Effects of TOD and GVD on single soliton pulse

We considered the single soliton pulse propagation by choosing the initial soliton conditions, of hyperbolic secant pulse profiles, given by

$$U(Z=0)=N\cos(\theta)\operatorname{sech}(T) \quad (3a)$$
$$V(Z=0)=N\sin(\theta)\operatorname{sech}(T) \quad (3b)$$

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The angle θ is the initial polarization angle, which determines the relative strengths of the two partial polarizing components. N is the order of the soliton [4] ,[9]. Numerical analysis of single pulse propagation through birefringent optical fibers in the presence of third-order dispersion is carried out by solving (2a) and (2b) using split-step Fourier algorithm [10] with initial conditions given by (3a) and (3b) respectively.

We analyse the variation in threshold of soliton trapping under the presence of third-order dispersion and the interaction between the adjacent solitons by considering the following quantities of the pulse:

The resultant soliton amplitude R which is related to two polarizing components U, V by the expression

$$R = \sqrt{|U(Z,T)|^2 + |V(Z,T)|^2} \text{ and} \quad (4)$$

The separation between the maximum locations of two polarizing components given by

$$\Delta T_{peak}(Z) = T^{U}_{peak}(Z) - T^{V}_{peak}(Z) \quad (5)$$

where $T^{U}_{peak}(Z)$ and $T^{V}_{peak}(Z)$ are the positions at which $U(Z,T)$ and $V(Z,T)$ have maximum values i.e. the Maximum locations of the two polarizing components $U(Z,T)$ and $V(Z,T)$. In our simulations, where the effect of GVD is analysed, we use the full width half maximum pulse width τ=4.452*ps* this gives an initial pulse width t_0 of 2.525*ps*, the normalized mode coupling length L_c/L_d=1, and the polarization-mode dispersion parameter of D_P = 0.2 $ps\sqrt{km}$. The length of the fast Fourier- transform vector, used in split-step Fourier algorithm, is taken as 1024.

Fig. 1 shows the evolution of resultant soliton amplitudes for different values of third-order dispersion parameters after travelling a normalized distance of 30 soliton periods (Z=15π). It is also observed that soliton trapping is more severe near zero-dispersion wavelength and large GVD counter balances this deviation of soliton due to third-order dispersion as shown in Fig. 2.

The surface plot and its 3-dimentional plot showing the variation in soliton amplitude over the normalized distance of 40 soliton periods (Z=20π) with the TOD parameters β=0 are depicted in Figs. 3(a) and 3(b) respectively and Figs. 4(a) and 4(b) depicts similar plots with β=1. From these plots it is observed that the soliton pulse deviates more from its reference position and sheds its energy with increase in TOD parameter β. Figs. 5(a) and 5(b) depicts the surface plot and its 3-dimensional plot representing the variation of soliton amplitude over the normalized distance Z=20π at zero-dispersion wavelength with β=0.6, where as Fig. 6(a) and 6(b) show the similar plots with GVD parameter D=1. These

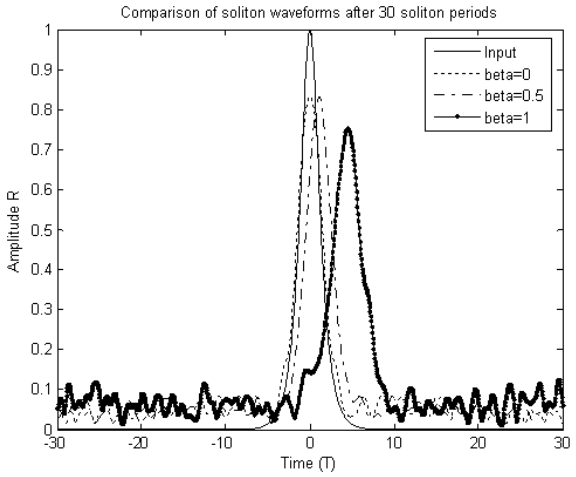


Fig. 1 Comparison of resultant soliton amplitudes for different values of β and Input soliton with δ=0.15 after 30 soliton periods (Z=15π).

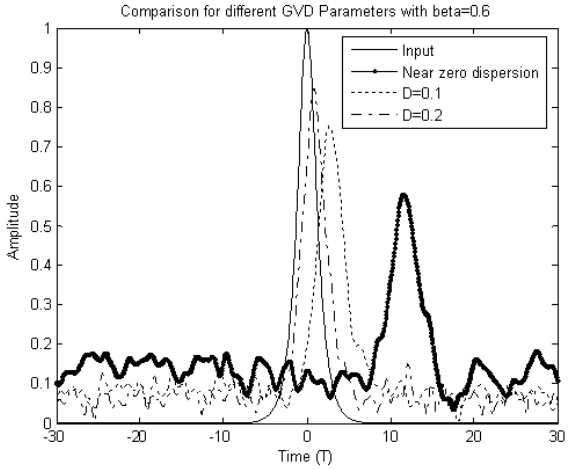


Fig. 2 Comparison of resultant soliton amplitudes for different values of group velocity dispersion parameters with β=0.6 after 30 soliton periods (Z=15π).

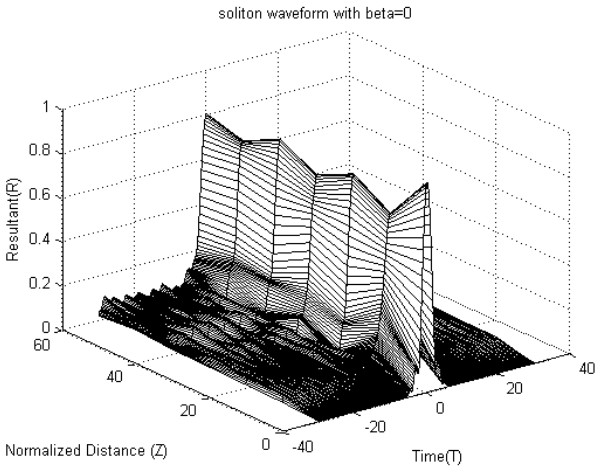


Fig. 3(a) Surface plot of resultant soliton amplitude without third-order dispersion parameter β=0 and δ=0.5 for 40 soliton periods (Z=20π).

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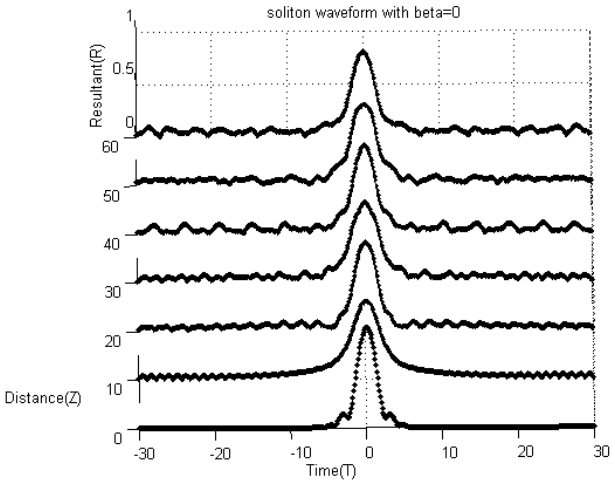


Fig. 3(b) 3-Dimentinal plot of resultant soliton amplitude without third-order dispersion parameter β=0 and δ=0.5 for 40 soliton periods (Z=20π).

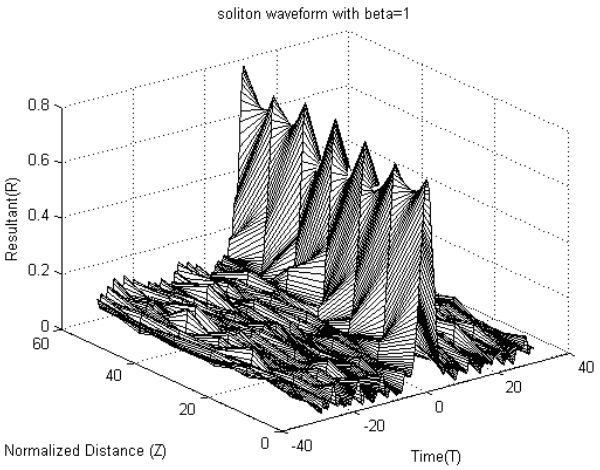


Fig. 4(a) Surface plot of resultant soliton amplitude without third-order dispersion parameter β=1 and δ=0.5 for 40 soliton periods (Z=20π).

results confirm that the affect of third-order dispersion is more prominent near zero-dispersion wavelength. But as the operating wavelength of the fibers deviate from zero-dispersion, the robustness of solitons to third-order dispersion can be improved significantly.

Fig. 7 illustrates the maximum locations of two polarizing components $T^{U}_{peak}(Z)$ and $T^{V}_{peak}(Z)$ along the length of fiber over a normalized distance of 30 soliton periods for different values of TOD parameters (β=0, 0.5 and 1) with δ=0.5. When there is no third-order dispersion (β=0) the maximum locations of U and V are close together, and deviates significantly with increase in the value of β. The separation between the maximum locations of the two polarizing components $\Delta T_{peak}(Z)$ decreases with increase in soliton order and illustrated in Fig. 8. These results are in agreement with the results obtained in [4], [6].

B. Soliton Interaction due to TOD

In our simulations to find the interaction between adjacent solitons we use two parallel polarized hyperbolic secant pulse profiles with normalized initial time shift of 5ps around the vicinity of origin as initial soliton conditions given by

$$U(Z=0) = N\left[\cos(\theta)\sec h(T-5) + \cos(\theta)\sec h(T+5)\right] \quad (6a)$$

$$V(Z=0) = N\left[\sin(\theta)\sec h(T-5) + \sin(\theta)\sec h(T+5)\right] \quad (6b)$$

The interaction between the neighbouring soliton pulses is analysed by using initial soliton pair conditions of (6a) and (6b) to solve the modified CNLSE (2a) and (2b) respectively.

From the numerical results obtained it can be seen that, interaction between the soliton becomes severe with increasing third-order dispersion parameter and it becomes worse at zero-dispersion wavelength. The initial normalized time shift between the solitons is assumed as 10ps which corresponds to a bit rate of 100$Gbps$. The dispersion length assumed is L_D=20km. this corresponds to an actual distance of approximately 1000km.if Z=15π and 1300km. if Z=20π.

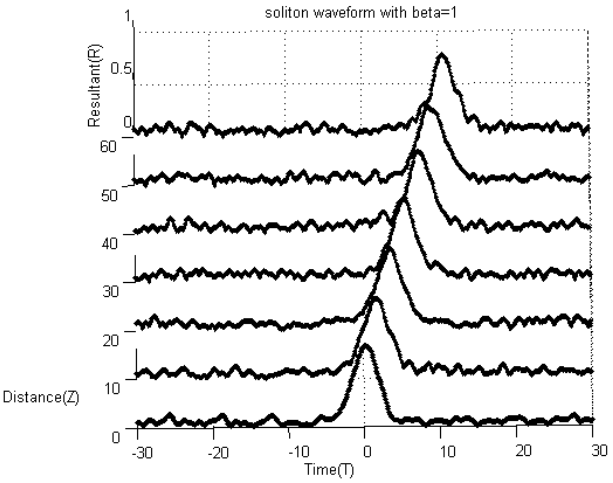


Fig. 4(b) 3-Dimentinal plot of resultant soliton amplitude with TOD parameter β=1 and δ=0.5 for 40 soliton periods (Z=20π).

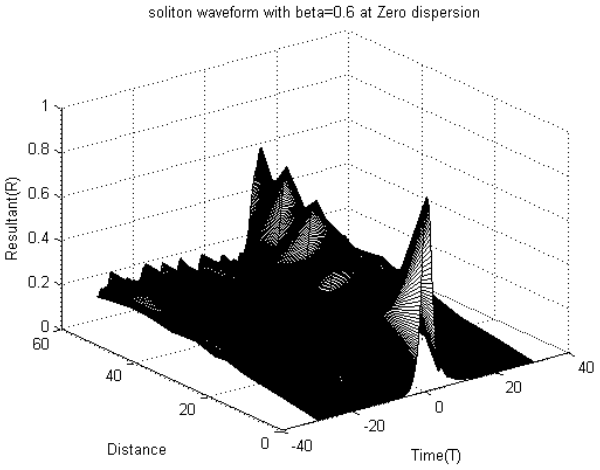


Fig. 5(a) Surface plot of resultant soliton amplitude at zero-dispersion wavelength with TOD parameter β=0.6 for 40 soliton periods (Z=20π).

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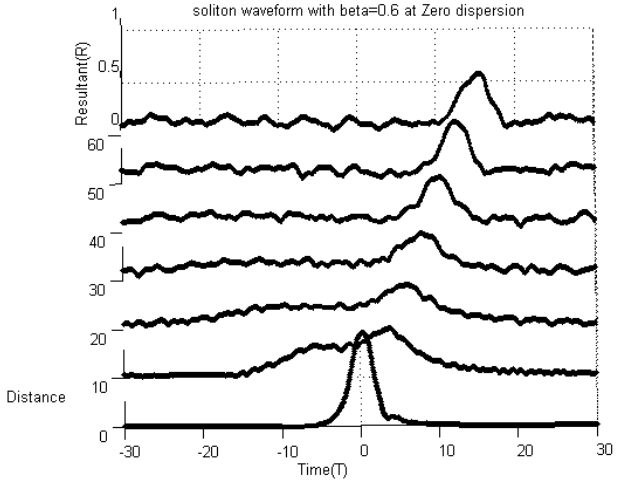


Fig. 5(b) 3-Dimentinal plot of resultant soliton amplitude at zero-dispersion wavelength with TOD parameter β=0.6 for 40 soliton periods (Z=20π).

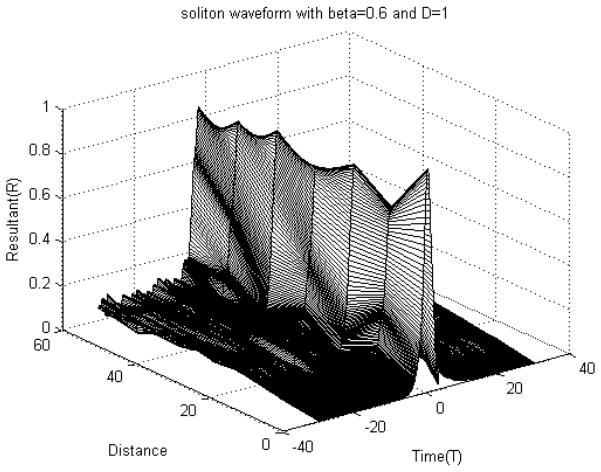


Fig. 6(a) 3-Dimentinal plot of resultant soliton amplitude with GVD parameter D=1 and TOD parameter β=0.6 for 40 soliton periods (Z=20π).

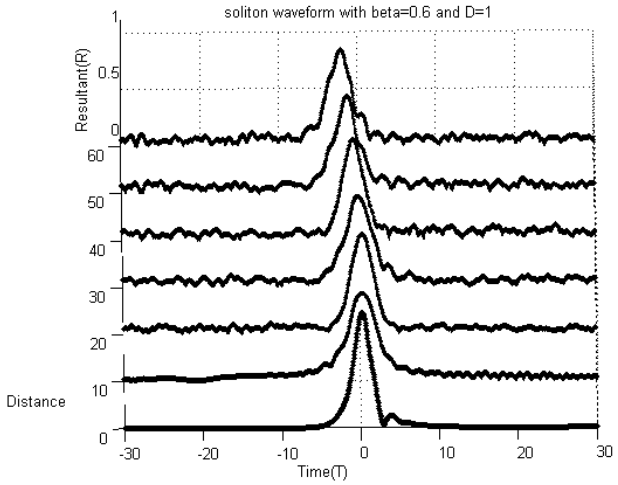


Fig. 6(b) 3-Dimentinal plot of resultant soliton amplitude with GVD parameter D=1 and TOD parameter β=0.6 for 40 soliton periods (Z=20π).

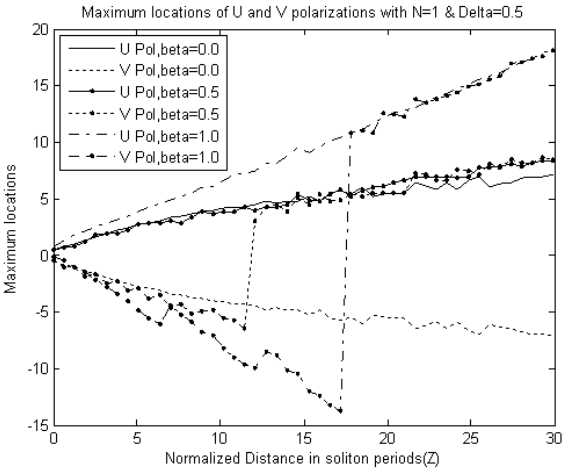


Fig. 7 Maximum locations of U and V components ($T^{U}_{peak}(Z)$, $T^{V}_{peak}(Z)$) with δ=0.5, soliton order N=1 and for different TOD parameters β=0.0, β=0.5, β=1.0.

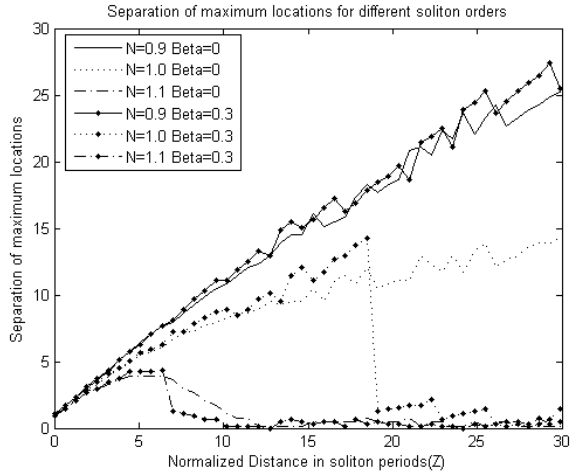


Fig. 8 Separation between the maximum locations of the two polarizing components $\Delta T_{peak}(Z)$ for different soliton orders N and β with δ=0.5

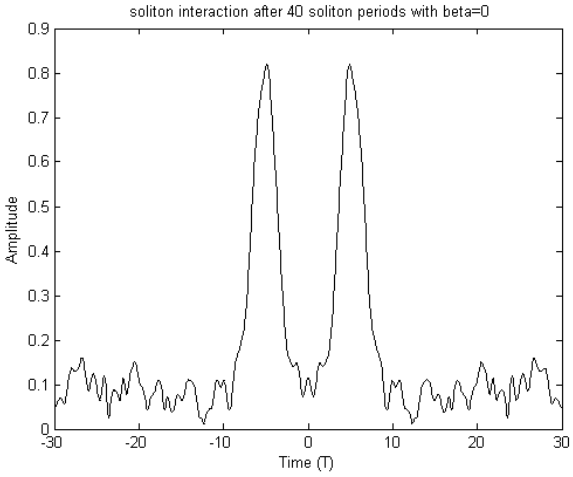


Fig. 9 Interaction between the adjacent soliton pulses without TOD (β=0) and δ=0.5 after 40 soliton periods (Z=20π).

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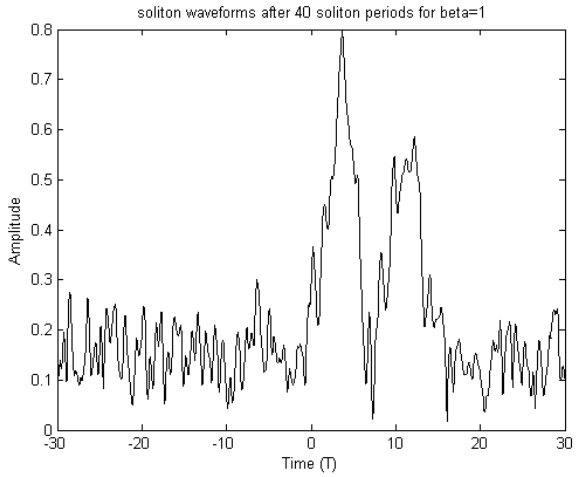


Fig.10 Interaction between the adjacent soliton pulses with TOD β=1 and δ=0.5 after 40 soliton periods (Z=20π).

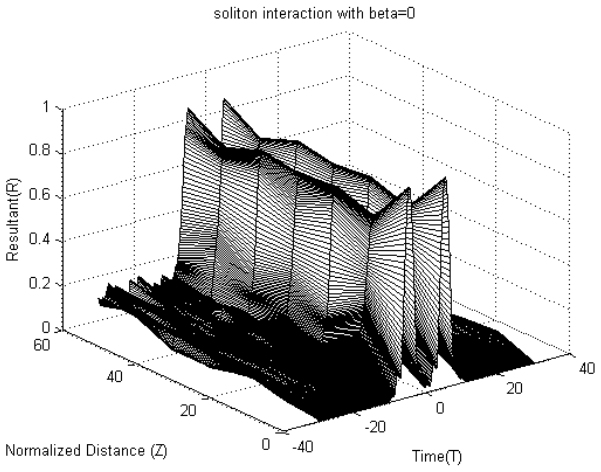


Fig. 11(a) Surface plot for interaction between the adjacent soliton pulses without TOD (β=0) and δ=0.5 after 40 soliton periods (Z=20π).

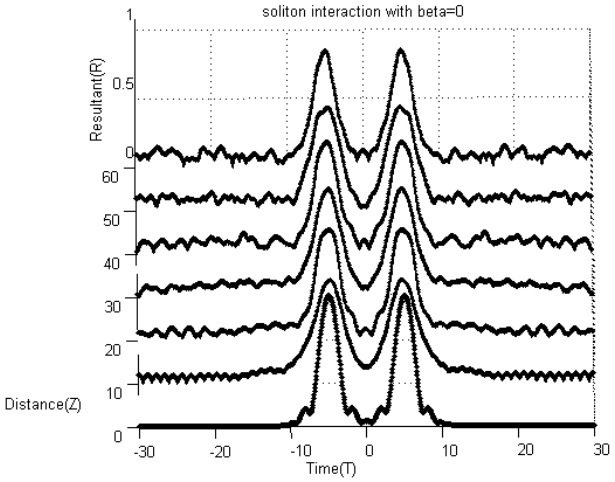


Fig. 11(b) 3-Dimentinal plot for interaction between the adjacent soliton pulses without TOD (β=0) and δ=0.5 after 40 soliton periods (Z=20π).

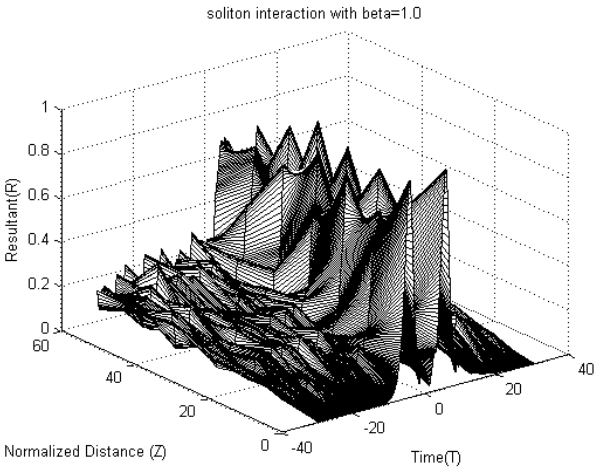


Fig. 12(a) Surface plot for interaction between the adjacent soliton pulses with TOD β=1 and δ=0.5 after 40 soliton periods (Z=20π).

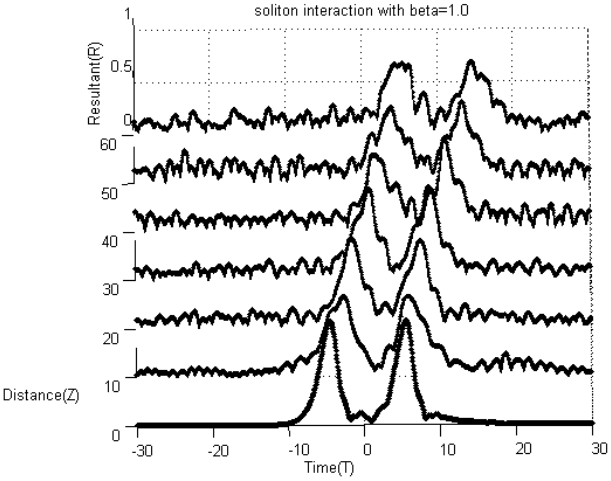


Fig. 12(b) 3-Dimentional plot for interaction between the adjacent soliton pulses with TOD β=1 and δ=0.5 after 40 soliton periods (Z=20π).

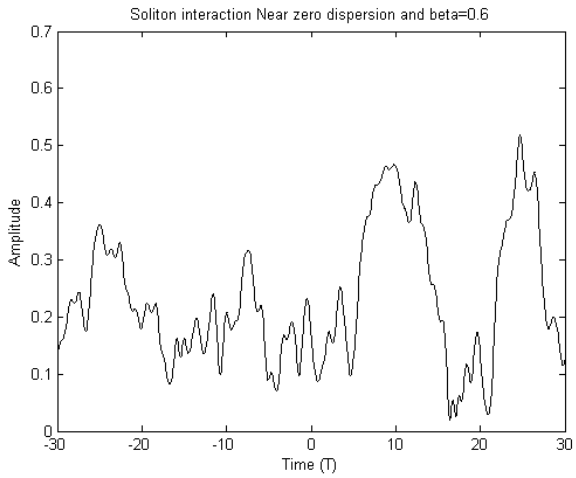


Fig. 13 Interaction between the adjacent soliton pulses at zero-dispersion wavelength with β=0.6 after 40 soliton periods (Z=20π).

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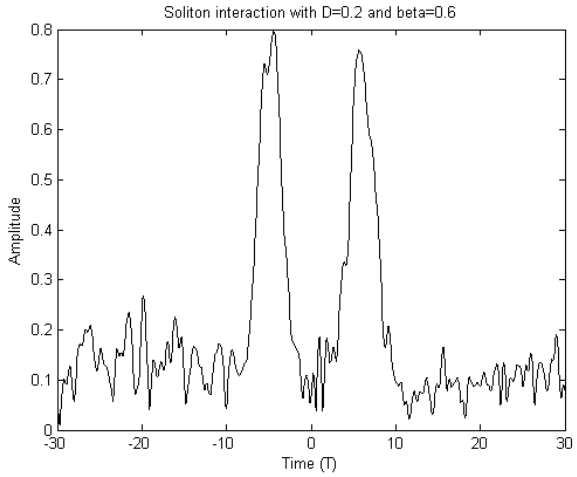


Fig. 14 Interaction between the adjacent soliton pulses with D=0.2 and β=0.6 after 40 soliton periods (Z=20π).

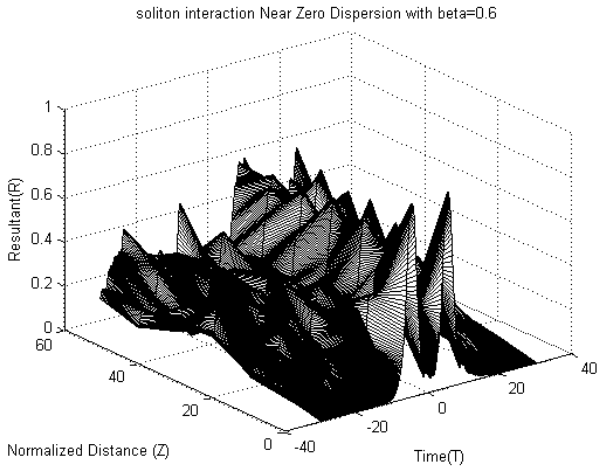


Fig. 15(a) Surface plot for interaction between the adjacent soliton pulses at zero-dispersion wavelength with β=0.6 after 40 soliton periods (Z=20π).

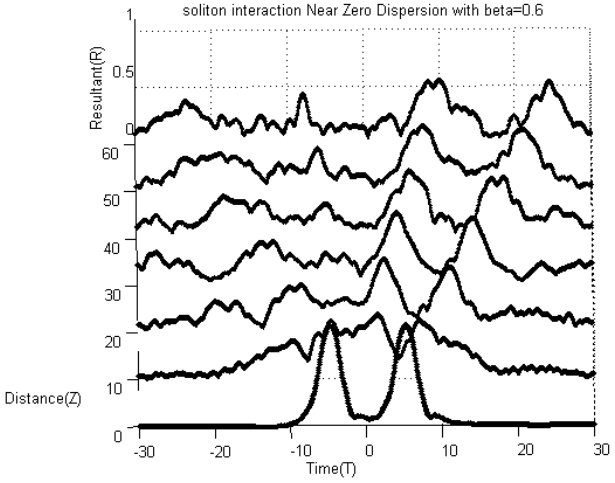


Fig. 15(b) 3-Dimentional plot for interaction between the adjacent soliton pulses at zero-dispersion wavelength with β=0.6 after 40 soliton periods (Z=20π).

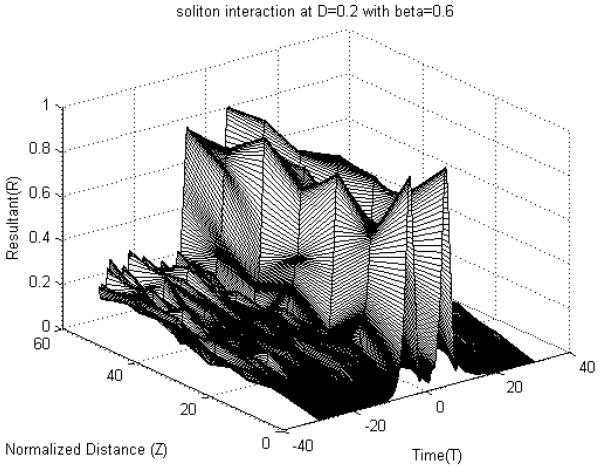


Fig. 16(a) Surface plot for interaction between the adjacent soliton pulses with D=0.2 and β=0.6 after 40 soliton periods (Z=20π).

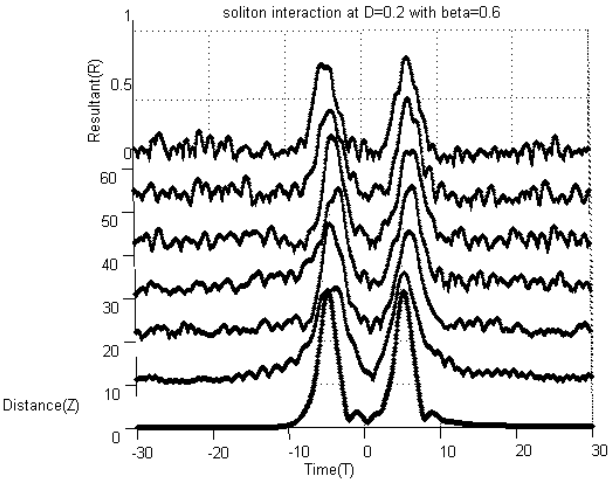


Fig. 16(b) 3-Dimentional plot for interaction between the adjacent soliton pulses with D=0.2 and β=0.6 after 40 soliton periods (Z=20π).

Fig. 9 is the plot showing the interaction between neighbouring solitons without third-order dispersion (β=0), and Fig. 10 is with third-order dispersion parameter β=1 after travelling a normalized distance of 40 soliton periods (Z=20π). The surface plot and its 3-dimentional plot showing the interaction of solitons without TOD (β=0) are given in Figs. 11(a) and 11(b) respectively, and the similar plots with β=1 are depicted in Figs. 12(a) and 12(b) respectively. From these results it can be observed that increase in third-order dispersion enhances the interaction between neighbouring solitons.

Fig. 13 shows the soliton interaction at zero-dispersion wavelength with third-order dispersion parameter β=0.6, and Fig. 14 shows with GVD parameter D=0.2. The surface plot and its 3-dimentional plot showing the interaction of solitons at zero-dispersion wavelength are given in Figs. 15(a) and 15(b) respectively, and the similar plots with D=0.2 are depicted in Figs. 16(a) and 16(b) respectively. From these results we observed that the soliton interaction due to TOD becomes severe at zero-dispersion wavelength, and it can be

reduced if the fiber optic system operates far from zero-dispersion wavelength.

IV. Conclusion

In this paper we have examined the behavior of optical pulses in single mode fibers with third-order dispersion. The modified coupled nonlinear Schrödinger equations, describing soliton propagation with third-order dispersion are numerically solved using split-step Fourier algorithm. From the simulation results it is observed that the resultant soliton deviates more from its reference position and hence enhances the interaction between neighboring solitons in the presence of third-order dispersion, than in its absence. It is further observed that the affects of TOD are severe near zero-dispersion wavelength and can be reduced by increasing GVD parameter. Hence we conclude that the soliton interaction in single mode fibers completely destroys the soliton condition near zero-dispersion, and hence increases the bit error rate. However at wavelengths other than zero-dispersion, the solitons are more robust to third-order dispersion.

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