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A Simplified Approach for Load Flow Analysis of Radial Distribution Network

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***Abstract*—**This paper presents a simple approach for load flow analysis of a radial distribution network. The proposed approach utilizes forward and backward sweep algorithm based on Kirchoff's current law (KCL) and Kirchoff's voltage law (KVL) for evaluating the node voltages iteratively. In this approach, computation of branch current depends only on the current injected at the neighbouring node and the current in the adjacent branch. This approach starts from the end nodes of sub lateral line, lateral line and main line and moves towards the root node during branch current computation. The node voltage evaluation begins from the root node and moves towards the nodes located at the far end of the main, lateral and sub lateral lines. The proposed approach has been tested using four radial distribution systems of different size and configuration and found to be computationally efficient.

***Keywords*—**constant current load, constant impedance load, constant power load, forward–backward sweep, load flow analysis, radial distribution system.

List of symbols

N_n = Total number of nodes in the given radial distribution network
N_b = Total number of branches in the given radial distribution network
N_l = Total number of lateral lines in the given radial distribution network
N_{sl} = Total number of sub lateral lines in the radial distribution network
N_m = Total number of minor lines in the given radial distribution network
en_M = Ending node number in the main line
Z_b = Impedance of the branch b
R_b = Resistance of the branch b
X_b = Reactance of the branch b
y_b = Line charging admittance of the branch b
V_n = Voltage at the node n
S_n = Complex load power at the node n
I_{Ln} = Load current injections at the node n
I_b = Branch current of the branch b
I_{cb} = Line charging current of the branch b
$I_{bl,l}$ = Branch current in the l^{th} lateral line
$I_{bsl,sl}$ = Branch current in the sl^{th} sub lateral line
$I_{bm,m}$ = Branch current in the m^{th} minor line
n_{Ml} = Node number in the main line from which this l^{th} lateral line begins
n_{lsl} = Node number in the lateral line from which this sl^{th} sub lateral begins
n_{slm} = Node number in the sub lateral line from which this m^{th} minor begins
b_{Ml} = Branch number connecting the main line with the l^{th} lateral line
b_{lsl} = Branch number connecting the lateral line with the sl^{th} sub lateral line
b_{slm} = Branch number connecting the sub lateral line with the m^{th} minor line
snl_l = Starting node number of l^{th} lateral line
$snsl_{sl}$ = Starting node number of sl^{th} sub lateral line
snm_m = Starting node number of m^{th} minor line
enl_l = Ending node number of l^{th} lateral line
$ensl_{sl}$ = Ending node number of sl^{th} sub lateral line
enm_m = Ending node number of m^{th} minor line
l = denotes a lateral line, l = 1,2,3,……………… N_l
sl = denotes a sub lateral line, sl =1,2,3…..………. N_{sl}
m = denotes a minor line, m = 1,2,3………………. N_m

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I. INTRODUCTION

THOUGH the conventional load flow methods like Newton's method and fast-decoupled method are simple, due to the radial nature and high $\frac{R}{X}$ ratio of the distribution lines, they cannot be effectively used for the load flow analysis of radial distribution systems. Few researchers have modified the Newton method and fast decoupled method to suit the nature of the distribution network [1]–[3]. They are neither computationally efficient nor convergent for ill-conditioned systems. A compensation-based technique has been proposed in [4]. This technique requires adoption of a methodology for numbering every branch and also the current in any branch is computed as the sum of load current injections at all nodes located beyond the branch under consideration. This is true for all branches irrespective of their location in the network. Hence it is quite evident that repetitive mathematical operations are required and thus the compensation based technique needs longer computational time.

Moreover, a number of attempts have been made using ladder network theory for load flow analysis of a radial network [5],[6]. In [7], Stevens *et al* have shown that this ladder network theory is computationally fast but did not converge in five out of twelve cases studied. In [8], [9] a different approach for solving load flow problem involving the following two major steps has been proposed:

i) Identification of all nodes located beyond each branch.

ii) Calculation of branch currents and node voltages.

Identification of nodes in a large system with multiple branches is tedious and takes a longer duration to determine. In addition, the shortcomings associated with [4] hold for this approach also.

In this context, a novel simple approach for the forward-backward sweep algorithm is proposed in this paper, which overcomes all the above drawbacks for balanced radial distribution network. In the proposed approach, load flow analysis of a radial network is performed by treating every lateral and sub lateral line as an individual main line. The branch current evaluation starts from the far end of each of the sub lateral, lateral and main lines and moves towards the root node. Computation of branch current depends only on the current injected at the neighbouring node and the current in the adjacent branch. This avoids repetitive computations at each branch and thus makes the approach computationally simple and efficient.

Once the branch currents are determined, the node voltage evaluation begins from the root node and moves towards the nodes located at the far end of the main, lateral and sub lateral lines.

This paper is organized as follows: Section-II presents the possible configuration of the radial distribution network. Section-III describes the proposed approach for load flow analysis of radial distribution network and provides the flow chart of the proposed approach. In Section-IV, the proposed approach is illustrated using a simple network. Section–V validates the proposed approach and provides the test results.

II. Radial Distribution Network

A typical radial distribution network consisting of root node, main line, lateral line, sub lateral line and minor line is shown in Fig.1.

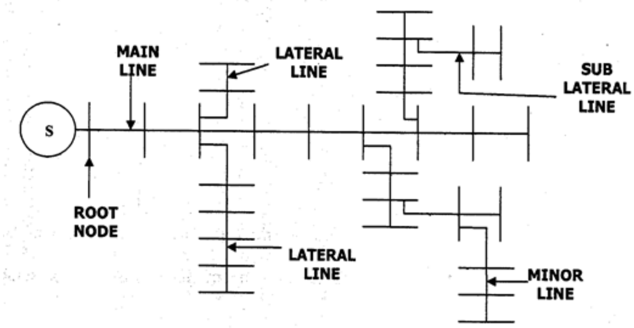

Fig. 1 Single Line Diagram of a Radial Distribution Network

Root node:	The node connected to the voltage regulating station/substation in the radial distribution network.
Main line:	Line emanating from the root node.
Lateral line:	Line emanating from the main line.
Sub lateral line	Line emanating from the lateral line
Minor line:	Line emanating from the sub lateral line

The approach proposed in this paper, assuming balanced load condition, is presented in the following section.

III. Proposed Approach

i) Current injections at any node n can be written as,

$$I_{Ln} = \frac{S_n^*}{V_n^*} \tag{1}$$

Where $n = 1,2,3,\ldots\ldots\ldots\ldots\ldots\ldots N_n$

ii) Line charging current in any branch b can be written as,

$$I_{cb} = \frac{1}{2} y_b V_b + \frac{1}{2} y_b V_{b+1} \tag{2}$$

Where $b = 1,2,3,\ldots\ldots\ldots\ldots\ldots\ldots N_b$

$V_b, V_{b+1} =$ node voltage of node b and $(b+1)$ respectively.

y_b = line charging admittance of branch b

iii) a) Branch current in any branch b in the minor line can be written as,

$$I_b = I_{b+1} + I_{L(b+1)} + I_{cb} \tag{3}$$

Where $\begin{cases} b = (snm_m - 1) \text{ to } (enm_m - 1) \ \forall \ m, \\ m = 1,2,\ldots\ldots\ldots\ldots\ldots\ldots N_m \\ I_{b+1} = 0, if\,(b+1) = enm_m \end{cases}$

$$I_{bm,m} = I_b \text{, if } b = b_{slm} \tag{4}$$

b) Branch current in any branch b in the sub lateral line can be written as,

$$I_b = I_{b+1} + I_{L(b+1)} + I_{cb} + \sum_{m=1}^{N_m} I_{bm,m} \tag{5}$$

Where
$$\begin{cases} b = (snsl_{sl} - 1) \text{ to } (ensl_{sl} - 1) \ \forall\, sl, \\ sl = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots N_{sl}; \\ I_{bm,m} = 0 \text{ if } (b+1) \neq n_{slm} \ \forall\, m, \\ m = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots N_m \\ I_{b+1} = 0 \text{ if } (b+1) = ensl_{sl} \end{cases}$$

$$I_{bsl,sl} = I_b \text{ if } b = b_{lsl} \tag{6}$$

c) Branch current in any branch b in the lateral line can be written as,

$$I_b = I_{b+1} + I_{L(b+1)} + I_{cb} + \sum_{sl=1}^{N_{sl}} I_{bsl,sl} \tag{7}$$

$$\begin{cases} \text{Where} \quad b = (snl_l - 1) \text{ to } (enl_l - 1) \ \forall\, l \\ l = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots N_l; \\ I_{bsl,sl} = 0 \text{ if } (b+1) \neq n_{lsl} \ \forall\, sl \\ sl = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots N_{sl} \\ I_{b+1} = 0 \text{ if } (b+1) = enl_l \end{cases}$$

$$I_{bl,l} = I_b \text{ if } b = b_{Ml} \tag{8}$$

d) Branch current in any branch b in the main line can be written as,

$$I_b = I_{b+1} + I_{L(b+1)} + I_{cb} + \sum_{l=1}^{N_l} I_{bl,l} \tag{9}$$

Where
$$\begin{cases} b = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (en_M - 1) \\ I_{bl,l} = 0 \text{ if } (b+1) \neq n_{Ml} \ \forall\, l \\ l = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots N_l \\ I_{b+1} = 0 \text{ if } (b+1) = en_M \end{cases}$$

iv) Voltage of any node n is given by,

$$V_n = V_{n-1} - I_b Z_b \tag{10}$$

Where V_{n-1} = voltage at (n-1)$^{\text{th}}$ node.

$b = (n-1)$

I_b = Current in the branch b

Z_b = Impedance of the branch b

The approach begins with the assumption of flat voltage start at all nodes. The node current injections, line-charging current and all branch currents are evaluated using (1) to (9). The node voltages are evaluated using (10). The node voltages evaluated are compared with the previous values of node voltages. If the differences in the node voltages between successive iterations are not within the specified tolerance then the above procedure is repeated until convergence in node voltages.

The real and reactive power loss in the network is given by,

$$\left.\begin{aligned} &\text{Real power loss,} \quad P = \sum_{b=1}^{N_b} \left|I_b\right|^2 R_b \\ &\text{Reactive power loss,} \quad Q = \sum_{b=1}^{N_b} \left|I_b\right|^2 X_b \end{aligned}\right\} \tag{11}$$

The flow chart shown in Fig.2 depicts the step-by-step procedure of the proposed approach.

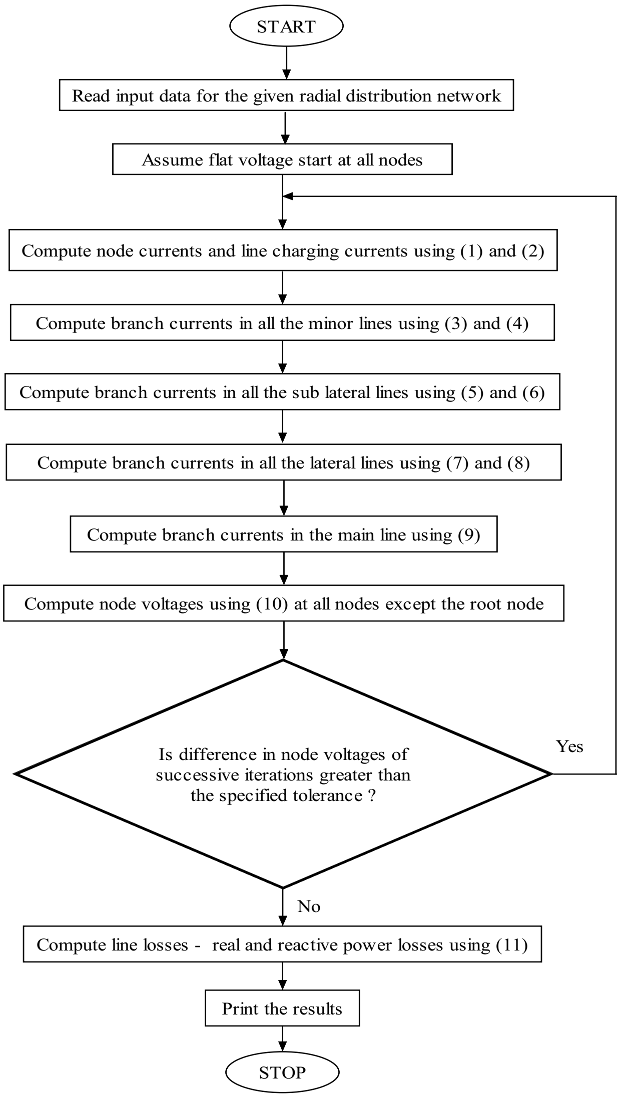

Fig. 2 Flow Chart for the Proposed Approach

IV. ILLUSTRATION

The computational steps involved in the proposed approach are illustrated with the help of a simple radial distribution network given in [8] and is shown in Fig.3 for ease of understanding. For the radial distribution network shown in Fig.3, assuming a flat voltage start of

$$V_n = (1+j0) \text{ p.u,} \tag{12}$$

Where $n = 1, 2, ...12$

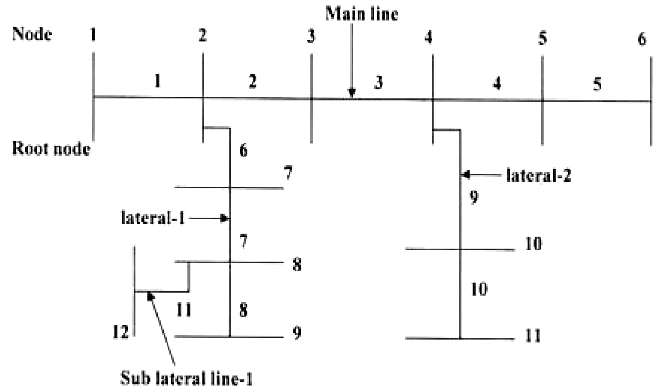

Fig. 3 Typical Radial Distribution Network – An Illustration

The network details are:

$N_n = 12; N_b = 11; N_l = 2; N_{sl} = 1; N_m = 0; en_M = 6$

Lateral line details are:

l	n_{Ml}	b_{Ml}	snl_l	enl_l
1	2	6	7	9
2	4	9	10	11

Sub lateral line details are: (13)

sl	n_{lsl}	b_{lsl}	$snsl_l$	$ensl_l$
1	8	11	12	12

The following quantities were computed:

(i) Current injections at any node n is computed using (1) namely, $I_{L1}, I_{L2}, I_{L3}, ...I_{L12}$.

(ii) Charging current in any branch b is computed using (2) namely, $I_{C1}, I_{C2}, I_{C3}, ...I_{C11}$

(iii) Branch currents in the sub lateral line-1 i.e., $(sl = 1)$ is computed using (5) and (6),

$$I_{11} = I_{12} + I_{L12} + I_{C11} + \sum_{m=1}^{N_m} I_{bm,m} \tag{14}$$

$I_{12} = 0$ because $(b+1) = ensl_1 = 12$.

$\sum_{m=1}^{N_m} I_{bm,m} = 0$ since $N_m = 0$,

$$I_{bsl,1} = I_{11} \text{ as } b = b_{l1} = 11 \tag{15}$$

(iv) Branch currents in the lateral line (i.e. l = 1,2) is computed using (7) and (8),

For lateral line–2 ($l = 2$),

The branch current in branch –10 can be written as:

$$I_{10} = I_{11} + I_{L11} + I_{C10} + I_{bsl,1} \tag{16}$$

$I_{11} = 0$ because $(b+1) = enl_2 = 11$

$I_{bsl,1} = 0$ since $(b+1) \neq n_{lsl} \forall\ sl$,

i.e. $(b+1) = 11$ and $n_{l1} = 8$

The branch current in branch –9 can be written as:

$$I_9 = I_{10} + I_{L10} + I_{C9} + I_{bsl,1} \tag{17}$$

$I_{bsl,1} = 0$ since $(b+1) \neq n_{lsl} \forall\ sl$,

i.e. $(b+1) = 10$ and $n_{l1} = 8$

$$I_{bl,2} = I_9 \text{ since } b = b_{M2} = 9 \tag{18}$$

<u>For lateral line –1</u> ($l = 1$),

The branch current in branch-8 can be written as:

$$I_8 = I_9 + I_{L9} + I_{C8} + I_{bsl,1} \tag{19}$$

$I_9 = 0$ because $(b+1) = enl_1 = 9$

$I_{bsl,1} = 0$ since $(b+1) \neq n_{lsl} \forall\ sl$,

i.e. $(b+1) = 9$ and $n_{l1} = 8$

The branch current in branch –7 can be written as:

$$I_7 = I_8 + I_{L8} + I_{C7} + I_{bsl,1} \tag{20}$$

$I_{bsl,1}$ is obtained using (15) since (b+1) = $n_{l1} = 8$

The branch current in branch –6 can be written as:

$$I_6 = I_7 + I_{L7} + I_{C6} + I_{bsl,1} \tag{21}$$

$I_{bsl,1} = 0$ since $(b+1) \neq n_{lsl} \forall\ sl$,

i.e. $(b+1) = 7$ and $n_{l1} = 8$

$$I_{bl,1} = I_6 \text{ as } b = b_{M1} = 6 \tag{22}$$

(v) Branch currents in the main line is computed using (9)

The branch current in branch-5 can be written as:

$$I_5 = I_6 + I_{L6} + I_{C5} + I_{bl,1} + I_{bl,2} \tag{23}$$

$I_{bl,1}, I_{bl,2} = 0$ since $(b+1) \neq n_{Ml} \forall\ l$,

i.e. $(b+1) = 6$, $n_{M1} = 2$ and $n_{M2} = 4$

$I_6 = 0$ because $(b+1) = en_M = 6$.

The branch current in branch-4 can be written as,

$$I_4 = I_5 + I_{L5} + I_{C4} + I_{bl,1} + I_{bl,2} \tag{24}$$

$I_{bl,1}, I_{bl,2} = 0$ since $(b+1) \neq n_{Ml} \forall\ l$,

i.e. $(b+1) = 5$, $n_{M1} = 2$ and $n_{M2} = 4$

The branch current in branch-3 can be written as,

$$I_3 = I_4 + I_{L4} + I_{C3} + I_{bl,1} + I_{bl,2} \tag{25}$$

$I_{bl,2}$ is obtained using (18) since (b+1) = $n_{M2} = 4$

$I_{bl,1} = 0$ since $(b+1) \neq n_{M1}$, i.e. $(b+1) = 4$ and $n_{M1} = 2$

The branch current in branch-2 can be written as,

$$I_2 = I_3 + I_{L3} + I_{C2} + I_{bl,1} + I_{bl,2} \quad (26)$$

$I_{bl,1}, I_{bl,2} = 0$ since $(b+1) \neq n_{Ml} \forall \; l$,
i.e. $(b+1) = 3, n_{M1} = 2$ and $n_{M2} = 4$

The branch current in branch-1 can be written as,

$$I_1 = I_2 + I_{L2} + I_{C1} + I_{bl,1} + I_{bl,2} \quad (27)$$

$I_{bl,1}$ is obtained using (22) since $(b+1) = n_{M1} = 2$

$I_{bl,2} = 0$ since $(b+1) \neq n_{M2}$, i.e. $(b+1) = 2$ and $n_{M2} = 4$

(vi)The node voltages are computed using (10).

Now the voltages computed using (10) in the present iteration and that using (12) are compared. If the difference is more than the specified tolerance (in this paper the tolerance is taken as 0.0005 p.u), all the above six steps are repeated iteratively until convergence.

(vii) The line losses are computed using (11).

V. RESULTS

The proposed approach has been implemented using MATLAB and tested on a P-IV, 3.20GHz 1MB RAM computer. The computational efficiency of the present approach has been tested using 28, 33, 69 and modified IEEE 34 node radial distribution networks. The data for 28-node system is given in [8]. The data for 33 and 69 node systems are given in [9]. The data for IEEE 34 node system is given in [10] and has been reproduced in the appendix of this paper assuming balanced load conditions. Table–I gives the size and configuration of the systems under study. Tables-II to V gives the load flow results for 28, 33, modified IEEE 34 and 69 node radial distribution system respectively. It is evident from the load flow analysis results of 28 node radial distribution system shown in Table – II that the high $\left(\frac{R}{X}\right)$ ratio of the distribution lines leads to the low voltage magnitude at few nodes of the system. This may lead to voltage collapse for higher loading conditions.

The rate of convergence of the proposed approach is tested using 28, 33, 69 and modified IEEE 34 node radial distribution systems with varying load conditions ranging from 0.5 to 3.0 times of the given load condition. The CPU time and number of iterations obtained using proposed approach has been compared with those obtained using the approach described in [8]. The comparisons between the existing method [8] and proposed method, based on the CPU time in seconds and number of iterations for convergence, under various loading conditions are furnished in Table-VI.

The proposed approach has also been examined with constant current and constant impedance load models. It can be observed from Table-VII that the present approach is computationally more efficient than the approach in [8] for different load models also.

VI. CONCLUSION

A novel approach for load flow analysis of a radial distribution network, which is simple to implement and efficient in computation has been proposed and described in detail in this paper. The computational efficiency and speed of the proposed method has been tested using 28, 33, 69 and modified IEEE 34-node radial distribution networks. The comparison between the proposed and existing method ensures the speed and accuracy of the proposed approach in terms of CPU time both for varying load conditions and systems of different sizes and configurations. It can be concluded that the *simplification made in the branch current computation* of the proposed approach has resulted in improved computational speed of load flow analysis of radial distribution network.

APPENDIX

Fig. A-I shows the IEEE 34 node radial distribution network [10]. Assuming balanced conditions, the line data and load data are given in Tables-A-I and A-II respectively.

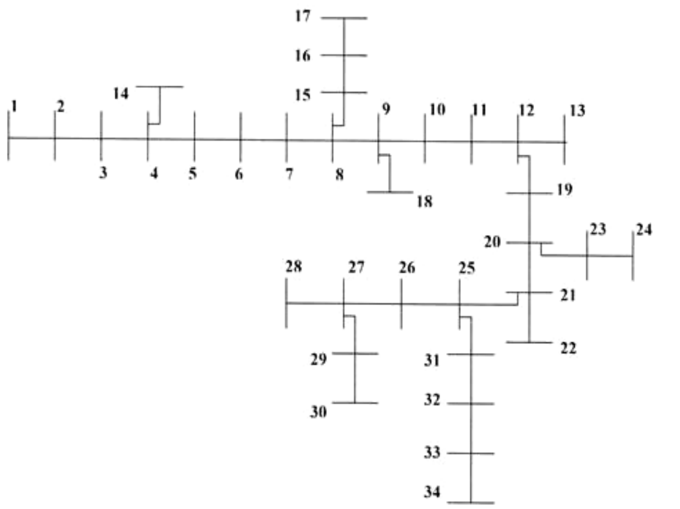

Fig. A-I IEEE 34 Node Radial Distribution Network

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TABLE I NETWORK SIZE AND CONFIGURATION OF THE SYSTEM UNDER STUDY

	28 node	33 node	69 node	IEEE 34 node
Number of nodes in main lines	18	18	27	13
Total number of branches	27	32	68	33
Number of lateral lines	3	3	7	4
Number of sub lateral lines	0	0	0	2
Number of minor lines	0	0	0	2

TABLE II LOAD FLOW SOLUTION OF 28 NODE RADIAL DISTRIBUTION SYSTEM–CONSTANT POWER LOAD

Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)
1	1.0000	8	0.7255	15	0.5399	22	0.9373
2	0.9511	9	0.6897	16	0.5299	23	0.8937
3	0.8997	10	0.6465	17	0.5214	24	0.8902
4	0.8720	11	0.6194	18	0.5184	25	0.8867
5	0.8544	12	0.6075	19	0.9438	26	0.7846
6	0.7886	13	0.5771	20	0.9418	27	0.7833
7	0.7463	14	0.5538	21	0.9393	28	0.7826

TABLE III LOAD FLOW SOLUTION OF 33 NODE RADIAL DISTRIBUTION SYSTEM – CONSTANT POWER LOAD

Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)
1	1.0000	10	0.9296	19	0.9965	28	0.9341
2	0.9970	11	0.9288	20	0.9929	29	0.9271
3	0.9830	12	0.9273	21	0.9922	30	0.9236
4	0.9756	13	0.9212	22	0.9916	31	0.9194
5	0.9682	14	0.9190	23	0.9794	32	0.9185
6	0.9499	15	0.9176	24	0.9728	33	0.9182
7	0.9465	16	0.9162	25	0.9695		
8	0.9416	17	0.9142	26	0.9480		
9	0.9354	18	0.9136	27	0.9455		

TABLE IV LOAD FLOW SOLUTION OF MODIFIED IEEE 34 NODE RADIAL SYSTEM – CONSTANT POWER LOAD

Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)
1	1.0000	10	0.9503	19	0.9208	28	0.9126
2	0.9988	11	0.9495	20	0.9208	29	0.9126
3	0.9980	12	0.9494	21	0.9193	30	0.9126
4	0.9827	13	0.9494	22	0.9193	31	0.9135
5	0.9650	14	0.9827	23	0.8679	32	0.9126
6	0.9510	15	0.9495	24	0.8623	33	0.9123
7	0.9510	16	0.9160	25	0.9136	34	0.9123
8	0.9508	17	0.9118	26	0.9130		
9	0.9504	18	0.9504	27	0.9126		

TABLE V LOAD FLOW SOLUTION OF 69 NODE RADIAL DISTRIBUTION SYSTEM – CONSTANT POWER LOAD

Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)	Node Number	Voltage magnitude (in p.u.)
1	1.0000	19	0.9600	37	0.9998	55	0.9694
2	1.0000	20	0.9597	38	0.9996	56	0.9655
3	0.9999	21	0.9592	39	0.9996	57	0.9451
4	0.9999	22	0.9592	40	0.9995	58	0.9350
5	0.9991	23	0.9592	41	0.9989	59	0.9312
6	0.9908	24	0.9590	42	0.9986	60	0.9266
7	0.9821	25	0.9589	43	0.9985	61	0.9199
8	0.9801	26	0.9588	44	0.9985	62	0.9196
9	0.9790	27	0.9588	45	0.9984	63	0.9193
10	0.9742	28	0.9999	46	0.9984	64	0.9175
11	0.9731	29	0.9999	47	0.9998	65	0.9170
12	0.9701	30	0.9997	48	0.9986	66	0.9731
13	0.9673	31	0.9997	49	0.9948	67	0.9731
14	0.9645	32	0.9996	50	0.9942	68	0.9698
15	0.9618	33	0.9994	51	0.9800	69	0.9698
16	0.9613	34	0.9990	52	0.9800		
17	0.9604	35	0.9990	53	0.9765		
18	0.9604	36	0.9999	54	0.9735		

TABLE VI COMPARISON OF PROPOSED APPROACH WITH [8] FOR VARYING LOAD CONDITIONS

Loading condition	28 node				33 node				69 node				IEEE 34 node			
	CPU Time in seconds		Number of iterations		CPU Time in seconds		Number of iterations		CPU Time in seconds		Number of iterations		CPU Time in seconds		Number of iterations	
	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]	Proposed method	Existing method [8]
0.50	0.047	0.063	3	3	0.047	0.063	2	2	0.031	0.062	2	2	0.094	0.140	2	2
0.70	0.062	0.078	4	4	0.047	0.063	2	2	0.031	0.062	2	2	0.094	0.140	2	2
0.90	0.062	0.078	5	5	0.047	0.063	2	2	0.031	0.062	2	2	0.094	0.140	2	2
1.00	0.078	0.097	7	7	0.047	0.063	2	2	0.031	0.062	2	2	0.094	0.140	2	2
1.10			NC	NC	0.047	0.063	2	2	0.031	0.062	2	2	0.094	0.140	2	2
1.25			NC	NC	0.047	0.063	2	2	0.031	0.062	2	2	0.109	0.140	2	2
1.50			NC	NC	0.047	0.063	2	2	0.031	0.062	2	2	0.109	0.140	2	2
2.00			NC	NC	0.047	0.063	3	3	0.031	0.062	2	2	0.109	0.140	2	2
2.50			NC	NC	0.047	0.063	3	3	0.031	0.062	2	2	0.109	0.172	2	8
3.00			NC	NC	0.047	0.063	3	3	0.031	0.062	2	2	0.109	0.156	2	3

NC – Not converged within 10 iterations during heavily loading conditions due to voltage instability.

TABLE VII COMPARISON FOR CONSTANT CURRENT AND CONSTANT IMPEDANCE LOADS

System and its parameter			Constant current load	Constant impedance load
28 node	CPU time in seconds	Proposed approach	0.047	0.016
		Existing method [8]	0.062	0.032
	Number of iterations	Proposed approach	3	3
		Existing method [8]	3	3
33 node	CPU time in seconds	Proposed approach	0.047	0.016
		Existing method [8]	0.062	0.032
	Number of iterations	Proposed approach	2	2
		Existing method [8]	2	2
69 node	CPU time in seconds	Proposed approach	0.047	0.016
		Existing method [8]	0.062	0.032
	Number of iterations	Proposed approach	2	2
		Existing method [8]	2	2
IEEE 34	CPU time in seconds	Proposed approach	0.110	0.062
		Existing method [8]	0.188	0.125
	Number of iterations	Proposed approach	2	2
		Existing method [8]	2	2

TABLE A-I LINE DATA FOR MODIFIED IEEE 34 NODE RADIAL DISTRIBUTION NETWORK

Branch Number	Starting node	Ending node	Resistance in p.u.	Reactance in p.u.	Branch Number	Starting node	Ending node	Resistance in p.u.	Reactance in p.u.
1	1	2	0.4075	0.4067	18	12	19	8.3983	6.1421
2	2	3	0.2732	0.2727	19	19	20	0.0023	0.0017
3	3	4	5.0905	5.0810	20	20	21	1.1173	0.8172
4	4	5	5.9229	5.9118	21	21	22	0.5358	0.2843
5	5	6	4.6956	4.6869	22	20	23	14.6997	31.5706
6	6	7	0.0023	0.0017	23	23	24	2.3976	1.7732
7	7	8	0.0707	0.0517	24	21	25	1.3294	0.9723
8	8	9	2.3282	1.7027	25	25	26	0.4606	0.3369
9	9	10	0.1915	0.1401	26	26	27	0.6111	0.4469
10	10	11	4.6609	3.4088	27	27	28	0.1961	0.1434
11	11	12	0.1186	0.0867	28	27	29	0.0639	0.0467
12	12	13	7.7166	4.0947	29	29	30	1.1035	0.8161
13	4	14	1.9197	1.0187	30	25	31	0.0639	0.0467
14	8	15	0.5656	0.3001	31	31	32	0.3078	0.2251
15	15	16	15.9261	8.4509	32	32	33	0.8300	0.6070
16	16	17	4.5447	2.4115	33	33	34	0.1209	0.0884
17	9	18	1.0022	0.5318					

TABLE A-II LOAD DATA FOR MODIFIED IEEE 34 NODE RADIAL DISTRIBUTION NETWORK

Node Number	P (in p.u.)	Q (in p.u.)	Node Number	P (in p.u.)	Q (in p.u.)
1	0.000000	0.000000	18	0.000000	0.000000
2	0.000000	0.000000	19	0.000000	0.000000
3	0.000000	0.000000	20	0.000035	0.000015
4	0.000000	0.000000	21	0.000065	0.000030
5	0.000000	0.000000	22	0.000010	0.000005
6	0.000000	0.000000	23	0.000000	0.000000
7	0.000000	0.000000	24	0.001500	0.000750
8	0.000000	0.000000	25	0.000100	0.000050
9	0.000000	0.000000	26	0.000430	0.000275
10	0.000000	0.000000	27	0.000240	0.000120
11	0.000100	0.000050	28	0.000180	0.000115
12	0.000000	0.000000	29	0.000000	0.000000
13	0.000000	0.000000	30	0.000000	0.000000
14	0.000000	0.000000	31	0.000045	0.000025
15	0.000170	0.000085	32	0.001395	0.001075
16	0.000845	0.000435	33	0.000000	0.000000
17	0.000675	0.000350	34	0.000200	0.000160

The per unit values are obtained on a base of 100 MVA and 12.66 kV