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Stability of Alliances between Service Providers

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Abstract—Three service providers in competition, try to optimize their quality of service / content level and their service access price. But, they have to deal with uncertainty on the consumers' preferences. To reduce their uncertainty, they have the opportunity to buy information and to build alliances. We determine the Shapley value which is a fair way to allocate the grand coalition's revenue between the service providers. Then, we identify the values of β (consumers' sensitivity coefficient to the quality of service / contents) for which allocating the grand coalition's revenue using the Shapley value guarantees the system stability. For other values of β, we prove that it is possible for the regulator to impose a per-period interest rate maximizing the market coverage under equal allocation rules.

Keywords—Alliance, Shapley value, Stability, Repeated game, Interest rate.

I. INTRODUCTION

Alliance definition We speak about alliances[1] when firms on a market agree with one another to realize profits which are superior to the *standard* profits that they should receive under competition. The *standard* profits are those obtained at the non cooperative Nash equilibrium where each firm tries to maximize selfishly his utility[2].

Alliances can be explicit when firms agree explicitly with one another on prices, quantities, production capacities, investments, etc., via contracts, for instance. Alliances are said tacit provided this is not the case.

Under competition, firms maximize their profits. But, sometimes, firms realize that coordination might increase their joint profit. Hence, firms on a market have natural incentives to agree together in order to increase their market power and their profit. Competition between firms can then be compared with a prisonner dilemma:

- A firm chooses his strategy in order to maximize his profit but, does not care about the effect of his decision on the other firms.
- In an alliance, firms take into account how their own decisions affect the others' profits.

There exists of course various forms of alliances: joint price determination, joint quantity setting, geographic allocation, etc. [2].

Alliances do not emerge on every market since collusion may be forbidden (competition policy) and since the firms in the alliance might have incentives to cheat, deviating from the cooperative equilibrium. Firms may for instance, deviate unilaterally from the collusion agreement by proposing a price inferior to the expected one in the hope to capture the larger part of the market demand. If each firm forecasts that the others might cheat, cooperation is not imaginable anymore. Punishment mechanisms should then be implemented to prevent the firms in the alliance from cheating. Courts punish explicit accords whose objectives are clearly to decrease the competition. Heavy sanctions have been applied to international accords on the vitamin market (855 millions of euros), on lysine and citric acid (200 millions of dollars and imprisonment years) [2] and more recently, on the memory chip market (331 millions of euros).

Alliance instability Cases of collusion (explicit or tacit) have been reported in the telecommunications literature, both in the long distance market [10] and in the mobile sector [7]. In practice, alliances are difficult to build since firms might suspect that their partner would defect and then, adopt tit for tat strategy which, in a short time-scale, does not give incentives to firms for cooperation. Besides, it is difficult to deal with the selfish tendency of the alliance partners. These latter might for instance, enter a learning race where the firm who learns the quickest wins and then, leaves the alliance. Furthermore, alliances may be highly unstable due to changes in customers' demand, political relation and alliance management. de Man et al. propose a robust framework based on a win-win strategy to guarantee the long-term alliance stability in the airline industry [3]. However, reputation phenomenon and guarantee provisioning[3] might give firms incentives to stay in the alliance [5].

Cooperation in supply chains The different relations between the economic actors can be modeled by a supply chain i.e., a complex network containing a large number of entities who sometimes compete and sometimes cooperate to fulfill customers' needs. Selfish behaviors generating a loss of efficiency, decisions should be centralized in order to maximize the global profit. Hence, building alliances appears as a successful strategy in modern supply chain networks. Cooperation implies then, a better exploitation, benefits from large economies of scope, the decrease of total costs and the increase of total savings. The problem is to identify properly which coalitions can be expected to form and how the alliance members should share the pie i.e., the total benefit to guarantee the stability of the formed alliance [1].

How to share the pie? Indeed, one of the main difficulty studied in cooperative game theory [11] is the pie sharing to guarantee that none of the alliance member would have incen-

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[1]The term of collusion exists also in the literature; these two terms will be used undistinctly in the article.

[2]As usual in game theory, firms and service providers will be associated with male or female players.

[3]Firms provide usually guarantees of their willingness to cooperate by investing money in common technologies, data-bases, leboratories, etc., or through the sharing of their high-level experts' knowledge, or by making available skilled labor force, etc.

tives to leave it. Various allocations of the grand coalition's revenue rules exist such as the Shapley value which guarantees a fair sharing, the nucleolus which provides an allocation that minimizes the dissatisfaction of the players from the allocation that they can receive, the proportional allocation which shares the total revenue depending on the players' intial investment costs, the equal allocation which gives an equal proportion to each player, etc. [1], [11], [15].

Article originality and organization The article aims at studying the alliances that might emerge between three service providers in competition, who have to cope with uncertainty on the consumers' preferences. The originality of our approach is to incorporate uncertainty about the consumers' preferences in the game between the operators. Besides, to our knowledge, this is the first article proposing solutions to deal with alliance instability in the telecommunications framework and modelling explicitly information exchange / investment between the operators.

The article is organized as follows. We start by introducing the two level game between the service providers in Section II. Then, in Section III, we compute the Shapley value of the cooperative game which enables us to allocate fairly the grand coalition's revenue between the three service providers. Depending on the consumers' sensitivity to the quality of service / contents (β), we determine then, whether the Shapley value belongs to the core of the game i.e., guarantees the alliance stability. If the grand coalition is stable then, the regulatory authority's advices would be without effect on the providers since these latter would have no incentives at all to leave the alliance where the total revenue is shared according to the Shapley value. Finally, in Section IV, we consider the values of β for which the Shapley value is not in the core. Having no guarantee on the system's dynamic behavior, the regulator should intervene to assure the consumers' welfare. We prove that it is possible for the regulator to impose a per-period interest rate maximizing the market coverage under equal allocation of the coalitions' revenues. We conclude in Section V.

Notations The main notations used throughout the article are stored in the table below.

A_i	firm or service provider i
p_i^N	access price chosen by firm A_i under complete competition
$p_i^{A_j A_k}$	access price chosen by firm A_i when A_j and A_k form a coalition
p_i^*	access price chosen by firm A_i when all the firms cooperate
U_i	firm A_i's utility
α_i	firm A_i's information level
q_i	firm A_i's quality of service / content level
$q_i^{\max}$	firm A_i's maximum quality of service / content level due to the capacity constraint
$\mathcal{I}(.)$	investment cost function
n_i	number of consumers subscribing to A_i's service
n_0	number of consumers who choose to not subscribe to any service
$c_i(.)$	consumers' opportunity cost for firm A_i
β	consumers' sensitivity to the quality of service / content level
$\mathcal{U}_i$	consumers' intrinsic utility for firm A_i
ξ_k	consumer k's maximum admissible opportunity cost
ν	grand coalition's characteristic function
$\phi_i(\nu)$	part of the grand coalition's revenue allocated to firm A_i
$\mathcal{S}$	set of all the possible coalitions
N	total number of consumers on the market
δ	discount factor
r	interest rate

II. Game model and information sharing

Description of the providers' utilities We consider three service providers: A_1, A_2 and A_3. They can belong to various categories. It can be content providers, Internet access providers, or TV channels, etc. We assume that these three firms are interconnected via the Internet backbone. Besides, they have the opportunity to exchange information[4] (cf. Figure 1). The information can be either bought from other providers or the providers can choose to invest together to gather data through a common data-base.

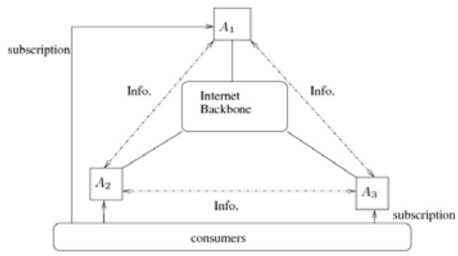


Fig. 1. Description of the relations between the three service providers.

Consider firm $A_i,\ i = 1, 2, 3$, his utility is of the form

$$U_i = p_i n_i - \mathcal{I}(\alpha_i)$$

with p_i the acess price to the firm i's network / service for the consumers[5], n_i the total number of clients for firm A_i. α_i represents the level of information collected / bought by firm A_i while $\mathcal{I}(.)$ is the cost of information acquisition.

The quality of service (QoS) / content level q_i can be seen as a function of firm A_i's information level i.e., $\alpha_i = \vartheta(q_i)$ with $\vartheta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ an invertible function on $\mathbb{R}_+$. Furthermore, we assume that $\mathcal{I} o \vartheta$ is convex on $\mathbb{R}_+$. In economics, Schlee [16], Johnson and Myatt [6] use convex marginal cost functions, in different contexts.

The quality of service / content level is tightly related to the information level acquired by the firm. Indeed, if firm A_i manages to extract the most pertinent information about the network topologies or more generally, about the consumers' preferences, he will be able to optimise q_i more easily. Of course the consumers' perception of firm A_i will by turns depend on the firm's information level since, in this case,

[4]The exchanged information might concern the providers' network topology, data acquired about the consumers' preferences, etc.

[5]This access price is a flat rate i.e., a fixed price which do not depend on the quantity of traffic really sent by the clients.

the firm's information level conditions his quality of service / content level. To give an illustration, in Figure 2, the firms' information levels are expressed as linear and quadratic (with stars) functions of the quality of service / content level; we have set: $\alpha_i = 4q_i$ and $\alpha_i = \frac{1}{2}q_i^2$ (with stars). Furthermore, we assume that the firm is limited in the improvement of his quality of service / content level by the access network capacity; thus: $q_i \leq q_i^{\max},\ i = 1, 2, 3$.

The idea that the more information the provider acquires, the easier it is for him to optimize his quality of service / content level can be found again in the concept of entropy which has been introduced by Shannon. The entropy is a mathematical function which corresponds to the quantity of information being contained or delievered by a source (the consumers, for instance) [14]. The more information is received by the receptor (provider A_i), the more the entropy (or uncertainty) about the message (consumers' preferences) decreases, due to this information gain.

In practice, service providers reserve bandwidth on transit providers' networks who route their traffics. These transit operators have approximately identical marginal costs. It is then essential for the service providers to book the optimal bandwidth volume on transit providers' networks using the acquired information to infer consumers' preferences. Indeed, service providers will have to pay for the quantity of data transfered. Besides, it is essential for service providers to reserve the exact quantity of bandwidth; otherwise transit providers punish them either by evening out their traffic if the reserved bandwidth is insufficient or by overbooking and deprioritarizing if there is a waste. It might then altered the quality of service / content level perceived by the consumers.

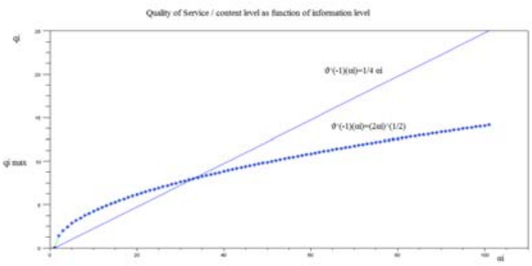


Fig. 2. Quality of service / content level as a function of firm A_i's information level.

Game description for firm A_i The three service providers play simultaneously the two level game described below. Then, depending on their choices, the consumers subscribe to a service or report their decision. The two levels in the game result from the difference of timing between the access price determination and the contract of bandwidth reservation.

(1) A_i sets his quality of service / content level q_i

(2) A_i chooses the access price to his service p_i

Description of the consumers' choice model We suppose that the consumers have the opportunity to choose their service provider or to not subscribe to any service. In order to make their choice, they should compute the opportunity costs [13] associated with each service provider. For firm A_i, the opportunity cost is $c_i = p_i + \beta q_i$ with $-1 \leq \beta \leq 0$[6] coefficient characterizing consumers' sensitivity to the quality of service / content level.

Now, the consumer choice model description requires the introduction of intrinsic utilities [4] for the consumers, associated with each provider, being independent of the opportunity costs. Consumers' intrinsic utility for firm A_i will be denoted $\mathcal{U}_i$. Furthermore, we assume that the consumers have an a priori preference for the firms which is characterized by the following order $\mathcal{U}_1 \leq \mathcal{U}_2 \leq \mathcal{U}_3$. The order is arbitrary and it can be modified. We assume that $0 \leq c_1(.) < c_2(.) < c_3(.) < 1$. This hypothesis guarantees the existence of positive market shares for each of the three firms. An approching choice model for the consumers has already been detailed in [9].

Then, we introduce the maximum admissible opportunity cost for consumer k, $k = 1, 2, ..., N$: Ξ_k, whose realization ξ_k is generated according to the uniform density on $[0; 1]$[7]. This choice of a uniform density to model the consumers' intrinsic utility is motivated by the assumption that the firms have a priori no information on the consumers' preferences. Necessity to introduce a maximum admissible opportunity cost results from the following observation: a consumer will refuse to subscribe to the provider's service or will report his purchase to a later date for the follwing reasons, either the access price is too high, either the quality of service level is not sufficient, or the contents are not enough diversified compared to what he expects.

Consumer k's utility for firm $A_i,\ i = 1, 2, 3$ is

$$u_{k,i} = \begin{cases} \mathcal{U}_i \text{ if } \xi_k \geq c_i(.), \\ 0 \text{ otherwise.} \end{cases}$$

This means that if $\xi_k \in [0; c_1(.)[$ consumer k does not choose any offer, if $\xi_k \in [c_1(.); c_2(.)[$, consumer k selects firm A_1, if $\xi_k \in [c_2(.); c_3(.)[$, consumer k chooses firm A_2, finally, if $\xi_k \in [c_3(.); 1]$, the consumer selects firm A_3.

The main assumptions related to the providers' game and to the consumer choice model are listed below.

Hypotheses

- $\vartheta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is an invertible application on $\mathbb{R}_+$,
- $\mathcal{I}o\vartheta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a convex application on $\mathbb{R}_+$,
- $\mathcal{U}_1 \leq \mathcal{U}_2 \leq \mathcal{U}_3$,
- $\Xi_k \sim \mathcal{U}_{[0;1]}$,
- $0 \leq c_1(.) < c_2(.) < c_3(.) < 1$,
- firms are limited in their investment in the quality of service / contents by the capacity of their access networks i.e., $q_i \leq q_i^{\max},\ i = 1, 2, 3$.

Then, we can compute the number of clients for each firm. Let n_0 be the total number of consumers who choose to not subscribe to any service, n_i, the total number of clients for firm A_i and N, the total number of consumers on the market.

[6] To simplfy the numerical analysis, β is supposed to be normalized.

[7] To simplify the analytical game resolution, we have chosen the unit interval. However, the model can be extended without additional difficulties if we assume that the consumers' maximum admissible opportunity costs are distributed according to a uniform density on the interval $[0; f],\ f > 0$.

Lemma 1. *Consumers are distributed between the three service providers according to the following rule*

- $n_0 = N\Big[p_1 + \beta q_1\Big]$ *do not subscribe to any service,*
- $n_1 = N\Big[(p_2 - p_1) + \beta(q_2 - q_1)\Big]$ *choose firm A_1,*
- $n_2 = N\Big[(p_3 - p_2) + \beta(q_3 - q_2)\Big]$ *prefer firm A_2,*
- $n_3 = N\Big[1 - p_3 - \beta q_3\Big]$ *select firm A_3.*

Proof of Lemma 1 We have $n_0 = c_1 N$, $n_1 = (c_2 - c_1)N$, $n_2 = (c_3 - c_2)N$ and $n_3 = (1 - c_3)N$. □

III. Sharing mechanism of the grand coalition's revenue and stability

The game contains $2^3 - 1 = 7$ distinct coalitions. We assume that the game is with transferable utility i.e., the total revenue associated with each coalition can be freely shared between the coalition members. It implicitly means that the coalition members can freely give, receive or even, burn money [11]. Games with transferable utility require the introduction of a characteristic function $\nu : 2^3 - 1 \to \mathbb{R}$ which associates a global revenue with each coalition. Function ν valuates on a coalition, measures its worth [8].

To valuate the characteristic function on each coalition, we have chosen a representation by defensive equilibria [11].

Let $\mathcal{S}$ be the set of the 7 possible coalitions and $s \in \mathcal{S}$ a given coalition. We note $\sigma_s^{\text{opt}} = (p_i^{\text{opt}}, q_i^{\text{opt}})_{i\in s}$ with $p_i^{\text{opt}},\ q_i^{\text{opt}} \in \mathbb{R}_+^2,\ \forall i \in s$, the strategy on prices, quality of service / content level chosen by the coalition s members and $\sigma_{\mathcal{S}-s}^{\text{opt}} = (p_j^{\text{opt}}, q_j^{\text{opt}})_{j\in\mathcal{S}-s}$ with $p_j^{\text{opt}},\ q_j^{\text{opt}} \in \mathbb{R}_+^2,\ \forall j \in \mathcal{S}-s$, the strategy selected by the firms who do not belong to coalition s. For each coalition $s \in \mathcal{S}$, we have to solve the following system to determine the providers' optimal strategies

$$\begin{cases} \sigma_s^{\text{opt}} \in \arg \max\limits_{\sigma_s = \left\{(p_i,q_i)\in\mathbb{R}_+^2 | i\in s\right\}} \sum\limits_{i\in s} U_i(\sigma_s, \sigma_{\mathcal{S}-s}^{\text{opt}}) \\ \sigma_{\mathcal{S}-s}^{\text{opt}} \in \arg \max\limits_{\sigma_{\mathcal{S}-s} = \left\{(p_j,q_j)\in\mathbb{R}_+^2 | j\in \mathcal{S}-s\right\}} \sum\limits_{j\in \mathcal{S}-s} U_j(\sigma_s^{\text{opt}}, \sigma_{\mathcal{S}-s}) \end{cases}$$

Using coalition s optimal strategy expression, we obtain the value of the characteristic function ν on coalition s: $\nu(s) = \sum_{i\in s} U_i(\sigma_s^{\text{opt}}, \sigma_{\mathcal{S}-s}^{\text{opt}})$ and on $\mathcal{S} - s$: $\nu(\mathcal{S} - s) = \sum_{j\in\mathcal{S}-s} U_j(\sigma_s^{\text{opt}}, \sigma_{\mathcal{S}-s}^{\text{opt}})$.

Theorem 2. *For each possible coalition, the two level game between the three providers admits a unique pure Nash equilibrium on prices and quality of service / content level. The characterisitc function is then defined uniquely on each possible coalition.*

Proof of Theorem 2 In order to fix the ideas, we assume in the rest of the article that $\mathcal{I}(.)$ is the identity application and suppose that the information level is either quadratic in the QoS / content level i.e., $\alpha_i = \vartheta(q_i) = \frac{q_i^2}{2}$ (example 1), or that it is linear in the QoS / content level i.e., $\alpha_i = \vartheta(q_i) = q_i$ (example 2). However, the proof can be generalized to far more complex functions satisfying the hypotheses of Section II.

For each possible coalition, we have to solve for every provider in the coalition, a two level game in price and quality of service / content. Going backward, we start by determining the optimal prices for each provider in the coalition by differentiating the coalition's utility with respect to the prices. Then, we substitute the optimal prices in the coalition's utility and differentiate it once more, with respect to the quality of service / content level. Finally, we obtain the optimal quality of service / content level and the optimal prices for each provider in the coalition. The resulting strategies form a Nash equilibrium for the players since none of them has incentives to deviate from it unilaterally. Details about the analytical determination of the Nash equilibrium and its unicity are provided in Appendix A. The characteristic function ν is also evaluated on each coalition. □

A cooperative game is an interactive decision model based on the behavior of groups of players or coalitions. In a cooperative game, one of the most difficult problem to solve is the sharing of the coalition's total revenue between its members. Shapley has proposed a fair sharing rule for a n player cooperative game. Another solution concept should require that the coalition members had no incentives to deviate to increase their revenue. Such a solution concept should guarantee the system *stability*. The set of the allocations of the grand coalition's revenue which should guarantee the system stability is the core of the cooperative game; formally, it is the set of the global revenue allocations $x = (x_i)_{i=1,2,...,n}$ such that $\sum_{i=1,2,...,n} x_i = \nu(1, 2, ..., n)$ (feasible) and $\sum_{i\in s} x_i \geq \nu(s),\ \forall s \subseteq \mathcal{S}$.

Computation of the Shapley value The core of a cooperative game can be empty or very large. It explains partly, why this notion is so difficult to apply to predict the players' behavior. An alternative approach might be to identify a unique mapping $\phi : \mathcal{S} \to \mathbb{R}^3$ such that for the cooperative game defined by the characterisitc function ν, the expected revenue of each provider i is $\phi_i(\nu)$.

Shapley has approched this problem axiomatically by defining a solution that prescribes a single payoff for each player which is the average of all marginal contributions of that player to each coalition he is member of. It satisfies four main axioms: efficiency i.e., the payoffs must add up to $\nu(\{1, 2, ..., n\})$ which means that all the grand coalition's surplus is allocated; symmetry i.e., if two players are substitutable because they contribute the same to each coalition the solution should treat them equally; additivity i.e., the solution to the sum of two games with transferable utility must be the sum of what it awards to each of the two games; dummy player i.e., if a player contributes to nothing to every coalition the solution should pay him nothing.

Theorem 3. *(Shapley [11]) There exists exactly one application $\phi : \mathcal{S} \to \mathbb{R}$ satisfying the four axioms cited above. In a cooperative game with n players and transferable utility, this function satisfies the following equation for each player i and for any characteristic function ν*

$$\phi_i(\nu) = \sum_{\{s\subseteq\mathcal{S} | i\notin s\}} \frac{|s|!(n - |s| - 1)!}{n!} \Big(\nu(s \cup \{i\}) - \nu(s)\Big)$$

Formula in Theorem 3 can be interpreted the following way. Suppose that we plan to assemble the grand coalition in a room but, the door to the room is only large enough for one player to enter at a time, so the players randomly line up in a queue at the door. The are $n!$ different ways that the players might be ordered in the queue. For any set s that does not contain player i, there are $|s|!(n-|s|-1)!$ different ways to order the players so that s is the set of players who are ahead of player i in the queue. Thus, if the various orderings are equally likely, $\frac{|s|!(n-|s|-1)!}{n!}$ is the probability that, when player i enters the room, he will find the coalition s there ahead of him. If i finds s ahead of him when he enters then, his marginal contribution to the worth of the coalition in the room when he enters is $\Big(\nu(s \cup \{i\}) - \nu(s)\Big)$. Thus, under this story of randomly ordered entry, the Shapley value of any player is his expected marginal contribution when he enters.

For each of the firms, we can compute the Shapley value which corresponds to a fair sharing of the grand coalition's total benefit between the firms.

- For firm A_1, we have $\phi_1(\nu) = \frac{1}{3}\Big(\nu(A_1, A_2, A_3) - \nu(A_2, A_3)\Big) + \frac{1}{6}\Big(\nu(A_1, A_2) - \nu(A_2)\Big) + \frac{1}{6}\Big(\nu(A_1, A_3) - \nu(A_3)\Big)$.
- For firm A_2, we set $\phi_2(\nu) = \frac{1}{3}\Big(\nu(A_1, A_2, A_3) - \nu(A_1, A_3)\Big) + \frac{1}{6}\Big(\nu(A_1, A_2) - \nu(A_1)\Big) + \frac{1}{6}\Big(\nu(A_2, A_3) - \nu(A_3)\Big)$.
- For firm A_3, we obtain $\phi_3(\nu) = \frac{1}{3}\Big(\nu(A_1, A_2, A_3) - \nu(A_1, A_2)\Big) + \frac{1}{6}\Big(\nu(A_1, A_3) - \nu(A_1)\Big) + \frac{1}{6}\Big(\nu(A_2, A_3) - \nu(A_2)\Big)$.

Analytical expressions of Shapley value allocations for each provider when the information level is quadratic (respectively linear) in the quality of service / content level are detailed in Appendix B.

Influence of β on the system stability with Shapley value as sharing rule Presently, we want to characterize the influence of the consumers' sensitivity coefficient to the quality of service / content level (β) on the mechanism of fair revenue sharing for the grand coalition.

We have represented the allocations given by the Shapley value for providers A_1, A_2, A_3, for different number of consumers on the market ($N = 5,\ N = 10,\ N = 50,\ N = 120$) as functions of $\beta \in [-1; 0]$. Each provider's allocation issued from the Shapley value are plotted on Figures 3 and 4 with information cost function being quadratic and linear in the quality of service / content level, respectively.

We observe that for small sensitivity coefficients[8], firms do not invest very much in quality of service / content and prefer proposing small access prices. In this case, we observe in Figure 3, that A_3 dominates the market. Indeed, access prices being small, providers' revenues are small and hardly compensate the information cost which is quadratic in the quality of service / content level. Thus, it is the favorite firm (A_3) which starts with a certain advantage and dominates the market. For an information cost linear in the quality of service / content, we note in Figure 4, that compensation is less difficult and that A_1 and A_3 alternately dominate the market. Indeed, A_1 is a priori the firm who differentiates himself the most from the others by proposing the smallest prices while A_3 is the favorite provider for the consumers.

For intermediate sensitivity coefficients, we observe on Figures 3 and 4, oscillations on the providers' revenues. Indeed, these latter are involved in a price war.

Finally, when the sensitivity coefficient is high, providers invest massively in the quality of service / content. The problem is then that the providers' access networks are limited in capacity. Thus, they cannot invest more than a fixed quantity $q_i^{\max},\ i = 1, 2, 3$. For a large number of consumers, each provider invests at the maximum of his capacity and tries to differentiate with the access prices. At the end, the game converges when β decreases towards a perfect competition situation where the access prices are fixed such that the firms' profits are near 1 or even null[9].

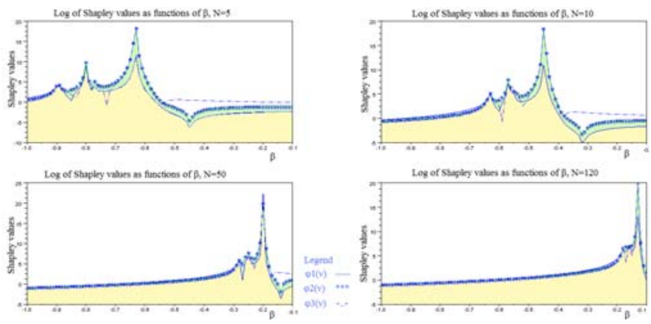


Fig. 3. Shapley value allocations for A_1, A_2, A_3 as functions of β with information quadratic in the quality of service / content level.

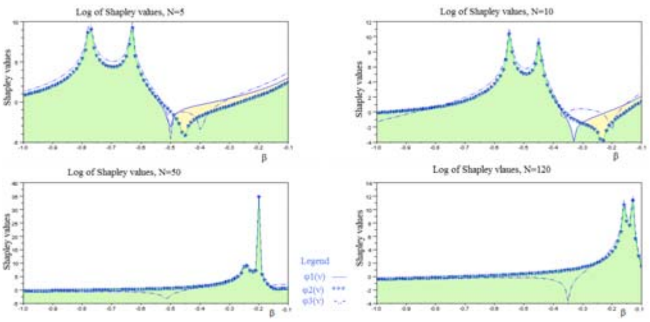


Fig. 4. Shapley value allocations for A_1, A_2, A_3 as functions of β with information linear in the quality of service / content level.

Sufficient conditions guaranteeing the core existence To determine the core of the game, we have to solve the following

[8]Note that in the economic interpretation, we consider the absolute value of β.

[9]We have considered in the numerical illustrations, the logarithm of the providers' revenues; but $\log(1) = 0$ and $\lim_{x \to 0^+} log(x) = -\infty$.

system of equations

$$(\mathcal{C}) = \begin{cases} x_1 + x_2 + x_3 = \nu(A_1, A_2, A_3) \\ x_1 \geq \nu(A_1) \\ x_2 \geq \nu(A_2) \\ x_3 \geq \nu(A_3) \\ x_1 + x_2 \geq \nu(A_1, A_2) \\ x_1 + x_3 \geq \nu(A_1, A_3) \\ x_2 + x_3 \geq \nu(A_2, A_3) \end{cases}$$

Proposition 4. *For $N = 450.10^3$ if $\mathcal{I}(\alpha_i) = \frac{q_i^2}{2}$ (resp. $\mathcal{I}(\alpha_i) = q_i$) and $\beta \in [-1; -0.78]$ (resp. $\beta \in [-1; -0.49]$) then, if the grand coalition's total revenue is shared according to the Shapley value, the grand coalition is stable i.e., no provider has incentives to leave it.*

Proof of Propositon 4 To start, we want to determine if there exists β values for which the Shapley value is in the core. To perform this point, we draw the 7 functions of β, each one of them describing System $(\mathcal{C})$ equality / inequalities. These functions are

$$\begin{aligned} f_0(\beta) &= \phi_1(\nu) + \phi_2(\nu) + \phi_3(\nu) - \nu(A_1, A_2, A_3) \\ f_1(\beta) &= \phi_1(\nu) - \nu(A_1) \\ f_2(\beta) &= \phi_2(\nu) - \nu(A_2) \\ f_3(\nu) &= \phi_3(\nu) - \nu(A_3) \\ f_{12}(\nu) &= \phi_1(\nu) + \phi_2(\nu) - \nu(A_1, A_2) \\ f_{13}(\nu) &= \phi_1(\nu) + \phi_3(\nu) - \nu(A_1, A_3) \\ f_{23}(\nu) &= \phi_2(\nu) + \phi_3(\nu) - \nu(A_2, A_3) \end{aligned}$$

Suppose that the information level is quadratic in the QoS / content level. In Figure 5, we have plotted these 7 functions of β. We observe that the constraints defining the core of the game are satisfied if, and only if, $\beta \in [-1; -0.78]$ with an information level quadratic in the quality of service / content investment level.

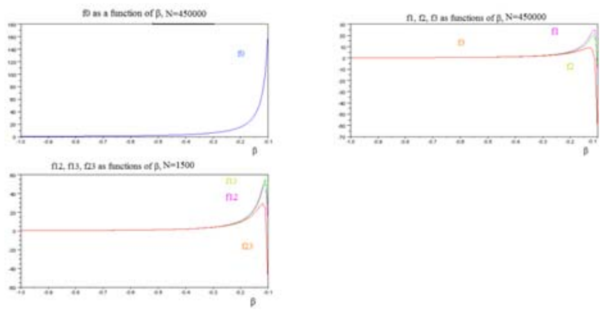


Fig. 5. Graphics of $f_0(\beta)$, $f_i(\beta)$, $f_{ij}(\beta)$, $i, j = 1, 2, 3$, $i \neq j$ as functions of β with information level being quadratic in the quality of service / content level.

In Figure 6, we have plotted these 7 functions of β assuming that the information level is this time, linear in the QoS / content level. We observe that if $\beta \in [-1; -0.49]$ then, the Shapley value satisfies the constraints characterizing the core of the game.

□

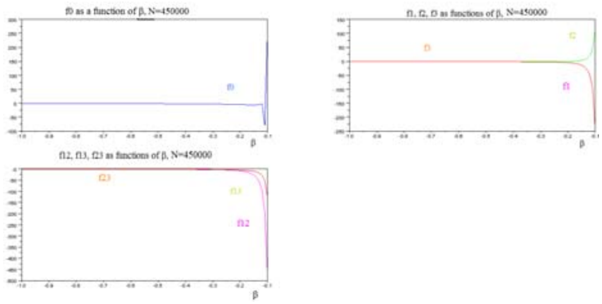


Fig. 6. Graphics of $f_0(\beta)$, $f_i(\beta)$, $f_{ij}(\beta)$, $i, j = 1, 2, 3$, $i \neq j$ as functions of β with information level being linear in the quality of service / content level.

If the grand coalition is stable then, the regulatory authority's advices would not be followed by the providers. Indeed, the Shapley value being in the core of the game, the providers would rather follow this fair way to share the grand coalition's profit than deviating from it by following the regualtor's advices and risking to lose money. However, if the Shapley value is not in the core of the game, the providers' behaviors might be unpredictable and the regulator should intervene to control the system.

IV. Intervention of the regulatory authority to maximize the consumers' welfare

In order to fix the ideas, we assume in the rest of the article that $\mathcal{I}(\alpha_i) = \frac{q_i^2}{2}$. We have seen in Section III that in this case, if $\beta \in [-1; -0.78]$, the Shapley value is in the core of the game. Thus, if the three firms agree to implement this sharing mechanism, the grand coalition should be stable and none of the firms would deviate.

But, if $\beta > -0.78$, the System of equations $(\mathcal{C})$ being impossible to solve analytically, we cannot conclude on the existence of a core for the game. The considered ecosystem might then be highly unstable, penalizing heavily the firms and the consumers. Then, the regulatory authority should intervene. His aim is to increase the consumers' welfare while guaranteeing a maximal coverage of the market. In practice, it means that the regulatory authority will try to determine the system organization minimizing $n_0 = N\Big(p_1 + \beta q_1\Big)$ i.e., the total number of consumers who choose to not subscribe to any service, as a function of β and N.

Contrary to Section III, for each coalition, the regulatory authority being unbiased, divides *equally* its total revenue between its members. This is the simplest allocation rule [1]. It means that the regulatory authority does not want to take into account the providers' worth in the coalition to allocate the total revenue. Besides, since $\beta > -0.78$, the Shapley value is not in the core anymore; hence the providers would most certainly deviate from this sharing rule if the regulator imposed it. Consequentely, it is not a solution for the regulator to impose the Shapley value as sharing rule in this section.

Determination of the domain of definition for (β, N) depending on the proposed services In Figure 7, we have

TABLE I
POSSIBLE INTERACTIONS BETWEEN THE PROVIDERS WHILE A_1 REFUSES TO COOPERATE.

A_2 \ A_3	cooperation	no cooperation
cooperation	$\{A_2, A_3\}$	no cooperation
no cooperation	no cooperation	no cooperation

TABLE II
POSSIBLE INTERACTIONS BETWEEN THE PROVIDERS WHILE A_1 AGREES TO COOPERATE.

A_2 \ A_3	cooperation	no cooperation
cooperation	$\{A_1, A_2, A_3\}$	$\{A_1, A_2\}$
no cooperation	$\{A_1, A_3\}$	no cooperation

depicted which system organization should maximize the market coverage i.e., minimize the number of consumers without subscription ($n_0 = N\Big(p_1 + \beta q_1\Big)$) as a function of β and N. The different possible system organizations are: complete competition (M_0), cooperation between A_1 and A_2 solely (M_1), cooperation between A_1 and A_3 (M_2), cooperation between A_2 and A_3 solely (M_3) and cooperation between the three operators (M_4).

The applications that we consider in this article, are highly sensitive to the quality of service like for instance, video over demand, music downloadings, interactive multiplayer games, etc.; hence we assume that $\beta < -0.7$. Besides, the total number of consumers on the market being rather large, Figure 7 tells that the regulatory authority should break all the emerging alliances (M_0) to guarantee perfect competition between the providers and maximize the market converage.

The relation between N and $\beta < -0.7$ delimiting the area where no alliance should emerge can be interpolated by a second order polynomial equation in β and N: $N = 1\ 100 + 20\ 833(0.8+\beta)^2$. Now and in the rest of the article, we assume to be in the domain of definition $\mathcal{D}$, characterized by the relations $\mathcal{D} = \Big\{\{\beta > -0.78\} \cap \{N \geq 1\ 100 + 20\ 833(0.8+\beta)^2\}\Big\}$. The regulatory authority should then intervene to guarantee perfect competition between the providers.

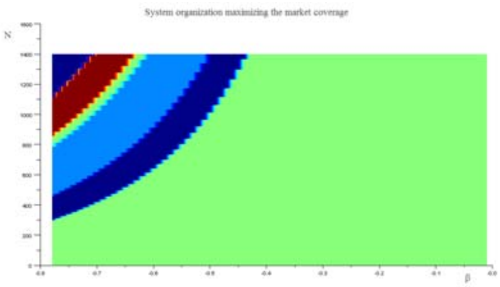


Fig. 7. System organization minimizing the total number of consumers without subscription as function of β and N.

Conditions on (β, N) guaranteeing that (p_1^N, p_2^N, p_3^N) is a Nash equilibrium We want to determine conditions on β and N guaranteeing that (p_1^N, p_2^N, p_3^N) is a Nash equilibrium for the three service providers. It means that none of them has an incentive to deviate from it.

Suppose that the providers *cannot communicate* (to avoid cheating or reputation phenomenon for instance) and that they play simultaneously the two-level game described in Section II. Initially, they are all members of a common alliance. The no-communication assumption means that a two-player coalition might be formed only if both players decide at the same instant to form a sub-coalition without prior arrangement. Furthermore, border situations might appear where one provider continues to play as if he were in an alliance although the other players have already defected. But of course, the observation of the resulting payoffs gives indications to the providers about their allies' past choices[10]. In Tables I and II, we have listed all the joint possible actions for the providers. In fact, each provider is confronted to a prisonner dilemma: either he tries to cooperate or, he defects. Contrary to the classical version of the dilemma [18], we have to cope here with three players in competition. To be more precise, we describe the game setting. If the three providers choose to cooperate, they form a grand coalition and their access prices are $p_i^*,\ i = 1,2,3$ as defined in Appendix A. If only two of the providers accept to cooperate (for instance A_i and A_j), they form (secretly) a coalition against $A_k,\ i,j,k = 1,2,3,\ i \neq j,\ i \neq k,\ j \neq k$ who, being not informed of this alliance, continues to play as under total competition. The access prices are denoted $p_i^{A_iA_j}$ and $p_j^{A_iA_j}$ (resp. p_k^N) for A_i and A_j (resp. A_k) as defined in Appendix A.

In Tables I and II, (p_1^N, p_2^N, p_3^N) is a Nash equilibrium if, and only if, the providers have no incentives to leave this equilibrium to form coalitions. Formally, the three following inequalities should be satisfied

(1) $\min\Big\{U_1(p_1^N,p_2^N,p_3^N) - U_1(p_1^{A_1A_2},p_2^{A_1A_2},p_3^N); U_1(p_1^N,p_2^N,p_3^N) - U_1(p_1^{A_1A_3},p_2^N,p_3^{A_1A_3}); U_1(p_1^N,p_2^N,p_3^N) - U_1(p_1^*,p_2^*,p_3^*)\Big\} \geq 0$

(2) $\min\Big\{U_2(p_1^N,p_2^N,p_3^N) - U_2(p_1^{A_1A_2},p_2^{A_1A_2},p_3^N); U_2(p_1^N,p_2^N,p_3^N) - U_2(p_1^N,p_2^{A_2A_3},p_3^{A_2A_3}); U_2(p_1^N,p_2^N,p_3^N) - U_2(p_1^*,p_2^*,p_3^*)\Big\} \geq 0$

(3) $\min\Big\{U_3(p_1^N,p_2^N,p_3^N) - U_3(p_1^{A_1A_3},p_2^N,p_3^{A_1A_3}); U_3(p_1^N,p_2^N,p_3^N) - U_3(p_1^N,p_2^{A_2A_3},p_3^{A_2A_3}); U_3(p_1^N,p_2^N,p_3^N) - U_2(p_1^*,p_2^*,p_3^*)\Big\} \geq 0$

Proposition 5. *(p_1^N, p_2^N, p_3^N) is a Nash equilibrium for the three providers on the domain of definition $\mathcal{D}$ if,and only if, $N \geq 14\ 100 + 500.10^3\beta^2$.*

Proof of Proposition 5 In Figure 8, we have depicted

[10] We assume here that the providers detect that one of their allies has defected in at most two steps. Generalizations where the detection time is random might be considered as future research avenues.

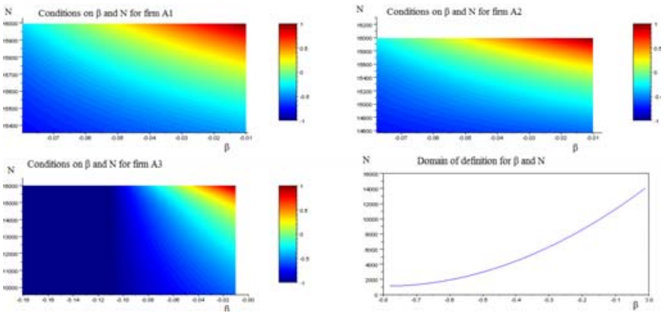


Fig. 8. Conditions for each provider on β and N guaranteeing the existence of a Nash equilibrium in (p_1^N, p_2^N, p_3^N).

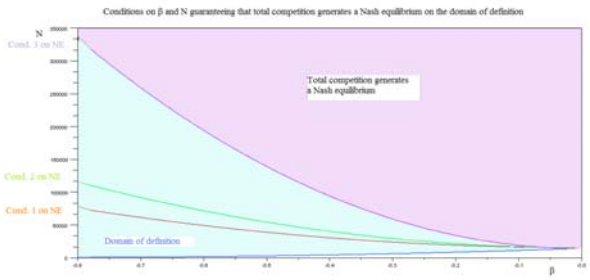


Fig. 9. Conditions on β and N guaranteeing the existence of a Nash equilibrium in (p_1^N, p_2^N, p_3^N) on the domain of definition.

the areas in β and N where the Nash equilibrium conditions for (p_1^N, p_2^N, p_3^N) are satisfied for each provider. We observe that these areas can be delimited by second order polynomial equations. For firm A_1, the area guaranteed that (p_1^N, p_2^N, p_3^N) is a Nash equilibrium is delimited by the equation $N = 15\ 600 + 93\ 750\beta^2$. Identically, for firm A_2, we get: $N = 15\ 400 + 156\ 250\beta^2$ and for firm A_3, we obtain: $N = 14\ 100 + 500.10^3\beta^2$. In Figure 9, we have pictured the three resulting curves as functions of β and N as well as the domain of definition $\mathcal{D}$. We then get numerically that (p_1^N, p_2^N, p_3^N) is a Nash equilibrium for the three providers if, and only if (β, N) lie in the area delimited by the polynomial equation in β: $N = 14\ 100 + 500.10^3\beta^2$.

□

Controled dynamic evolution from cooperation to total competition As already stated, initially, the three providers form a common alliance. We assume that the conditions obtained in Proposition 5 are satisfied and that two providers cannot simultaneously leave the alliance; besides, the total number of consumers on the market is fixed and known by all the providers. The cooperative game introduced in Section III is now repeated under an infinite horizon and discrete time steps. In a dynamic framework, we need to introduce a discount factor $\delta \in]0;1[$ which enables us to valuate a future profit, today [2], [8], [18]. Three evolution scenarios may raise at time instant $T > 0$[11]

- Scenario 1 $\{A_1, A_2, A_3\} \rightsquigarrow \{A_1, A_2\}$ i.e., A_3 deviates at time instant T. Then at time instant $T+1$[12], three cases might appear: either A_1 and A_2 continue to cooperate in the rest of the game; either A_1 deviates; or A_2 breaks the alliances he had with A_1. If A_1 or A_2 breaks the alliance with the other provider, cooperation cannot be envisaged anymore in the rest of the game since the providers' prices stabilize forever in the Nash equilibrium (p_1^N, p_2^N, p_3^N).
- Scenario 2 $\{A_1, A_2, A_3\} \rightsquigarrow \{A_1, A_3\}$ i.e., A_2 leaves the grand coaltion at time instant T. Then at time instant $T+1$, three cases are possible: either A_1 and A_3 continue to cooperate in the rest of the game; either A_1 breaks the alliance he had with A_3; or A_3 breaks the alliance he had with A_1. In these two latter cases, no more cooperation can emerge in the rest of the game.
- Scenario 3 $\{A_1, A_2, A_3\} \rightsquigarrow \{A_2, A_3\}$ i.e., A_1 leaves the grand coalition at time instant T. Then at time instant $T+1$, three cases can be envisaged: either A_2 and A_3 continue to cooperate in the rest of the game; either A_2 breaks the alliance he had with A_3; or A_3 breaks the alliance he had with A_2. In these two latter cases, competition becomes total forever.

The main hypotheses of these section are recalled below.

Hypotheses

- $\mathcal{I}(\alpha_i) = \frac{q_i^2}{2}$,
- two providers cannot simultaneously leave the alliance,
- contrary to Section III, for each coalition, the regulatory authority divides equally its total revenue between its members,
- $\delta \in]0;1[$ is the discount factor of the repeated game.

Study of Scenario 1 We let U_{ij}^* be the coalition $\{A_i, A_j\}$ maximum revenue when both A_i and A_j cooperate; it is simply defined as the maximized sum of A_i and A_j's utilities. Similarly, U^* denote the grand coalition maximum revenue when the three providers cooperate; it coincide with the maximized sum of the three providers utilities. If

$$\frac{1}{2}U_{12}^* \geq \max\Big\{(1-\delta)U_1(p_1^N, p_2^*, p_3^N) + \delta U_1(p_1^N, p_2^N, p_3^N); \\ (1-\delta)U_2(p_1^*, p_2^N, p_3^N) + \delta U_2(p_1^N, p_2^N, p_3^N)\Big\}$$

and

$$U_3(p_1^*, p_2^*, p_3^N) - \frac{1}{3}U^* > 0 \tag{1}$$

then, A_3 deviates at time instant T and then, A_1 and A_2 cooperate for the rest of the game.

If

$$\frac{1}{2}U_{12}^* < (1-\delta)U_1(p_1^N, p_2^*, p_3^N) + \delta U_1(p_1^N, p_2^N, p_3^N)$$

and

$$(1-\delta)U_3(p_1^*, p_2^*, p_3^N) + (1-\delta)\delta U_3(p_1^N, p_2^*, p_3^N) \\ + \quad \delta^2 U_3(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \tag{2}$$

[11] T, is fixed arbitrarily.

[12] We assume that the discrete time steps are large enough so that providers have time to realize that one of them has left the alliance.

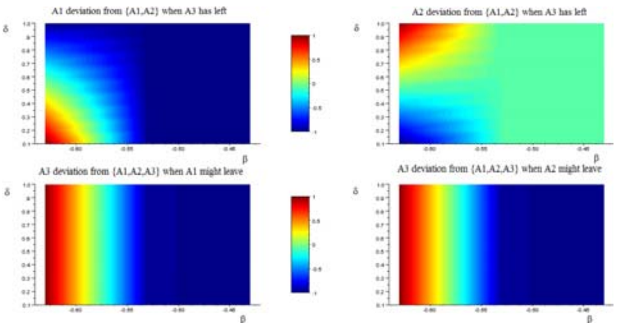


Fig. 10. Conditions on β and δ under which A_3 leaves the grand coalition and no cooperation between the providers can emerge, $N = 450.10^3$.

then, A_3 deviates at time instant T and then, A_1 deviates at time instant $T+1$ and no cooperation cannot be envisaged anymore for the rest of the game.

If

$$\frac{1}{2}U_{12}^* < (1-\delta)U_2(p_1^*, p_2^N, p_3^N) + \delta U_2(p_1^N, p_2^N, p_3^N)$$
and
$$(1-\delta)U_3(p_1^*, p_2^*, p_3^N) + \delta(1-\delta)U_3(p_1^*, p_2^N, p_3^N) + \delta^2 U_3(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \quad (3)$$

then, A_3 leaves the grand coalition at time instant T and then, A_2 deviates at time instant $T+1$ and the providers do not cooperate anymore in the rest of the game.

Proposition 6. *If provider A_3 leaves the grand coalition then, there does not exist any value of $\beta \in [-0.78; 0]$ and $\delta \in]0; 1[$ guaranteeing that providers A_1 and A_2 would cooperate.*

Provider A_1 refuses to cooperate with provider A_2 after provider A_3 has left the grand coalition if, and only if, $\delta \in]0; 0.5[$ and $\beta \in [-0.78; -0.58]$.

Provider A_2 refuses to cooperate with provider A_1 after provider A_3 has left the grand coalition if, and only if, $\delta \in [0.5; 1[$ and $\beta \in [-0.78; -0.58]$.

Proof of Proposition 6 A_3 leaves the grand coalition at time instant T if the profit he expects to receive from this deviation is superior to the profit that he would receive if he continues to cooperate with A_1 and A_2. In order to valuate A_3's worth of deviation, this latter should distinguish between three cases

- At time instant $T+1$, A_1 and A_2 understanding that A_3 has deviated punish him by forming a coalition. In this case, A_3 deviates in spite of his rival's threats if, and only if $\frac{1}{3}\sum_{t=0}^{+\infty}\delta^t U^* < \frac{1}{3}\sum_{t=0}^{T-1}\delta^t U^* + \delta^T U_3(p_1^*, p_2^*, p_3^N) + \underbrace{\sum_{t=T+1}^{+\infty}\delta^t U_3(p_1^*, p_2^*, p_3^N)}_{\text{punishment}}$.
- At time instant $T+1$, A_1 understands that A_3 has deviated and to punish him refuses to cooperate once more with A_2. A_3 deviates in spite of A_1's threat if, and only if $\frac{1}{3}\sum_{t=0}^{+\infty}\delta^t U^* < \frac{1}{3}\sum_{t=0}^{T-1}\delta^t U^* + \delta^T U_3(p_1^*, p_2^*, p_3^N) + \delta^{T+1}U_3(p_1^N, p_2^*, p_3^N) + \underbrace{\sum_{t=T+2}^{+\infty}\delta^t U_3(p_1^N, p_2^N, p_3^N)}_{\text{punishment}}$.
- At time instant $T+1$, A_2 understands that A_3 has deviated and to punish him refuses to cooperate once more with A_1. A_3 deviates in spite of A_2's threat if, and only if $\frac{1}{3}\sum_{t=0}^{+\infty}\delta^t U^* < \frac{1}{3}\sum_{t=0}^{T-1}\delta^t U^* + \delta^T U_3(p_1^*, p_2^*, p_3^N) + \delta^{T+1}U_3(p_1^*, p_2^N, p_3^N) + \underbrace{\sum_{t=T+2}^{+\infty}\delta^t U_3(p_1^N, p_2^N, p_3^N)}_{\text{punishment}}$.

A few simplifications later, we obtain Inequalities (1), (2), (3).

We let $F_0 = \frac{1}{2}U_{12}^*$, $F_1 = (1-\delta)U_1(p_1^N, p_2^*, p_3^N) + \delta U_1(p_1^N, p_2^N, p_3^N)$, $F_2 = (1-\delta)U_2(p_1^*, p_2^N, p_3^N) + \delta U_2(p_1^N, p_2^N, p_3^N)$. $F_0 - F_1 > 0$ if, and only if A_1 has no incentives to leave $\{A_1, A_2\}$. $F_0 - F_2 > 0$ if, and only if A_2 has no incentives to leave $\{A_1, A_2\}$. At the top of Figure 10, we have depicted $F_0 - F_1$ (left) and $F_0 - F_2$ (right). Since $\{F_0 - F_1\} \cap \{F_0 - F_2\} = \emptyset$, A_1 and A_2 would not cooperate if A_3 deviates. At the bottom of Figure 10, we have depicted $F_4 = (1-\delta)U_3(p_1^*, p_2^*, p_3^N) + \delta(1-\delta)U_3(p_1^N, p_2^*, p_3^N) + \delta^2 U_3(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^*$ (left) and $F_5 = (1-\delta)U_3(p_1^*, p_2^*, p_3^N) + \delta(1-\delta)U_3(p_1^*, p_2^N, p_3^N) + \delta^2 U_3(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^*$ (right). $F_4 > 0$ if, and only if A_3 deviates eventhough he thinks that A_1 will quickly realize it and to punish him, will refuse to cooperate with A_2 anymore. Identically, $F_5 > 0$ if, and only if A_3 deviates eventhough he thinks that A_2 will quickly relaize it and to punish him, will refuse to cooperate once more with A_1. Depending on δ and β values, we identify the system evolution during the course of the repeated game and obtain Proposition 6.

□

Study of Scenario 2 If

$$\frac{1}{2}U_{13}^* \geq \max\Big\{(1-\delta)U_1(p_1^N, p_2^N, p_3^*) + \delta U_1(p_1^N, p_2^N, p_3^N); (1-\delta)U_3(p_1^*, p_2^N, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)\Big\}$$
and
$$U_2(p_1^*, p_2^N, p_3^*) - \frac{1}{3}U^* > 0 \quad (4)$$

then, A_2 deviates and then, A_1 and A_3 cooperate during the rest of the game.

If

$$\frac{1}{2}U_{13}^* < (1-\delta)U_1(p_1^N, p_2^N, p_3^*) + \delta U_1(p_1^N, p_2^N, p_3^N)$$
and
$$(1-\delta)U_2(p_1^*, p_2^N, p_3^*) + \delta(1-\delta)U_2(p_1^N, p_2^N, p_3^*) + \delta^2 U_2(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \quad (5)$$

then, A_2 deviates and then, A_1 refuses to cooperate with A_3 solely and the providers are in competition during the rest of the game.

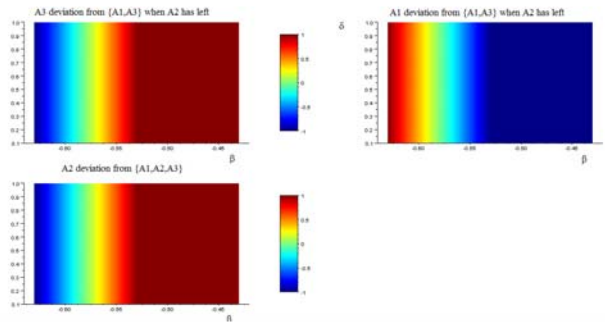


Fig. 11. Conditions on β and N guaranteeing that A_2 leaves the grand coalition and no cooperation occurs between A_1 and A_3, $N = 450.10^3$.

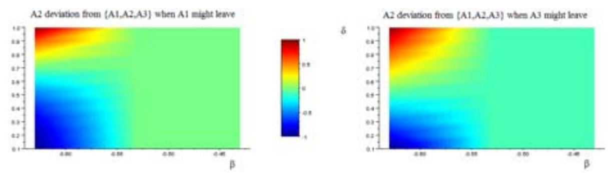


Fig. 12. Conditions on β and δ giving incentives for A_1 or A_3 to not cooperate when A_2 has already left, $N = 450.10^3$.

If

$$\frac{1}{2}U^*_{13} < (1-\delta)U_3(p_1^*, p_2^N, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)$$
and
$$(1-\delta)U_2(p_1^*, p_2^N, p_3^*) + \delta(1-\delta)U_2(p_1^*, p_2^N, p_3^N)$$
$$+\delta^2 U_2(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \qquad (6)$$

then, A_2 leaves the grand coalition and then, A_3 deviates and no more cooperation can emerge during the rest of the game.

Proposition 7. *If $\beta \in [-0.78; -0.57[$ then, provider A_2 does not deviate provided he thinks that providers A_1 and A_3 are going to cooperate. In this case, the three providers stay in the grand coalition.*

On the contrary, if provider A_2 thinks that the other firms have incentives to deviate then, his strategy is different.

If $\beta \in [-0.78; -0.57]$ and $\delta \in [0.65; 0.8[$ then, provider A_2 and then, provider A_3 deviate. But, if $\beta \in [-0.78; -0.57]$ and $\delta \in [0.8; 1[$ or $\beta \in [-0.57; 0]$ and $\delta \in]0; 1[$ then, provider A_2 and then, provider A_3 or provider A_1 deviates.

Proof of Proposition 7 A_2 deviates at time instant T from the grand coalition if the profit he receives from this deviation is superior to the profit that he should have received if he had continued to cooperate with A_1 and A_3. In the same spirit as in the proof of Proposition 6, we need to distinguish between three cases.

- A_2 leaves the grand coalition at time instant T then at time instant $T+1$, A_1 and A_3 realize it and decide to cooperate. A_2 chooses to deviate in spite of $\{A_1, A_2\}$'s threat if, and only if $\frac{1}{3}\sum_{t=0}^{+\infty}\delta^t U^* < \frac{1}{3}\sum_{t=0}^{T-1}\delta^t U^* + \underbrace{\sum_{t=T}^{+\infty}\delta^t U_2(p_1^*, p_2^N, p_3^*)}_{\text{punishment}}$.
- A_2 leaves the grand coalition at time instant T and then at time instant $T+1$, A_1 realizing it, refuses to cooperate once more with A_3. The three providers are then in competition for the rest of the game. A_2 deviates in spite of A_1's threat if and only if $\frac{1}{3}U^* < (1-\delta)U_2(p_1^*, p_2^N, p_3^*) + \delta(1-\delta)U_2(p_1^N, p_2^N, p_3^*) + \delta^2 U_2(p_1^N, p_2^N, p_3^N)$.
- A_2 leaves the grand coalition at time instant T and then at time instant $T+1$, A_3 realizing it, refuses to cooperate once more with A_1. The three providers are then in competition for the rest of the game. A_2 deviates in spite of A_3's threat if, and only if $\frac{1}{3}U^* < (1-\delta)U_2(p_1^*, p_2^N, p_3^*) + \delta(1-\delta)U_2(p_1^*, p_2^N, p_3^N) + \delta^2 U_2(p_1^N, p_2^N, p_3^N)$.

We let $F_0 = \frac{1}{2}U^*_{13}$, $F_1 = (1-\delta)U_1(p_1^N, p_2^N, p_3^*) + \delta U_1(p_1^N, p_2^N, p_3^N)$, $F_2 = (1-\delta)U_3(p_1^*, p_2^N, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)$. At the bottom of Figure 11, we have pictured F_3. $F_3 > 0$ for $\beta \in [-0.66; -0.56[$ i.e., A_2 has no incentives to leave the grand coalition for these values of β. At the top of Figure 11, we observe $F_0 - F_1$ (left) and $F_0 - F_2$ (right). We infer that if A_2 leaves the grand coaltion, A_1 and A_3 will never cooperate. Figure 12, we have depicted F_4 (left) and F_5 (right). We infer the system evolution towards complete competition as function of β and of δ.

□

Study of Scenario 3 If

$$\frac{1}{2}U^*_{23} \geq \max\{(1-\delta)U_2(p_1^N, p_2^N, p_3^*) + \delta U_2(p_1^N, p_2^N, p_3^N);$$
$$(1-\delta)U_3(p_1^N, p_2^*, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)\}$$
$$U_1(p_1^N, p_2^*, p_3^*) - \frac{1}{3}U^* > 0 \qquad (7)$$

then, A_1 deviates and then, A_2 and A_3 continue to cooperate during the rest of the game.

If

$$\frac{1}{2}U^*_{23} < (1-\delta)U_2(p_1^N, p_2^N, p_3^*) + \delta U_2(p_1^N, p_2^N, p_3^N)$$
$$(1-\delta)U_1(p_1^N, p_2^*, p_3^*) + \delta(1-\delta)U_1(p_1^N, p_2^N, p_3^*)$$
$$+\delta^2 U_1(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \qquad (8)$$

then, A_1 deviates and then, A_2 refuses to cooperate with A_3 only and leaves the coalition. The three providers are in competition for the rest of the game.

If

$$\frac{1}{2}U^*_{23} < (1-\delta)U_3(p_1^N, p_2^*, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)$$
$$(1-\delta)U_1(p_1^N, p_2^*, p_3^N) + \delta(1-\delta)U_1(p_1^N, p_2^*, p_3^N)$$
$$+\delta^2 U_1(p_1^N, p_2^N, p_3^N) - \frac{1}{3}U^* > 0 \qquad (9)$$

then, A_1 deviates and then, A_3 breaks the alliance with A_2. The three providers are in competition during the rest of the game.

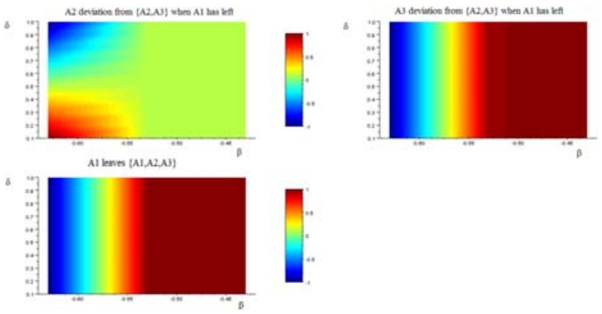


Fig. 13. Conditions on β and δ guaranteeing that A_1 will leave the grand coalition and then, A_2 and A_3 will refuse to cooperate, $N = 450.10^3$.

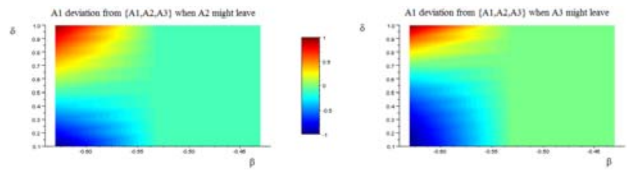


Fig. 14. Conditions on β and δ giving incentives for A_2 or A_3 to not cooperate when A_1 has already left, $N = 450.10^3$.

Proposition 8. *If provider A_1 thinks that providers A_2 and A_3 will cooperate, he deviates from the grand coalition if, and only if, $\beta \in [-0.56; 0]$.*

Provider A_1 leaves the grand coalition and then, provider A_2 deviates if, and only if, $\beta \in [-0.78; -0.56]$ and $\delta \in [0.67; 0.8[$.

Provider A_1 leaves the grand coalition and then, provider A_2 or provider A_3 deviates if, and only if $\beta \in [-0.78; -0.56]$ and $\delta \in [0.8; 1[$.

Proof of Proposition 8 In the same spirit as in the proofs of Propositions 6 and 7, A_1 deviates from the grand coalition if the profit he receives from this deviation is superior to the profit he would have received if he had continued to cooperate with A_2 and A_3. Three cases should be distinguised. The arguments are the same as thos detailed in the previous propositions; hence it will not be detailed.

Let $F_0 = \frac{1}{2}U_{23}^*$, $F_1 = (1-\delta)U_2(p_1^N, p_2^N, p_3^*) + \delta U_2(p_1^N, p_2^N, p_3^N)$, $F_2 = (1-\delta)U_3(p_1^N, p_2^*, p_3^N) + \delta U_3(p_1^N, p_2^N, p_3^N)$ and $F_3 = U_1(p_1^N, p_2^*, p_3^*) - \frac{1}{3}U^*$. At the botton of Figure 13, we have pictured F_3. $F_3 > 0$ for $\beta \in [-0.56; -0.43]$ if, and only if A_1 leaves the grand coalition eventhough he thinks that A_2 and A_3 will punish him by forming a coalition. Let $F_4 = (1-\delta)U_1(p_1^N, p_2^*, p_3^*) + \delta(1-\delta)U_1(p_1^N, p_2^N, p_3^*) + \delta^2 U_1(p_1^N, p_2^N, p_3^N)$ and $F_5 = (1-\delta)U_1(p_1^N, p_2^*, p_3^*) + \delta(1-\delta)U_1(p_1^N, p_2^*, p_3^N) + \delta^2 U_1(p_1^N, p_2^N, p_3^N)$. On figure 14, we observe F_4 (left) and F_5 (right). Depending on β and δ values, we determine the system dynamic evolution.

□

Proposition 9. *For the regulatory authority, it is sufficient to choose an interest rate smaller than 53% to maximize the market coverage and a fortiori, the consumer welfare.*

Proof of Proposition 9 Using Proposition 6, 7 and 8, we could infer the system's behavior as function of β and δ values.

Using Proposition 6

- If $\delta \in]0; 0.5[$ and $\beta \in [-0.78; -0.58[$ then, A_3 leaves the grand coalition and then, A_1 refuses to cooperate once more with A_2. No cooperation cannot be envisaged in the rest of the game.
- If $\delta \in [0.5; 1[$ and $\beta \in [-0.78; -0.58[$ then, A_3 leaves the grand coalition and then, A_2 refuses to cooperate once more with A_1. No cooperation cannot be envisaged in the rest of the game.
- If $\delta \in]0; 1[$ and $\beta \in [-0.58; 0]$ then, $\{A_1, A_2, A_3\}$ cooperate.

Then, using Proposition 7

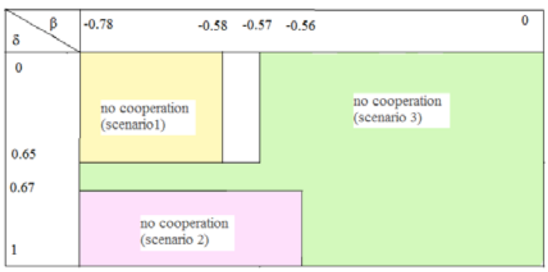
- If $\delta \in]0; 0.65[$ and $\beta \in [-0.78; -0.57[$ the three providers cooperate.
- If $\delta \in [0.65; 0.8[$ and $\beta \in [-0.578 - 0.57]$ then, A_2 leaves the grand coalition and then, A_3 breaks the alliance he had with A_1. No cooperation is possible in the rest of the game.
- If $\delta \in [0.8; 1[$ and $\beta \in [-0.78; -0.57]$ then, A_2 leaves the grand coalition and then, A_3 *or* A_1 refuses to cooperate. No cooperation can emerge in the rest of the game.

Finally, using Proposition 8

- If $\delta \in]0; 0.67[$ and $\beta \in [-0.78; -0.56[$ or $\delta \in [0.1]$ and $\beta \in [-0.56; 0]$ then, the three providers cooperate.
- If $\delta \in [0.67; 0.8[$ and $\beta \in [-0.78; -0.56]$ then, A_1 leaves the grand coalition and then, A_2 refuses to cooperate. No cooperation can emerge in the rest of the game.
- If $\delta \in [0.8; 1[$ and $\beta \in [-0.78; -0.56]$ then, A_1 leaves the grand coaliton and then, A_2 *or* A_3 refuses to cooperate. Cooperation cannot be envisaged in the rest of the game.

The discount factor can be expressed as a function of the interest rate: $\delta = \frac{1}{1+r}$ with $r > 0$ the per-period interest rate imposed by the regulatory authority [2]. It is equivalent with $r = \frac{1}{\delta} - 1$.

TABLE III
AREA OF β AND δ WHERE SCENARIOS HOLD.



Comparing Propositions 6, 7 and 8 results (cf. Table III), we observe that cooperation might emerge if, and only if, $\beta \in [-0.58; -0.57]$ and $\delta \in [0; 0.65]$. Hence, to prevent cooperation from occuring, the regulatory authority should impose that $0.65 < \delta < 1 \Leftrightarrow 0 < r < 0.53$. Hence, the interest rate r, should be smaller than 53% to insure that competition is total between the three service providers.

$\square$

V. Conclusions

In this article, we have studied the game that arises between three service providers in competition, who have the opportunity to cooperate in order to increase their profits. We prove that if the consumers' sensitivity to the quality of service / content coefficient (β) is inferior to -0.78 then, the Shapley value is a fair way to share the grand coalition revenue between the service providers and belongs to the core of the game i.e., provided the three service providers agree on this sharing rule the game remains stable. But if $\beta > -0.78$, the regulatory authority should intervene since the system behavior might be highly unstable penalizing the consumers and the weakest providers. We show that under equal allocations of the coalitions' revenues, if the regulatory authority imposes a per-period interest rate smaller than 53%, the consumer welfare is guaranteed and the market coverage is maximized.

Extensions of the article might consider other sharing rules such as the nucleolus or the proportional fairness criteria [15]. The game resolution can also be extended to deal with far many service providers in interaction and / or various sources of uncertainty. To perform this point, simulation and stochastic processes should be introduced [17].

Appendix

A Proofs

We just recall some basic notations that could be useful to understand the following proofs. Under complete competition, provider i's optimal decision will be denoted p_i^N for the access price and q_i^N for the optimal QoS / content level. Provider i's maximized utility is U_i^N. When firms A_i and A_j form a coalition against A_k, the firms optimal decisions will be denoted $p_i^{A_iA_j}, p_j^{A_iA_j}, p_k^{A_iA_j}$ for the prices and $q_i^{A_iA_j}, q_j^{A_iA_j}, q_k^{A_iA_j}$ for the QoS / content levels. Then, the coalition maximized utility is denoted U_{ij}^* while A_k maximized utility is U_k^*. Finally, when all the providers enter a grand coalition, its optimal utility is denoted U^*.

Computation of $\nu(A_1)$**,** $\nu(A_2)$**,** $\nu(A_3)$ In this case, the three providers are in competition and no cooperation can emerge. By differentiation of each provider's utility, we obtain that the optimal access prices for A_1, A_2 et A_3 are of the form

$$
\begin{aligned}
p_1 &= \frac{1}{8}\Big[\beta(q_3+2q_2-4q_1)+1\Big]\\
p_2 &= -\frac{1}{4}\Big[\beta(-q_3+2q_2)-1\Big]\\
p_3 &= -\frac{1}{2}\Big[\beta q_3-1\Big]
\end{aligned}
$$

By substitution of the price in the providers' utilities, we obtain

$$
\begin{aligned}
U_1 &= \frac{1}{8}((\beta q_3+2\beta q_2-4\beta q_1+1)(\frac{1}{4}(\beta q_3-2\beta q_2+1)\\
&+ \frac{1}{8}(-\beta q_3-2\beta q_2+4\beta q_1-1)\\
&+ \beta q_2-\beta q_1)N)-\mathcal{I}o\vartheta(q_1)\\
U_2 &= \frac{1}{4}((\beta q_3-2\beta q_2+1)(\frac{1}{4}(-\beta q_3+2\beta q_2-1)\\
&+ \frac{1}{2}(1-\beta q_3)+\beta q_3-\beta q_2)N)-\mathcal{I}o\vartheta(q2)\\
U_3 &= \frac{1}{2}((1-\beta q_3)((\beta q_3-1)/2-\beta q_3+1)N)-\mathcal{I}o\vartheta(q_3)
\end{aligned}
$$

Then, to get the optimal quality of service / content q_i^*, it is sufficient to differentiate U_i with respect to q_i and to solve the associated linear system.

As previously mentioned, in order to fix the ideas, we consider two examples. In the first example, the information level is quadratic in the quality of service / content level i.e., $\alpha_i = \frac{q_i^2}{2} \Leftrightarrow q_i = \sqrt{2\alpha_i}$ and the information cost function $\mathcal{I}(.)$ is the identity. In the second example, the information level is linear in the quality of service / content level i.e., $\alpha_i = q_i$ while the information cost function $\mathcal{I}(.)$ is still the identity.

Example 1. *Information level quadratic in the quality of service / content level* In this case, the maximized utilities for each firm $A_i,\ i=1,2,3$ can be written as

$$
\begin{aligned}
U_1^N &= \{-\beta^8N^5+4\beta^6N^4-6\beta^4N^3+4\beta^2N^2-N\}\\
&\dots\ \{2\beta^{10}N^5-20\beta^8N^4+80\beta^6N^3\\
&- 160\beta^4N^2+160\beta^2N-64\}^{-1}\\
U_2^N &= \frac{-\beta^4N^3+2\beta^2N^2-N}{2\beta^6N^3-12\beta^4N^2+24\beta^2N-16}\\
U_3^N &= \frac{1}{2}(N(\frac{1-(\beta^2N)}{\beta^2N-2})\frac{1}{2}(\frac{\beta^2N}{\beta^2N-2}-1)-\frac{\beta^2N}{\beta^2N-2}\\
&+ 1)-\frac{\beta^2N^2}{2(\beta^2N-2)^2}
\end{aligned}
$$

The optimal quality of service / content levels for each firm are

$$
\begin{aligned}
q_1^N &= \frac{\beta^5N^3-2\beta^3N^2+\beta N}{\beta^6N^3-6\beta^4N^2+12\beta^2N-8}\\
q_2^N &= \frac{\beta^3N^2-\beta N}{\beta^4N^2-4\beta^2N+4}\\
q_3^N &= \frac{\beta N}{\beta^2N-2}
\end{aligned}
$$

Finally, by substitution of the optimal prices in the previous QoS / content level, we obtain the access price final expressions

$$
\begin{aligned}
p_1^N &= \frac{1}{8}(-\frac{4\beta(\beta^5N^3-2\beta^3N^2+\beta N)}{\beta^6N^3-6\beta^4N^2+12\beta^2N-8}\\
&+ \frac{2\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+4}+\frac{\beta^2N}{\beta^2N-2}+1)\\
p_2^N &= \frac{1}{4}(-\frac{2\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+4}+\frac{\beta^2N}{\beta^2N-2}+1)\\
p_3^N &= \frac{1}{2}(1-\frac{\beta^2N}{\beta^2N-2})
\end{aligned}
$$

Example 2. *Information level quadratic in the quality of service / content level* The maximized utilities for each of the three providers are identical

$$U_1^N = U_2^N = U_3^N = \frac{-\beta N - 3}{\beta^2 N}$$

The quality of service / content levels are obtained

$$\begin{aligned} q_1^N &= \frac{\beta N + 4}{\beta^2 N} \\ q_2^N &= \frac{\beta N + 3}{\beta^2 N} \\ q_3^N &= \frac{\beta N + 2}{\beta^2 N} \end{aligned}$$

Finally, the optimal access prices for each firm are

$$\begin{aligned} p_1^N &= \frac{1}{8}(-\frac{4(\beta N + 4)}{\beta N} + (2\frac{\beta N + 3)}{\beta N} + \frac{\beta N + 2}{\beta N} + 1) \\ p_2^N &= \frac{1}{4}(-\frac{2(\beta N + 3)}{\beta N} + \frac{\beta N + 2}{\beta N} + 1) \\ p_3^N &= \frac{1}{2}(1 - \frac{\beta N + 2}{\beta N}) \end{aligned}$$

Computation of $\nu(A_1, A_2)$ In this case, firms A_1 and A_2 form an alliance against firm A_3 who remains alone. The optimal prices p_1 and p_2, are obtained by differentiation of the alliance's utility $U_1 + U_2$. A_3's optimal access price p_3, is obtained by differentiation of U_3. We get

$$\begin{aligned} p_1 &= \frac{1}{6}\Big[\beta(q_3 + 2q_2 - 4q_1) + 1\Big] \\ p_2 &= -\frac{1}{3}\Big[\beta(-q_3 + q_2 + q_1) - 1\Big] \\ p_3 &= -\frac{1}{2}\Big[\beta q_3 - 1\Big] \end{aligned}$$

By substitution of the access prices, the provider A_3 and the alliance utilities become

$$\begin{aligned} U_3 &= \frac{1}{4}\Big((\beta^2 q_3^2 - 2\beta q_3 + 1)N - 4\mathcal{I}o\vartheta(q_3)\Big) \\ U_1 + U_2 &= \frac{1}{12}\Big((\beta^2 q_3^2 + (-2\beta^2 q_2 - 2\beta^2 q_1 + 2\beta)q_3 \\ &+ 4\beta^2 q_2^2 + (-4\beta^2 q_1 - 2\beta)q_2 + 4\beta^2 q_1^2 - 2\beta q_1 \\ &+ 1)N - 12\mathcal{I}o\vartheta(q_2) - 12\mathcal{I}o\vartheta(q_1)\Big) \end{aligned}$$

Example 1. *Information level quadratic in the quality of service / content level* The maximized utilities for alliance $\{A_1, A_2\}$ and its rival A_3 are

$$\begin{aligned} U_{12}^* &= -\frac{\beta^4 N^3 - 2\beta^2 N^2 + N}{\beta^6 N^3 - 7\beta^4 N^2 + 16\beta^2 N - 12} \\ U_3^* &= -\frac{N}{2\beta^2 N - 4} \end{aligned}$$

The optimal quality of service / content levels are of the form

$$\begin{aligned} q_1^{A_1A_2} &= q_2^{A_1A_2} = \frac{\beta^3 N^2 - 2\beta N}{\beta^4 N^2 - 10\beta^2 N + 12} \\ q_3^{A_1A_2} &= \frac{\beta N}{\beta^2 N - 4} \end{aligned}$$

Finally, the optimal access prices for the partners in the alliance $\{A_1, A_2\}$ and for A_3 are

$$\begin{aligned} p_1^{A_1A_2} &= \frac{1}{6}(-\frac{2\beta(\beta^3 N^2 - 2\beta N)}{\beta^4 N^2 - 10\beta^2 N + 12} + \frac{\beta^2 N}{\beta^2 N - 4} + 1) \\ p_2^{A_1A_2} &= \frac{1}{3}(-\frac{2\beta(\beta^3 N^2 - 2\beta N)}{\beta^4 N^2 - 10\beta^2 N + 12} + \frac{\beta^2 N}{\beta^2 N - 4} + 1) \\ p_3^{A_1A_2} &= \frac{1}{2}(1 - \frac{\beta^2 N}{\beta^2 N - 2}) \end{aligned}$$

Example 2. *Information level linear in the quality of service / content level* The maximized utilities for coalition $\{A_1, A_2\}$ against A_3 are

$$\begin{aligned} U_{12}^* &= -(2\beta^7 N^4 + (-12\beta^5 - 3\beta^4)N^3 + (22\beta^3 + 6\beta^2)N^2 \\ &+ (-12\beta - 3)N)(\beta^8 N^4 - 10\beta^6 N^3 + 37\beta^4 N^2 \\ &- 60\beta^2 N + 36)^{-1} \\ U_3^* &= \frac{-\beta^3 N^2 - (-2\beta - 1)N}{\beta^4 N^2 - 4\beta^2 N + 4} \end{aligned}$$

The optimal quality of service / content levels are obtained

$$\begin{aligned} q_1^{A_1A_2} &= \frac{\beta^5 N^3 - 2\beta^3 N^2 + \beta N}{\beta^6 N^3 - 6\beta^4 N^2 + 11\beta^2 N - 6} \\ q_2^{A_1A_2} &= \frac{\beta^5 N^3 - 2\beta^3 N^2 + \beta N}{\beta^6 N^3 - 6\beta^4 N^2 + 11\beta^2 N - 6} \\ q_3^{A_1A_2} &= \frac{\beta N}{\beta^2 N - 2} \end{aligned}$$

Finally, the optimal prices become

$$\begin{aligned} p_1^{A_1A_2} &= \frac{1}{6}(-\frac{2\beta(\beta^5 N^3 - 2\beta^3 N^2 + \beta N)}{\beta^6 N^3 - 6\beta^4 N^2 + 11\beta^2 N - 6} + \frac{\beta^2 N}{\beta^2 N - 2} \\ &+ 1) \\ p_2^{A_1A_2} &= \frac{1}{3}(-\frac{2\beta(\beta^5 N^3 - 2\beta^3 N^2 + \beta N)}{\beta^6 N^3 - 6\beta^4 N^2 + 11\beta^2 N - 6} + \frac{\beta^2 N}{\beta^2 N - 2} \\ &+ 1) \\ p_3^{A_1A_2} &= \frac{1}{2}(1 - \frac{\beta^2 N}{\beta^2 N - 2}) \end{aligned}$$

Computation of $\nu(A_1, A_3)$

Firms A_1 and A_3 form a coalition against firm A_2 who is alone. By differentitation of $\{A_1, A_3\}$'s utilities and of firm A_2's utility, we obtain the optimal prices for the three providers

$$\begin{aligned} p_1 &= \frac{1}{8}\Big[\beta(q_3 + 2q_2 - 4q_1) + 1\Big] \\ p_2 &= -\frac{1}{4}\Big[\beta(-q_3 + 2q_2) - 1\Big] \\ p_3 &= -\frac{1}{2}\Big[\beta q_3 - 1\Big] \end{aligned}$$

Then, substituting these optimal prices, coalition $\{A_1, A_3\}$'s utility and A_2's utility become

$$\begin{aligned} U_1+U_3 &= \frac{1}{64}((17\beta^2 q_3^2+(4\beta^2 q_2-8\beta^2 q_1-30\beta)q_3 \\ &+ 4\beta^2 q_2^2+(4\beta-16\beta^2 q_1)q_2+16\beta^2 q_1^2 \\ &- 8\beta q_1+17)N-64\mathcal{I}o\vartheta(q_3)-64\mathcal{I}o\vartheta(q_1)) \\ U_2 &= \frac{1}{16}((\beta^2 q_3^2+(2\beta-4\beta^2 q_2)q_3+4\beta^2 q_2^2-4\beta q_2 \\ &+ 1)N-16\mathcal{I}o\vartheta(q2)) \end{aligned}$$

Example 1. *Information level quadratic in the quality of service / content level* The maximized utilities for coalition $\{A_1, A_3\}$ and firm A_2 are

$$\begin{aligned} U_2^* &= (-16\beta^10N^6+128\beta^8N^5-400\beta^6N^4+608\beta^4N^3 \\ -4 &\quad 48\beta^2N^2+128N)(32\beta^12N^6-400\beta^{10}N^5 \\ &+ 2034\beta^8N^4-5412\beta^6N^3+8002\beta^4N^2-6272\beta^2N \\ &+ 2048)^{-1} \\ U_{13}^* &= -(32\beta^{10}N^6-266\beta^8N^5+909\beta^6N^4-1604\beta^4N^3 \\ &+ 1457\beta^2N^2-544N)(32\beta^{12}N^6-400\beta^{10}N^5 \\ &+ 2034\beta^8N^4-5412\beta^6N^3+8002\beta^4N^2-6272\beta^2N \\ &+ 2048)^{-1} \end{aligned}$$

The optimal quality of service / content levels are

$$\begin{aligned} q_1^{A_1A_3} &= \frac{4\beta^5N^3-8\beta^3N^2+4\beta N}{4\beta^6N^3-25\beta^4N^2+49\beta^2N-32} \\ q_3^{A_1A_3} &= \frac{4\beta^5N^3-15\beta^3N^2+15\beta N}{4\beta^6N^3-25\beta^4N^2+49\beta^2N-32} \\ q_2^{A_1A_3} &= \frac{4\beta^5N^3-12\beta^3N^2+8\beta N}{4\beta^6N^3-25\beta^4N^2+49\beta^2N-32} \end{aligned}$$

Finally, the optimal prices for the providers take the following expression

$$\begin{aligned} p_1^{A_1A_3} &= \frac{1}{6}(-\frac{2\beta(\beta^5N^3-2\beta^3N^2+\beta N)}{\beta^6N^3-6\beta^4N^2+11\beta^2N-6} \\ &+ \frac{\beta^2N}{\beta^2N-2}+1) \\ p_2^{A_1A_3} &= \frac{1}{3}(-\frac{2\beta(\beta^5N^3-2\beta^3N^2+\beta N)}{\beta^6N^3-6\beta^4N^2+11\beta^2N-6} \\ &+ \frac{\beta^2N}{\beta^2N-2}+1) \\ p_3^{A_1A_3} &= \frac{1}{2}(1-\frac{\beta^2N}{\beta^2N}-2) \end{aligned}$$

Example 2. *Information level linear in the quality of service / content level* The maximized utilities for coalition $\{A_1, A_3\}$ and provider A_2 are

$$\begin{aligned} U_2^* &= \frac{-4\beta N-9}{4\beta^2N} \\ U_{13}^* &= \frac{-16\beta N-15}{16\beta^2N} \end{aligned}$$

The optimal quality of service / content levels are

$$\begin{aligned} q_1^{A_1A_3} &= \frac{4\beta N+17}{4\beta^2N} \\ q_3^{A_1A_3} &= \frac{2\beta N+5}{2\beta^2N} \\ q_2^{A_1A_3} &= \frac{4\beta N+13}{4\beta^2N} \end{aligned}$$

Finally, the optimal prices are

$$\begin{aligned} p_1^{A_1A_3} &= \frac{1}{8}(-\frac{4\beta N+17}{\beta N}+\frac{4\beta N+13}{2\beta N}+\frac{2\beta N+5}{2\beta N} \\ &+ 1) \\ p_2^{A_1A_3} &= \frac{1}{4}(-\frac{4\beta N+13}{2\beta N}+\frac{2\beta N+5}{2\beta N}+1) \\ p_3^{A_1A_3} &= \frac{1}{2}(1-\frac{2\beta N+5}{2\beta N}) \end{aligned}$$

Computation of $\nu(A_2, A_3)$ In this case, firms A_2 and A_3 form a coalition against firm A_1. The optimal prices are obatined by differentiation of coalition $\{A_2, A_3\}$'s utility and of firm A_1's utility

$$\begin{aligned} p_1 &= \frac{1}{6}\Big[\beta(q_3+q_2-3q_1)+1\Big] \\ p_2 &= \frac{1}{3}\Big[\beta(q_3-2q_2)+1\Big] \\ p_3 &= -\frac{1}{3}\Big[\beta(q_3+q_2)-2\Big] \end{aligned}$$

By substitution of the optimal prices, coalition $\{A_2, A_3\}$'s utility and A_1's utility can be expressed as functions of quality of service / content levels $q_i,\ i=1,2,3$ exclusively; which gives

$$\begin{aligned} U_1 &= \frac{1}{36}((\beta^2q_3^2+(2\beta^2q_2-6\beta^2q_1+2\beta)q_3+\beta^2q_2^2 \\ &+ (2\beta-6\beta^2q_1)q_2+9\beta^2q_1^2-6\beta q_1+1)N \\ &- 36\mathcal{I}o\vartheta(q_1)) \\ U_2+U_3 &= \frac{1}{3}((\beta^2q_3^2+(-\beta^2q_2-\beta)q_3+\beta^2q_2^2-\beta q_2+1)N \\ &- 3\mathcal{I}o\vartheta(q_3)-3\mathcal{I}o\vartheta(q_2)) \end{aligned}$$

Example 1. *Information level quadratic in the quality of service / content level*

The maximized utility for coalition $\{A_2, A_3\}$ and for firm A_1 are

$$\begin{aligned} U_{23}^* &= \frac{2N-2\beta^2N^2}{2\beta^4N^2-8\beta^2N+6} \\ U_1^* &= \frac{-\beta^4N^3+2\beta^2N^2-N}{2\beta^6N^3-16\beta^4N^2+42\beta^2N-36} \end{aligned}$$

The optimal quality of service / content levels are

$$\begin{aligned} q_2^{A_2A_3} &= \frac{\beta^3N^2-\beta N}{\beta^4N^2-4\beta^2N+3} \\ q_3^{A_2A_3} &= \frac{\beta^3N^2-\beta N}{\beta^4N^2-4\beta^2N+3} \\ q_1^{A_2A_3} &= \frac{\beta^3N^2-\beta N}{\beta^4N^2-4\beta^2N+3} \end{aligned}$$

Finally, the optimal access prices are obtained as follows

$$p_1^{A_2A_3} = \frac{1}{6}\left(\frac{2\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+3} - \frac{3\beta(\beta^3N^2-\beta N)}{\beta^4N^2-5\beta^2N+6} + 1\right)$$
$$p_2^{A_2A_3} = \frac{1}{3}\left(1 - \frac{\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+3}\right)$$
$$p_3^{A_2A_3} = \frac{1}{3}\left(2 - \frac{2\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+3}\right)$$

Example 2. *Information level linear in the quality of service / content level* The maximized utilities for coalition $\{A_2, A_3\}$ and firm A_1 are

$$U_{23}^* = \frac{-\beta N-3}{\beta^2N}$$
$$U_1^* = \frac{-\beta N-3}{\beta^2N}$$

The optimal quality of service / content levels are

$$q_2^{A_2A_3} = \frac{\beta N+3}{\beta^2N}$$
$$q_3^{A_2A_3} = \frac{\beta N+3}{\beta^2N}$$
$$q_1^{A_2A_3} = \frac{\beta N+4}{\beta^2N}$$

Finally, the optimal access prices are

$$p_1^{A_2A_3} = \frac{1}{6}\left(-\frac{3\beta(\beta^3N^2-\beta N)}{\beta^4N^2-5\beta^2N+6} + \frac{2(\beta N+3)}{\beta N+1}\right)$$
$$p_2^{A_2A_3} = \frac{1}{3}\left(1 - \frac{\beta(\beta^3N^2-\beta N)}{\beta^4N^2-4\beta^2N+3}\right)$$
$$p_3^{A_2A_3} = \frac{1}{3}\left(2 - \frac{2(\beta N+3)}{\beta N}\right)$$

Computation of $\nu(A_1, A_2, A_3)$ In this case, the three firms are in the grand coalition. The optimal access prices for the grand coalition are obtained by differentiation of $U_1+U_2+U_3$

$$p_1 = \frac{1}{4}\Big[\beta(q_3+q_2-3q_1)+1\Big]$$
$$p_2 = -\frac{1}{2}\Big[\beta(-q_3+q_2+q_1)-1\Big]$$
$$p_3 = -\frac{1}{4}\Big[\beta(q_3+q_2+q_1)-3\Big]$$

By substitution, the grand coalition's utility takes the form

$$\begin{aligned} U_1+U_2+U_3 &= \frac{1}{8}\Big[(3\beta^2q_3^2 + (-2\beta^2q_2 - 2\beta^2q_1 - 2\beta)q_3 \\ &+ 3\beta^2q_2^2 + (-2\beta^2q_1 - 2\beta)q_2 + 3\beta^2q_1^2 \\ &- 2\beta q_1 + 3)N - 8\sum_{i=1,2,3}\mathcal{I}o\vartheta(q_i)\Big] \end{aligned}$$

Example 1. *Information level quadratic in the quality of service / content level* The grand coalition's maximized utility is

$$U^* = \frac{3N-3\beta^2N^2}{2\beta^4N^2-10\beta^2N+8}$$

The optimal quality of service / content levels are identical for the three providers in the grand coalition

$$q_1^* = q_2^* = q_3^* = \frac{\beta^3N^2-\beta N}{\beta^4N^2-5\beta^2N+4}$$

The optimal access prices are identical for firm A_1 and A_2

$$p_1^* = p_2^* = \frac{1}{4}\left(1 - \frac{\beta(\beta^3N^2-\beta N)}{\beta^4N^2-5\beta^2N+4}\right)$$
$$p_3^* = \frac{1}{4}\left(3 - \frac{3\beta(\beta^3N^2-\beta N)}{\beta^4N^2-5\beta^2N+4}\right)$$

Example 2. *Information level linear in the quality of service / content level* The grand coalition's maximized utility is

$$U^* = \frac{-3\beta N-6}{\beta^2N}$$

The quality of service / content levels at the optimum are identical for the three service providers

$$q_1^* = q_2^* = q_3^* = \frac{\beta N+4}{\beta^2N}$$

Identically, the optimal access prices for the three providers coincide

$$p_1^* = p_2^* = p_3^* = \frac{1}{4}\left(1 - \frac{\beta N+4}{\beta N}\right)$$

B Computation of the Shapley value allocations

We determine the Shapley value of our game for two categories of information level functions. The Shapley value gives us a fair manner to share the grand coalition's total revenue between the three providers.

Example 1. *Information level quadratic in the quality of service / content level* Provider A_1's share of the grand coalition revenue is defined analytically as follows

$$\begin{aligned} \phi_1(\nu) &= -\Big(64\beta^{20}N^{11} - 1242\beta^{18}N^{10} + 10625\beta^{16}N^9 \\ &- 52839\beta^{14}N^8 + 169460\beta^{12}N^7 - 366930\beta^{10}N^6 \\ &+ 544327\beta^8N^5 - 547317\beta^6N^4 + 357612\beta^4N^3 \\ &- 137312\beta^2N^2 + 23552N\Big)\Big(192\beta^{22}N^{11} - 4896\beta^{20}N^{10} \\ &+ 56076\beta^{18}N^9 - 381012\beta^{16}N^8 + 1707612\beta^{14}N^7 \\ &- 5304828\beta^{12}N^6 + 11666904\beta^{10}N^5 - 18183408\beta^8N^4 \\ &+ 19702368\beta^6N^3 - 14150784\beta^4N^2 + 6070272\beta^2N \\ &- 1179648\Big)^{-1} \end{aligned}$$

For provider A_2, we obtain the following expression

$$\begin{aligned} \phi_2(\nu) &= -\Big(64\beta^{24}N^{13} - 1644\beta^{22}N^{12} + 19162\beta^{20}N^{11} \\ &- 133934\beta^{18}N^{10} + 625014\beta^{16}N^9 - 2051140\beta^{14}N^8 \\ &+ 4853783\beta^{12}N^7 - 8345493\beta^{10}N^6 + 10348649\beta^8N^5 \\ &\quad \Big(192\beta^{26}N^{13} - 9027025\beta^6N^4 + 5258292\beta^4N^3 \\ &- 1836544\beta^2N^2 + 290816N\Big)\Big(192\beta^{26}N^{13} - 5664\beta^{24}N^{12} \\ &+ 76428\beta^{22}N^{11} - 624900\beta^{20}N^{10} + 3455964\beta^{18}N^9 \\ &- 13659324\beta^{16}N^8 + 39716664\beta^{14}N^7 - 165693888\beta^8N^4 \\ &+ 141482880\beta^6N^3 82063872\beta^4N^2 + 28999680\beta^2N \\ &- 4718592\Big)^{-1} \end{aligned}$$

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Finally, for provider A_3 we get

$$\begin{array}{rcl}\phi_3(\nu) & = & -\Big(64\beta^{24}N^{13} - 1754\beta^{22}N^{12} + 21993\beta^{20}N^{11} \\ & - & 166403\beta^{18}N^{10} + 844492\beta^{16}N^{9} - 3024612\beta^{14}N^{8} \\ & + & 7834326\beta^{12}N^{7} - 14785310\beta^{10}N^{6} + 20184005\beta^{8}N^{5} \\ & - & 19448725\beta^{6}N^{4} + 12565220\beta^{4}N^{3} - 4891264\beta^{2}N^{2} \\ & + & 868352N\Big)\Big(192\beta^{26}N^{13} - 5664\beta^{24}N^{12} + 76428\beta^{22} \\ & \ldots & N^{11} - 624900\beta^{20}N^{10} + 3455964\beta^{18}N^{9} - 13659324 \\ & \ldots & \beta^{16}N^{8} + 39716664\beta^{14}N^{7} - 86070336\beta^{12}N^{6} \\ & + & 139103616\beta^{10}N^{5} - 165693888\beta^{8}N^{4} + 141482880 \\ & \ldots & \beta^{6}N^{3} - 82063872\beta^{4}N^{2} + 28999680\beta^{2}N \\ & - & 4718592\Big)^{-1}\end{array}$$

Example 2. *Information level linear in the quality of service / content level* Provider A_1's share of the grand coalition total revenue is

$$\begin{array}{rcl}\phi_1(\nu) & = & -\Big(64\beta^{9}N^{5} + (115\beta^{8} - 512\beta^{7} - 48\beta^{6})N^{4} \\ & + & (-1150\beta^{6} + 1536\beta^{5} + 96\beta^{4})N^{3} + (4255\beta^{4} - 2112\beta^{3} \\ & - & 48\beta^{2})N^{2} + (1152\beta - 6900\beta^{2})N + 4140\Big)\Big(96\beta^{10}N^{5} \\ & - & 960\beta^{8}N^{4} + 3552\beta^{6}N^{3} - 5760\beta^{4}N^{2} + 3456\beta^{2}N\Big)^{-1}\end{array}$$

For provider A_2, we obtain the following expression

$$\begin{array}{rcl}\phi_2(\nu) & = & -\Big(32\beta^{9}N^{5} + (37\beta^{8} - 256\beta^{7} - 24\beta^{6})N^{4} \\ & + & (-370\beta^{6} + 768\beta^{5} + 48\beta^{4})N^{3} + (1369\beta^{4} - 1056\beta^{3} \\ & - & 24\beta^{2})N^{2} + (576\beta - 2220\beta^{2})N + 1332\Big)\Big(48\beta^{10}N^{5} \\ & - & 480\beta^{8}N^{4} + 1776\beta^{6}N^{3} - 2880\beta^{4}N^{2} + 1728\beta^{2}N\Big)^{-1}\end{array}$$

Finally, for provider A_3 we get

$$\begin{array}{rcl}\phi_3(\nu) & = & \Big(64\beta^{9}N^{5} + (259\beta^{8} - 896\beta^{7} + 96\beta^{6})N^{4} \\ & + & (-2590\beta^{6} + 4032\beta^{5} - 192\beta^{4})N^{3} + (9583\beta^{4} \\ & - & 7296\beta^{3} + 96\beta^{2})N^{2} + (4608\beta - 15540\beta^{2})N \\ & + & 9324\Big)\Big(96\beta^{10}N^{5} - 960\beta^{8}N^{4} + 3552\beta^{6}N^{3} \\ & - & 5760\beta^{4}N^{2} + 3456\beta^{2}N\Big)^{-1}\end{array}$$

REFERENCES

[1] El Omri A., Cooperation in Supply chains: Alliance Formation and Profit Allocation among Independent Firms, Ph.D. thesis, Ecole Centrale de Paris, 2009
[2] Bourreau M., Coaltion, Industrial organization course, master IREN, 2010
[3] de Man A.-P., Roijakkers N., de Graauw H., Managing dynamics through robust alliance governance structures: The case of KLM and Northwest Airlines, European Management journal, in press, 2010
[4] Dupuit J., On the measurement of the utility of public works, Annales des Ponts et Chaussées, 1952
[5] Gulati R., Khanna T., Nohria N., Unilateral Commitments and the Importance of Process in Alliances, Sloan Management Review, MIT, 1994
[6] Johnson J., Myatt D., On the Simple Economics of Advertising, Marketing, and Product Design, American Economic Review, American Economic Association, vol. 96, pp. 756–784, 2006
[7] Grzybowski L., Karamti C., Competition in Mobile Telephony in France and Germany, working Paper 07-24, NET Institute, 2007
[8] Koessler F., Théorie des jeux, cours de Paris School of Economics, 2009
[9] Le Cadre H., Bouhtou M., An Interconnection Game between Mobile Network Operators: Hidden Information Forecasting using Expert Advice Fusion, Computer Networks, to appear, 2010
[10] MacAvoy P. W., Tacit Collusion under Regulation in the Pricing of Interstate Long-Distance Telephone Services, Journal of Economics and Management Strategy, vol.2, pp.147–185, 1995
[11] Myerson R., Game Theory, Analysis of Conflict, Harvard University Press, $6-$ th Edition, 2004
[12] Pénard T., Structures de marché et Pratiques facilitant la collusion: une approche par la théorie des jeux répétés, CREREG working paper, 2003
[13] Pohjola O.-P., Kilkky K., Value-based methodology to analyze communication services, Netnomics, vol.8, pp.135–151, 2007
[14] Rubinstein R., The Cross-Entropy Method, Springer Information Science and Statistics, 2004
[15] Saad W., Debbah M., Hjorungnes A., Basar T., Coalitional Game Theory for Communication networks, IEEE Signal Processing Magazine Tutorial, 2009
[16] Schlee E., The Value of Information About Product Quality, Rand Journal of Economics, the RAND Corporation, vol.27, pp.803–815, 1996
[17] Tijms A., A First Course in Stochastic Models, Wiley Interscience, 2003
[18] Yildizoglu M., Introduction à la théorie des jeux, Manuel et Exercices corrigés, Dunod, 2003

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