

Two Area Power Systems Economic Dispatch Problem Solving Considering Transmission Capacity Constraints

M. Zarei, A. Roozegar, R. Kazemzadeh, and J.M. Kauffmann

Abstract—This paper describes an efficient and practical method for economic dispatch problem in one and two area electrical power systems with considering the constraint of the tie transmission line capacity constraint. Direct search method (DSM) is used with some equality and inequality constraints of the production units with any kind of fuel cost function. By this method, it is possible to use several inequality constraints without having difficulty for complex cost functions or in the case of unavailability of the cost function derivative. To minimize the number of total iterations in searching, process multi-level convergence is incorporated in the DSM. Enhanced direct search method (EDSM) for two area power system will be investigated. The initial calculation step size that causes less iterations and then less calculation time is presented. Effect of the transmission tie line capacity, between areas, on economic dispatch problem and on total generation cost will be studied; line compensation and active power with reactive power dispatch are proposed to overcome the high generation costs for this multi-area system.

Keywords—Economic dispatch, Power System Operation, Direct Search Method, Transmission Capacity Constraint.

I. INTRODUCTION

DAILY production planning is one of the important and noticeable problems in electrical power system, and some researches in this matter have been done [2], [8]. Considering of the transmission capacity between power areas in multi area systems is one of the important problems in economic dispatch in power generation cost analysis, to be able to give a suitable index to site new power plants or to invest on transmission system in capacity constrained systems [5].

In this research a distributed system with more buses and lines is considered that has been divided in different areas which have been connected together. A portion of the total load exists in each area; a portion of this area's load is generated in it and what is left in area's load demand is fed and supported by other areas where production cost is cheaper due to technical problems.

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An electrical power system with two areas is shown in Fig. 1 where $x_i\%$ of total generation is the generation capacity of the area i with $\sum_{i=1}^n x_i = 100\%$ and $y_i\%$ of the total load is consumed in area i . It should be noted in some area, demanded load is more than its generation capacity or for this area $y_i > x_i$. Difference between the area generation and demand should be provided by other areas.

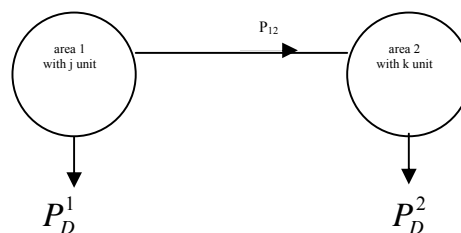


Fig. 1 Two area power systems

In conventional unit commitment problem all the bus load are summed together and transmission capacity constraints are usually not considered [8]. If the generation schedule ignores these requirements, the system may operate at a risky state since the critical transmission lines to transfer power area may be overloaded inadvertently. Therefore, outage of any committed unit or any transmission line may not be tolerated in such a risky state and clearly the transmission line outage and network trip may occur.

The main idea of this paper is investigation of a suitable algorithm for generation planning for multi areas power systems with considering power system reliability and security. Several calculation techniques and methods for optimization problem have been presented and used [8],[9],[10].

A powerful algorithm for the economic dispatch (ED) with considering the transmission capacity constraints is investigated. The basic structure of this algorithm is composed by Lagrange method and first or second order gradient and also dynamic programming [8]. The method like dynamic programming has no restriction on cost function and does a direct search in the solution space that is called direct search method (DSM).

II. PROBLEM FORMULATION

A. Principal of Basic Economic Dispatch Problem

The goal idea of the economic dispatch is the minimization of the total generation cost on the power system by satisfying the generation limits constraint and load and generation balancing. It means

$$\text{Minimize } F_T = \sum_{i=1}^N F_i(P_i) \quad (1)$$

Subject to:

$$\sum_{i=1}^N P_i = P_D \quad (2)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad \text{for } i=1, \dots, N \quad (3)$$

with:

- i = unit number
- N = total number of units in the system
- $F_i(\cdot)$ = fuel cost function of the units
- P_i = generation of unit i
- P_D = total system load
- P_i^{\max} = upper limit of the unit i
- P_i^{\min} = lower limit of the unit i

Generally this target function will be investigated by Lagrange method, first or second order gradient method but these methods have some difficulties for complex generation cost functions [5], [6], [7]. For this purpose advanced economic dispatch problem is investigated.

B. Advanced Economic Dispatch Problem

The economic dispatch problem can be extended by importing additional constraint like transmission line capacity limit. Fig. 1 presents a two area power system that have j, k generation units respectively. Each area has own special load and its spinning reserve. By Fig.1 the corresponding equations are:

$$\sum_{i=1}^j P_i^1 = P_D^1 + P_{12} \quad (4)$$

$$\sum_{i=1}^k P_i^2 = P_D^2 - P_{12} \quad (5)$$

with this constraints

$$-P_{12}^{\max} \leq P_{12} \leq P_{12}^{\max} \quad (6)$$

where:

- P_D^1, P_D^2 are the load demand of each area 1 and 2
- P_{12}, P_{12}^{\max} power transfer and its upper limit from area 1 to area 2.

III. DIRECT SEARCH METHOD FOR BASIC ECONOMIC DISPATCH PROBLEM

In this method the solving problem starts with an initial feasible solution and search for the optimal solution along a

trajectory that maintains a feasible solution at all time [1]. An initial feasible estimate and proper optimal searching direction is chosen that is rather than gradient method by having an additional constraint and a complex cost functions. The generation units without exceeding maximum or minimum generation limits are to increase or decrease their outputs by a predetermined step Δ for calculating their incremental costs (IC) and decrement costs (DC)[1]. This is shown as follows:

$$IC_i = \frac{F_i(P_i + \Delta) - F_i(P_i)}{\Delta} \quad \text{for } i=1, 2, \dots, N \quad (7)$$

$$DC_i = \frac{F_i(P_i) - F_i(P_i - \Delta)}{\Delta} \quad \text{for } i=1, 2, \dots, N \quad (8)$$

In condition that

$$P_i + \Delta \leq P_i^{\max} \quad \text{and} \quad P_i - \Delta \geq P_i^{\min} \quad (9)$$

where:

- IC_i = Unit i incremental cost
- DC_i = Unit i decremental cost
- Δ = Predetermined calculation step

Estimation of initial solution is the first step in DSM algorithm. The initial solution estimate steps have a simple procedure that is [1]:

- 1) Distribution of the part of the demanded load at first to the minimum generation limits of all committed units.
- 2) The unit with minimal $[F_i(P_i^{\max}) - F_i(P_i^{\min})] / (P_i^{\max} - P_i^{\min})$ is chosen to increase its output by the maximum step, one by one, until the load demand is satisfied.

The number of iterations could be reduced if the initial feasible solution is determined according to these criteria [1]. Particularly, units with discrepant incremental cost curves can be set to their maximum or minimum output properly by the proposed initial solution estimation. It overcomes the lambda-iteration method suffering from the oscillatory problem in its convergence.

The second step in DSM algorithm is direct search procedure.

Finding the best direction for improvement is the main objective in this method to solve this problem. To check the convergence condition, a predetermined resolution (e.g., $\Delta=1$ MW) is used to choose the fixed calculation step for each unit, and the computation steps of the direct search procedure are shown as follows:

- 1) Units without violating the maximum or minimum generation limits are chosen to increase or decrease their outputs by the predetermined step Δ for calculating their incremental costs and decrement costs.
- 2) All units are examined to check if there is any improvement? If no more improvement can be achieved, then stop; otherwise, go to step 3.
- 3) An independent unit with minimum incremental cost (assume unit x) is chosen to increase its output by the predetermined step Δ , and only the dependent unit (assume

unit $y, y \neq x$) without violating additional constraints while gaining the most reduction in the operating cost should be selected to reduce its output for satisfying the load balance equation.

4) This particular pair of units will adjust their output again by the predetermined step Δ if they do not violate their generation limits, and only the incremental cost of unit x and the decrement cost of unit y are recalculated.

5) Go to step 2.

The advantage of direct search procedure is to impose several inequality constraints without introducing any multipliers in the objective function and it can solve problems with unavailable derivatives or much more complicated fuel cost functions due to using the simple solution method without using the function derivative.

IV. ENHANCED DSM(EDSM) FOR EXTENDED ECONOMIC PROBLEM

A modified structure based on DSM is necessary for imposing additional constraints properly to solve the extended ED problem. Such a modified algorithm is called enhanced DSM (EDSM) and EDSM algorithm is detailed as follows [1]:

A. Initial Solution Estimate for Each Area

The efficiency of the proposed EDSM algorithm can be improved if proper initial solutions are obtained. The computation steps of the initial solution estimation of each area are shown as follows:

- 1) Distribution of the part of the demanded load at first to the minimum generation limits of the units in each area.
- 2) Units with low efficiency are chosen to satisfy their own area spinning reserve requirements.
- 3) Units with high efficiency are chosen to increase their outputs by maximum steps for satisfying their own area generation requirements. If the actual area generation is greater than the area load demand, export as much economical excess power as possible to other areas to satisfy other area generation requirements without violating transmission capacity constraints.

B. Additional Constraints Handling

To find a direction which reduces the operating cost and leads to a point within the feasible region, other procedures may be needed to improve the searching technique for extended ED problem. The computation steps of the direct search procedure are the same with the second step procedure in section II that has been numbered in 5 steps.

V. MUTI-LEVEL CONVERGENCE STRATEGY

Improving the performance of direct search procedure is the main purpose of multi-level convergence. Another attractive feature is to increase the possibility of occurrence of escaping from local optimal solution when the generator incremental cost curve is not monotonically increasing. Multi-level convergence strategy process is described as follows:

A. Number of Convergence Level Determination

Although the DSM or EDSM usually takes more steps to converge to the minimum of a given problem than the more sophisticated methods, it takes less computing time in each step. However, a too small calculation step in the direct search procedure often leads to slow convergence problem. To further speed up the solution process, a novel strategy with a two-level (or multi-level) convergence is used. It begins with a coarse convergence step, and ends with a refined convergence step. That is, a larger calculation step is always chosen first to perform a direct search procedure (DSM or EDSM). The step is then refined during each convergence level. The recommended value of convergence level is 3-5 [1].

B. Step Size for Each Convergence Level Selection

The selection of step size is also an important factor on convergence. A too larger step size may end up with divergence. And then small step size may need a large number of iterations to converge to the optimal solution. In this paper a new and efficient initial step size is chosen as follows:

$$\Delta l = 0.1 \times \text{Total demanded load} \quad i = 1, 2, \dots, N \quad (10)$$

By several example calculations it is shown that this determined step needs less iteration than the determined initial step size in reference [1]. After the first level converges, the step size is then successively refined with $\Delta = \Delta / k$ during each convergence level until the calculation step is less than the predetermined resolution (e.g., 1 MW). The flowchart of DSM or EDSM with multi-level convergence strategy is shown in Fig. 2.

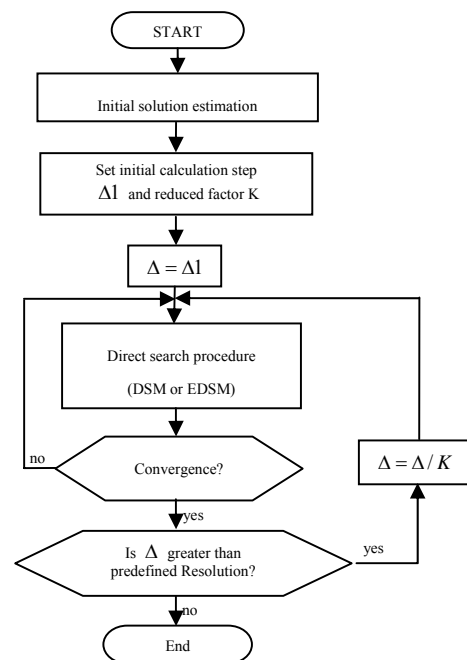


Fig. 2 Flowchart of DSM or EDSM with multi-level convergence strategy

VI. NUMERICAL EXAMPLES

Several test systems were studied to examine this method. Due to the physical operation limitations of power plant components, the generating units exhibit a greater variation in the fuel cost functions. Most analytical methods based on industrial algorithms, such as Lagrange multiplier method, may not be able to be directly applied to a physical-size power system ED problem but need to perform a direct search of the solution space due to no smooth fuel cost functions. However, the associated incremental costs of the units are assumed monotonically increasing for simplicity to concentrate on the effects of additional constraints in the studied cases.

In direct search procedure, the initial calculation step 1 is chosen to be 10% of the total demanded load by reduced factor 5. All the computation is performed on a PC Pentium IV, and a computer program has been developed in Matlab.

A. Test for a 3 Unit System

A system with 3 generating units is studied. The system unit data are given as follows:

$$F_1(P_1) = 712.1 + 6.60P_1 + 9.2 \times 10^{-4}P_1^2 + 1.21 \times 10^{-7}P_1^3$$

$$F_2(P_2) = 1221 + 6.7P_2 + 7.00 \times 10^{-4}P_2^2 + 6.13 \times 10^{-7}P_2^3$$

$$F_3(P_3) = 1454.5 + 6.20P_3 + 0.988 \times 10^{-4}P_3^2 + 9.48 \times 10^{-7}P_3^3$$

subject to

$$300 \leq P_1 \leq 800, 300 \leq P_2 \leq 1200, 275 \leq P_3 \leq 1100$$

DSM with Multi-level convergence process is used to minimize the number of total iterations. It needs only 8 iterations and Table I shows the numerical results for a 950MW demanded load in spite of the 300 iteration for $\Delta = 1$ MW. The four-level convergence process is illustrated in the first column of Table I, corresponding production costs and total number of iterations required are also shown in the second and third columns of Table I.

It is noted that this results has been tested by the existing Matlab power dispatch program in [9] and shows that production cost is not sensitive to the calculation step.

B. Test for a Two Area System

A two-area system with four generating units is studied. The percentage of the total load demand in area 1 is 70% and the unit data are given:

$$F_1(P_1) = 533 + 7.524P_1 + .001484P_1^2$$

$$150MW \leq P_1 \leq 600MW$$

$$F_3(P_3) = 74.1 + 7.57P_3 + .004579P_3^2$$

$$50MW \leq P_3 \leq 200MW$$

The load demand in area 2 is 30% of the total load demand and the unit data are given as follows:

$$F_2(P_2) = 294.5 + 7.46P_2 + .001845P_2^2$$

$$100MW \leq P_2 \leq 400MW$$

$$F_4(P_4) = 237.5 + 7.125P_4 + .00172P_4^2$$

$$50MW \leq P_4 \leq 350MW$$

For simplicity, network losses are neglected to understand the effect of transmission capacity limits in the studied cases. The system is transferring power from area 2 (cheap area) to area 1 (expensive area) since units in area 2 generate power cheaply. For the load of 950 MW, Table II gives a comparison of total number of iterations required and production costs considering the transmission capacity limits or not during each convergence level.

TABLE I
 ITERATIONS AND PRODUCTION COSTS FOR VARIOUS Δ USING DSM IN 3-UNIT SYSTEM

Convergence	Total cost(\$/h)	Iterations
$\Delta 1 = 95$ MW	9831.95	2
$\Delta 2 = 19$ MW	9824.98	3
$\Delta 3 = 3.8$ MW	9824.9	2
$\Delta 4 = 0.76$ MW	9824.9	1

TABLE II
 ITERATIONS AND PRODUCTION COSTS FOR 950 MW LOAD IN TWO-AREA POWER SYSTEM

Δ	DSM		EDSM	
	Iteration	Total cost(\$/h)	Iteration	Total cost(\$/h)
$\Delta 1 = 95$	3	8680.95	2	8654.43
$\Delta 2 = 19$	5	8670.31	3	8652.3
$\Delta 3 = 3.8$	2	8664.93	2	8651.7
$\Delta 4 = 0.76$	2	8664.83	1	8651.7
Total iterations	12		8	

VII. INVESTIGATION OF TRANSMISSION CAPACITY EFFECT

The effect of transmission capacity limits on generation scheduling is demonstrated in Table III showing the numerical results of our experiments under various load demand.

Fig. 3 shows the variation of total generation cost in two area power systems with respect to variation of the transmission capacity limit. It is shown that with increasing of this capacity, the total cost is decreased for a constant demanded load, because it is done by the power transmitted from cheap to expensive area.

From this figure it is seen that for high line capacity limit, the total cost is practically constant, and with a low variation while a constant power will flow between two areas.

It can be concluded that by power line compensation during the demanded load variation in systems, it is possible to change the total power generation cost.

TABLE III
 RESULTS COMPARISON IN TWO-AREA POWER SYSTEM WITH SEVERAL LINE CAPACITIES

Load (MW)	Flow Limit (MW)	Total cost (\$/h)
800	90	7444.9
	120	7422.2
	200	7389.6
950	90	8741.8
	120	8714.3
	200	8651.7
1030	90	9449.7
	120	9419.7
	200	9338.7
1120	90	10209
	120	10206
	200	10125

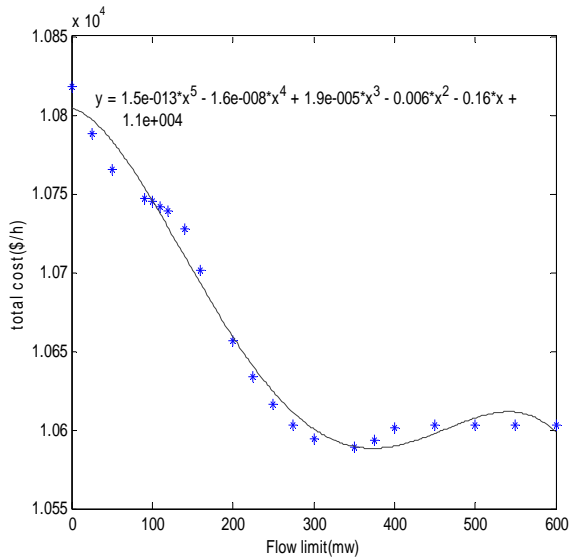


Fig. 3 Total generation cost variation with respect to line limit

By this means, we can consider and apply the reactive power control effect on economic dispatch and also on system reconfiguration; in other words it is possible to get the active and reactive power dispatch at the same time for economic dispatch solving.

Fig. 4 shows the variation of the power generation at generator 1 in area 1 and in area 2 with the variation of the transmission capacity. It is seen that by increasing the capacity, generated power in area 1 (expensive area) is decreased and it continues until the area 2 generation has reached its maximum upper limit. This is opposite for area 2 generation (cheap area) as seen in the figure.

Fig. 5 shows the total generation cost at areas 1 and 2 with respect to transmission capacity limit and shows that for expensive area (area 1) cost is decreasing with increasing the flow limit; it is opposite for area 2 as seen in this figure.

Fig. 6 shows the transmitted power from area 2 to the expensive area 1 with respect to the capacity limit. The power flow increases till the generation in cheap area is limited by its upper power limit and then it will be constant against the limit increasing.

Simulation shows that with zero amounts for line capacity of the transmission line, the power systems act as like two independent and separated areas and it is noted that for each area the economic dispatch is approved. But in this condition the total cost does not satisfy the minimum total cost. On the other hand the simulation shows that when each area load is equal to the minimum generation rate of the each area, in spite of the line capacity, the transmitted power will be zero.

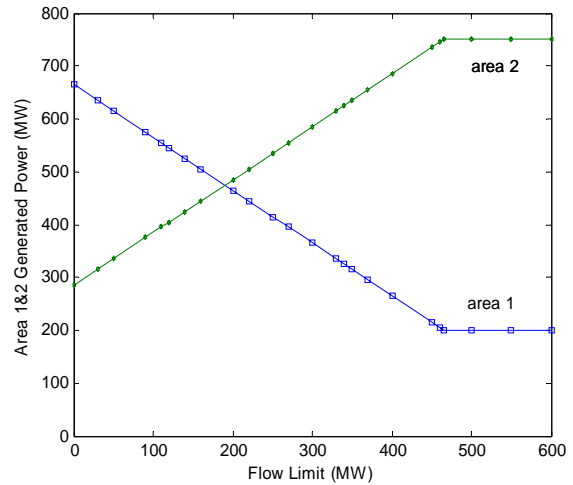


Fig. 4 Total generated powers at area 1 & 2 with respect to line flow limit

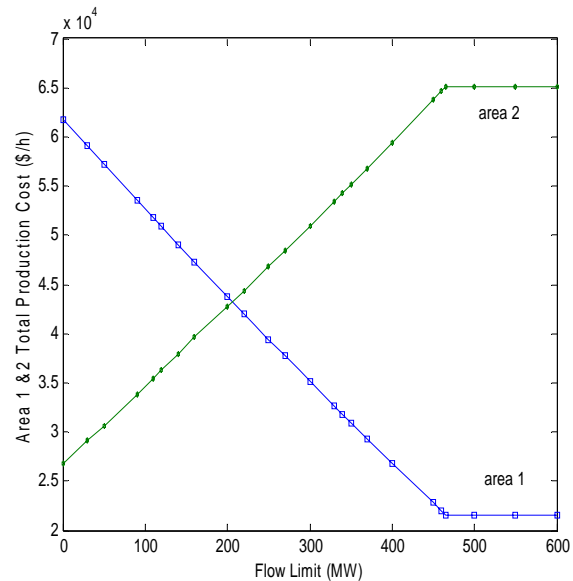


Fig. 5 Total generated cost at area 1 & 2 with respect to line limit

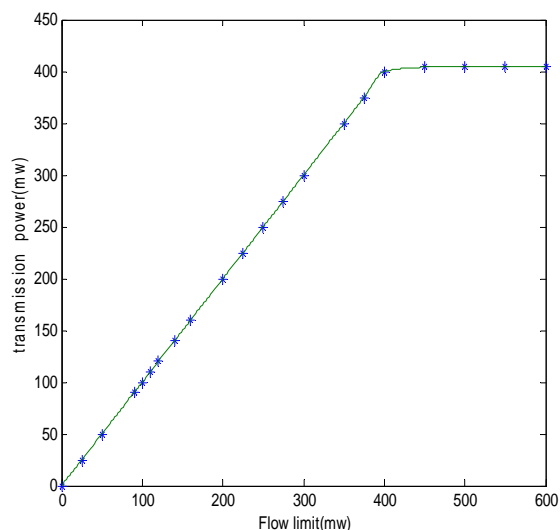


Fig. 6 Line transmitted power variation versus to line capacity limit

VIII. CONCLUSION

For studying the ED problem including transmission capacity constraints, a simple and novel approach is presented in this paper. Simulation and programming has been done in Matlab. In this approach, several example results show that there is not any restriction on generator cost function and it is possible to impose additional constraints properly by this method. On the other hand the examples show that chosen initial step in this paper causes less iteration than determined step value proposed in other reference.

Results show that the algorithm is an efficient approach for determining the optimal generation schedules by using the simple calculation method; computation time and required memory will be reduced. Numerical analysis shows effect of the transmission line capacity in multi- area power systems on

the economic dispatch. It is seen that capacity limit acts an important rule in two area power systems to have minimum total generation cost; system tends to use the all capacity of cheap area. It is an idea to tie line compensation on economic dispatch during the total demanded load variation. It is needed to compare the amount of funding needed for this purpose (compensation) against the economic dispatch saving. The study will be continued by considering system active losses and will be studied for a 6 area existing power system.

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