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Blind Channel Estimation Based on URV Decomposition Technique for Uplink of MC-CDMA

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***Abstract*—**In this paper, we investigate a blind channel estimation method for Multi-carrier CDMA systems that use a subspace decomposition technique. This technique exploits the orthogonality property between the noise subspace and the received user codes to obtain channel of each user. In the past we used Singular Value Decomposition (SVD) technique but SVD have most computational complexity so in this paper use a new algorithm called URV Decomposition, which serve as an intermediary between the QR decomposition and SVD, replaced in SVD technique to track the noise space of the received data. Because of the URV decomposition has almost the same estimation performance as the SVD, but has less computational complexity.

***Keywords*—**Channel estimation, MC-CDMA, SVD, URV

I. Introduction

The Muticarrier Code-division Multiple Access (MC-CDMA) scheme is widely considered as a promising technique for future wireless multimedia communications. In accordance with MC-CDMA systems is a combination of the CDMA systems and OFDM systems [1-3]. In the MC-CDMA system, every transmitted symbol is spreaded in frequency domain by using a given signature code such that every chip modulates a different sub-carrier.

On the reverse link transmission, every user has an independent channel fading and the received signal at base station is the sum of all user transmitted signals. On account of the channel fading, the orthogonality between signature codes is lost for every user. Because of many users share the same radio channel so Multiple Access Interference (MAI) is occurred at base station [4]. Therefore, a multiuser detector is required to deal with MAI. The orthogonality can be recovered through channel estimation [5].

In this paper proposed blind channel estimation based on Subspace estimation, since they don't require the transmission of training sequence to estimate channel so its higher spectral efficiency. Due to the Most subspace based blind channel estimation method need the singular value decomposition of the received data and the initialization calculation of any decomposition is expensive, it is desirable to calculate iteratively. Unfortunately the SVD is difficult to update because all SVD updating required $O(k^3)$ operations for a k-column matrix. In the URV Decomposition [6] can be updated in $O(k^2)$ operations and provide a basis for the noise space of the k-column matrix. In this paper, we show that URV Decomposition can be used to estimate the channel in MC-CDMA systems

This paper is organized as follow: The system model for MC-CDMA system is introduced in section 2. The Subspace-based blind channel estimation is developed in section 3.a. In the section 3.b, a blind estimation method based on URV Decomposition is proposed. The performance of channel estimation by URV Decomposition algorithm compare with SVD algorithm via simulation in section 4.

II. System Model

In this section, we describe the model of synchronous MC-CDMA system model shown in Fig.1. We assumed that K users are randomly distributed around a cell site. The number of subcarriers equal to the length of signature code G so all users shares the same subcarriers. The data vector for all users at symbol n-th is expressed as

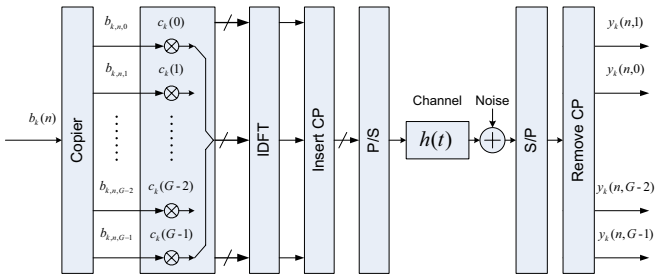


Fig. 1 MC-CDMA system model

$$b(n) = [b_1(n) \quad b_2(n) \quad \cdots \quad b_k(n)]^T \qquad (1)$$

Next, we define signature sequence code for the k-th user as a vector $c_k = (1, 2, 3, \cdots, \mathrm{K}-1, \mathrm{K})$, which can be written as

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$c_k = [c_k(0) \quad c_k(1) \quad \cdots \quad c_k(G-1)]^T$. From Figure 1, In the Baseband part can written received signal of every users at symbol n-th as

$$s(n) = C \times b(N) \tag{2}$$

where C is a matrix of all signature code for all users is shown as

$$C = [c_1 \quad c_2 \quad \cdots \quad c_k] \tag{3}$$

However, the transmitted signal have a channel fading so it have a linear convolution of the wireless channel impulse response (CIR) and the time-domain IDFT-transformed signature code. The time-domain CIR vector for the k-th user is described as

$$h_k = [h_k(0) \quad h_k(1) \quad \cdots \quad h_k(L_{ch})]^T \tag{4}$$

where L_{ch} is the channel length. According to the definition of linear convolution, the time-domain signature waveform for the k-th user is

$$\widetilde{w}_k(n) = \widetilde{c}_k * h_k(n) \tag{5}$$

where $*$ represents the linear convolution. In order to combat the resultant hostile Intersymbol Interference (ISI) is via addition of a cyclic prefix (CP) to each symbol, for this reason $\widetilde{c}_k$ is the IDFT-transformed of c_k can be written as

$$\widetilde{c}_k = [\widetilde{c}_k(0) \quad \widetilde{c}_k(1) \quad \cdots \quad \widetilde{c}_k(G+L_g-1)]^T \tag{6}$$

where L_g is a length of CP. The length L_w of $\widetilde{w}_k(n)$ is therefore $L_w = G + L_{ch} + L_g - 1$. After that, the received signal at base station was remove the cyclic prefix such as $\widetilde{w}_k(n)$ can be written new as

$$\widetilde{w}_{k,n} = A_k h_{k,n} \tag{7}$$

where $\widetilde{w}_{k,n}$ with dimension $G \times 1$ and A_k have dimension $G \times (L_{ch}+1)$ is given by

$$A_k = \begin{bmatrix} \widetilde{c}_k(0) & 0 & 0 & \cdots & 0 \\ \widetilde{c}_k(1) & \widetilde{c}_k(0) & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \widetilde{c}_k(G-2) & \widetilde{c}_k(G-3) & \ddots & \ddots & \vdots \\ \widetilde{c}_k(G-1) & \widetilde{c}_k(G-2) & \widetilde{c}_k(G-3) & \cdots & 0 \end{bmatrix} \tag{8}$$

When all K active user shares all subcarriers send data signal in same time, the reverse link-received data during n-th symbol can be given by

$$Y = W \times B + N \tag{9}$$

where the time-domain signature waveform matrix W with dimension $G \times K$ can be written as

$$W = [\widetilde{w}_1 \quad \widetilde{w}_2 \quad \cdots \quad \widetilde{w}_k] \tag{10}$$

The transmitted symbol matrix B with dimension $K \times N$ is define as

$$B = [b(1) \quad b(2) \quad \cdots \quad b(N)] \tag{11}$$

Next every entry noise matrix N with dimension $G \times N$ is the independent identically distributed (i.i.d.) complex zero mean Gaussian noise with variance σ_n^2.

III. Blind Channel Estimation

In this section, we review the Blind channel estimation based on Singular value decomposition. The disadvantage of the channel estimation based on SVD technique is difficult to update noise space so as to update channel when the channel variant in time change. Therefore we proposed channel estimation based on URV decomposition can update noise space and performance estimation channel has almost SVD but has less complexity than SVD. We also present the channel estimation based on SVD technique and URV decomposition technique in this section

A. Singular value decomposition blind estimation method

We perform SVD on the reverse link-received data Matrix

$$Y^H = \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{pmatrix} \Lambda_s & 0 \\ 0 & \Lambda_0 \end{pmatrix} \begin{bmatrix} V_s^H \\ V_n^H \end{bmatrix} \tag{12}$$

where $(\cdot)^H$ denotes Hermitian transpose. The column vector in V_s, associate with Λ_s singular values, span the signal space defined by the column of W. while vector V_n, associate with Λ_n singular values, span the orthogonal complement subspace of the signal subspace. Next, we have following orthogonality condition,

$$V_n^H \widetilde{w}_k = 0 \tag{13}$$

From the equation (13) has L_{ch} unknowns and G-K equations. It will be solvable when the number of equation is grater or equal than the number of unknowns, $L_{ch} \le G - K$. From this reason, the number of active users, K, is limited by the number of subcarriers, G, and the channel length, L_{ch}.

In order to solve the equations system (13), we can consider the following equivalent system

$$\begin{aligned} \left\| V_n^H \widetilde{w}_k \right\| &= (V_n^H \times A_k \times h_k)^H \times (V_n^H \times A_k \times h_k) \\ &= h_k^H \times A_k^H \times V_n \times V_n^H \times A_k \times h_k = 0 \end{aligned} \tag{14}$$

The solution to these equations can be found by solving the following minimization problem

$$h_k = \arg\min_{\|h\|=1} (h_k^H \times A_k^H \times V_n \times V_n^H \times A_k \times h_k) \tag{15}$$

When the new data vectors are received, then SVD updating is required to update noise subspace so as to estimate new channel. Unfortunately, SVD updating scheme required $O(G^3)$ operation. In the URV decomposition can be updated in $O(G^2)$ operation and provides basis of the noise subspace of the received data.

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B. URV decomposition blind estimation method

Suppose for the moment that Y has rank K. Then there are orthogonal matrices U and V such that

$$Y^H = U\begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix}V^H \tag{16}$$

where R is an upper triangular matrix of order K. We will call this decomposition a URV decomposition.

Now suppose that Y^H is nearly of rank K in the sense that its singular values satisfy

$$\sigma_1 \geq \cdots \geq \sigma_K > \sigma_{K+1} \cdots \geq \sigma_{G-K}$$

where σ_K is large compared to σ_{K+1}. It can be shown that there is a URV decomposition of Y^H of the form

$$Y^H = U\begin{pmatrix} R & F \\ 0 & G \end{pmatrix}V^H \tag{17}$$

Where

1) R and G are upper triangular.
2) $\inf(\mathrm{R}) \cong \sigma_{\mathrm{k}}$.
3) $\sqrt{\|F\|^2 + \|G\|^2} \cong \sqrt{\sigma_{K+1}^2 + \cdots + \sigma_{G-K}^2}$.

Next, we will show how to update URV decomposition of Y^H when the new data vectors $Y(N+1)^H$ is appended,

$$Y(N+1)^H = \begin{bmatrix} \beta Y^H \\ Y(N+1)^H \end{bmatrix} \tag{18}$$

where β is the "forgetting factor" which $1 \geq \beta \geq 0$. The forgetting factor damps out the effect of the previous data. Specifically, we suppose that Y^H has the URV decomposition (17), where V is known and $\omega = \sqrt{\|F\|^2 + \|G\|^2} \leq tol.$ *tol.* is a user supplied tolerance.

The first step is to compute

$$\begin{pmatrix} x^H & y^H \end{pmatrix} = Y(n+1)^H V \tag{19}$$

where x is of dimension K, i.e., the order of R. Our problem then becomes one of updating the matrix

$$A = \begin{pmatrix} R & F \\ 0 & G \\ x^H & y^H \end{pmatrix} \tag{20}$$

There are two cases to consider. The first case, occurs when

$$\sqrt{\omega^2 + \|y\|^2} \leq tol. \tag{21}$$

In this case the rank cannot increase but possible for the rank to decrease, then we reduce A to triangular form by a sequence of left rotations. Hence the rank must be checked. The first step is to determine the smallest singular value, denoted by $\inf(\mathrm{R})$ is less than *tol.* This problem has been extensively studied under the rubric of condition estimation [7], and there exist reliable algorithms that given a triangular matrix R, produce a K-dimension vector ϖ of norm one such that

$$\|\mathrm{R}\varpi\| \cong \inf(\mathrm{R}). \tag{22}$$

The next step is to determine a sequence $V_1^H, V_2^H, \cdots, V_{K-1}^H$ of rotations that eliminate the first K-1 component of ϖ, so that ϖ is zero except for its last component, which is one. Let $Q^H = V_{K-1}^H, V_{K-2}^H, \cdots, V_1^H$ denote the product of the rotations obtained from this step.

Next, we determine an orthogonal matrix P such that $P^H RQ$ is upper triangular. Then we will add the refinement step to improve the estimate of the orthogonal subspace. The procedure begin by reducing the first K-1 elements in the last column R to zero and then to reduce R back to triangular form. According to, all the above rotation must be multiplied into U and V. The second case, occurs when

$$\sqrt{\omega^2 + \|y\|^2} \geq tol. \tag{23}$$

There is a possibility that there is an increase in rank. Since the increase in rank can be at most one, the problem is to transform the matrix to triangular form without destroying all the small values in F and G

The first step is to reduce y^H so that it has only one nonzero component and G remains upper triangular. This is done by a sequence of rotation applied alternately from the right and the left. Then the entire matrix reduced to triangular form. Then K is increased by one. The second step, the new R is checked for degeneracy and if necessary reduced as described before. The result is the updated URV decomposition. Using the updated noise subspace, the channel can be estimated using (15).

IV. Simulation Results

This section is represented to evaluate the performance of URV decomposition based channel estimation technique. In order to compare with the SVD based channel estimation technique. But no updating is used for the SVD based channel estimation method.

The computer simulations are conducted assuming Binary Phase Shift Keying (BPSK) modulation. We assume that there are 10 active users, assigned hadarmard signature sequence of length 16 sharing a channel with 16 sub-carriers. The simulated channel is the 4-path Rayleigh distributed channel. The simulation parameters are summarized in Table I.

To measure the performance, we define the root mean square error (RMSE) as

$$RMSE = \frac{1}{\|h\|}\sqrt{\frac{1}{N_t(L_{ch}+1)}\sum_{i=1}^{N_t}\left\|\hat{h}(i) - h\right\|^2} \tag{24}$$

where N_t is a number of Monte-Carlo simulations. and $\hat{h}(i)$ is channel estimation for each user. Next, we define the channel average as

$$Channel\,average = \frac{1}{N_t(L_{ch}+1)}\sum_{l=0}^{L_{ch}}\left|\sum_{i=1}^{N_t}\hat{h}_i(l) - h(l)\right| \quad (25)$$

Figure 2 shows the RMSE (a) and the channel average (b) which used to compare channel estimation performance between the URV decomposition based channel estimation technique and SVD based channel estimation technique. From figure 2, we fix the number of data bits and vary signal to noise ratio (SNR) from 0 dB to 30 dB. From these curves show that, the URV decomposition has almost the same estimation performance as the SVD based channel estimation technique but URV decomposition based channel estimation has less computational complexity.

V. Conclusions

The channel estimation based on URV decomposition technique, was proposed to show that has less computation complexity than the SVD based channel estimation. But they have almost the same estimation performance as SVD based channel estimation. Not only that the other disadvantage of the channel estimation based on SVD technique is difficult to update noise space but channel estimation based on URV decomposition can update noise space Performance simulation results demonstrated the efficiency of the proposed channel estimation method.

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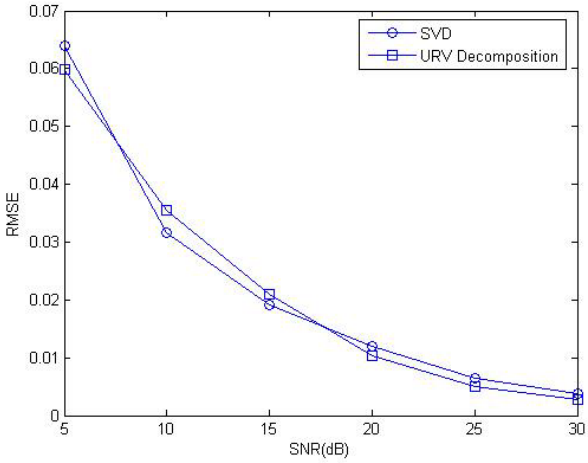
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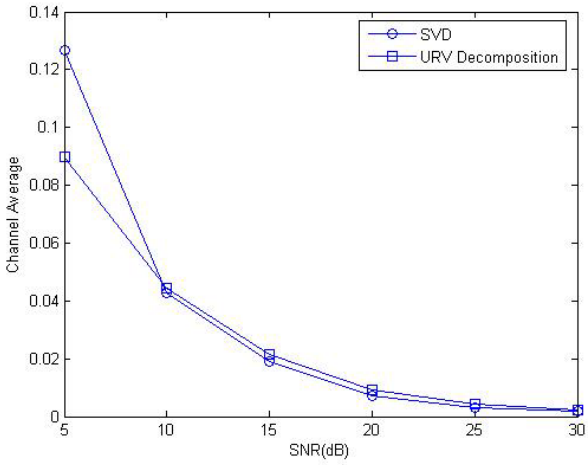
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TABLE I
SIMULATION PARAMETERS

Parameters	Value
Number of user	12
Length of signature sequence	16
Number of sub-carrier (*N*)	16
Frequency selective fading :	
Number of paths or CIR length (L_g)	4
Components	Rayleigh
Guard period (L_g)	4
Number of data bit	300
Modulation scheme	BPSK
FFT / IFFT	16 points
Number of Monte-Carlo simulation	10000



(a)



(b)

Fig.2 Channel estimation performance via Signal to noise ration (SNR)

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