A Heuristic Algorithm Approach for Scheduling of Multi-criteria Unrelated Parallel Machines

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Abstract**—**In this paper we address a multi-objective scheduling problem for unrelated parallel machines. In unrelated parallel systems, the processing cost/time of a given job on different machines may vary. The objective of scheduling is to simultaneously determine the job-machine assignment and job sequencing on each machine. In such a way the total cost of the schedule is minimized. The cost function consists of three components, namely; machining cost, earliness/tardiness penalties and makespan related cost. Such scheduling problem is combinatorial in nature. Therefore, a Simulated Annealing approach is employed to provide good solutions within reasonable computational times. Computational results show that the proposed approach can efficiently solve such complicated problems.

Keywords**—**Makespan, Parallel machines, Scheduling, Simulated Annealing

I. Introduction

SCHEDULING of parallel machines is one of the most important subjects in multi-machine manufacturing environments. Generally this problem consists of jobs allocation (which are simultaneously available) to machines with similar, but not necessarily identical, capabilities. Efficient scheduling leads to increased efficiency; thereby reducing the time required to complete jobs and consequently increasing the profitability of the organization in today's extremely competitive environment.

Traditional scheduling algorithms are mainly concerned with completion-time-related objectives (e.g., makespan), and aim to reduce production time and increase facility utilization. In modern manufacturing management, On-time delivery is also a critical factor towards realizing customer satisfaction. Hence, scheduling problems with due-date-related objectives have attracted increasing attention from managers and researchers [1]. Widely used performance measures in due-date-related scheduling problems include maximum tardiness, total or mean tardiness, total weighted tardiness and the number of tardy jobs [2-3]. In this regard, a rich body of research exists for single criterion. However, very few studies deal with multi criteria objectives in general scheduling problems [4-5].

Generally, machine scheduling falls into two main categories, single machine and multi-machine problems. Many scheduling algorithms for single machines proved to be effective and/or optimum. The most important of these algorithms are SPT and EDD orders that optimize such performance measures as mean flow time and maximum tardiness, respectively. Although, single machines are easier to solve, they hardly happen in real manufacturing systems. On the other hand, most scheduling problems for parallel machines have real occurrences in industrial systems. It's proved that scheduling problems for parallel machines is NP-hard [6]. The parallel machines scheduling is a growing area of research in recent years, and many papers have been published in this field [5, 7-9]. As due-date-related problems, especially for multi-machine environments, are usually computationally complex, most existing results are typically for problems with small sizes or simple settings, such as identical parallel machines [9-11]. As an example, the shifting bottleneck method which was originally designed to minimize makespan has been revised to decompose the multi-machine problems into a series of single-machine problems [12-13].

Researches on unrelated parallel-machine scheduling problems are very limited. Among the few papers, Li and Yang [8], proposed a research on non-identical parallel machine. Their single-criterion objective was to minimize the total weighted completion time on unrelated machines. Also Gurel and Akturk [14] proposed an improved branch and bound algorithm to solve a bi-criteria allocation and processing time problem for unrelated parallel CNC machines.

Reducing the makespan may results in increasing other costs such as machining cost or JIT related penalties [15-16]. Given the above, it may be necessary to consider more than one objective in manufacturing scheduling. Relative to scheduling problems that optimize makespan or flow time, due-date-related problems are usually much more computationally complex and are classified as strongly NP-hard [6]. Such complicated problems are difficult to solve optimally. In many situations, a ''good'' solution obtained by a heuristic algorithm in reasonably short computational time is often desirable. Currently the most widely used heuristic techniques in combinatorial optimization are Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithms (GAs), and Ant Colony Optimization (ACOs) algorithms.

In this paper, an efficient procedure has been developed to solve a multi-criteria scheduling problem for unrelated parallel machines. The objective is to simultaneously minimize the total machining costs, JIT related costs (earliness and tardiness

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penalties) and the cost associated with the system utilization, i.e., makespan.

It is notable that, this study consists of jobs allocation to machines with similar, but not necessarily identical, capabilities.

II. Problem Description

In this study, we investigate a system with ***M*** unrelated machines with similar capabilities which can process either of the ***N*** jobs. For such a problem there is $(n!)^m$ possible schedule. All jobs and machines are available at time zero. Each job has a distinct due date. However, the processing time (and cost) of a job may vary on different machines. This generalized property of the problem, to a large extend, makes it more realistic. In turn, all machines are capable of processing either of the ***N*** jobs, although with different processing costs and times. Each job can be assigned to one machine only and no preemption of jobs is allowed.

Given the above system description, the objective is to simultaneously determine the job-machine assignments and the jobs sequencing on each machine so as to minimize the total weighted processing cost. Here, the processing cost consists of three components as follows:

a) Earliness and tardiness penalties: Completing a job before its due date will lead to increasing inventory cost, shop floor congestion, etc. The tardiness penalty usually occurs due to the loss of goodwill if the job is to be delivered to the customer or due to the waiting time if the job is to be processed by the next manufacturing stage. Whereas these criterions have different importance, they are weighted. Usually, the earliness penalty is considered less than the tardiness penalty.

b) Makespan cost: Makespan is one of the most widely studied objectives in the literatures and reducing it is of great interest. Also makespan is a measure of utilization and relates directly to the system's efficiency. In multi-machine environment makespan is sequence dependent. Since reducing makespan results in increased system utilization, it may be one of the main objectives in process scheduling.

c) Machining cost: This is the most obvious cost components which is directly proportional to the time taken by each job on a given machine. In multi-machine manufacturing systems, the unit-time value of different processors may quite vary because of the technology differences, energy or labor requirements, tool usage and failure rates. Therefore, the job-machine assignment is very important and can affect the total processing costs.

III. The Solution Procedure-Simulated Annealing Approach

For real and large size optimization problems, the traditional optimization methods are often inefficient and time consuming. With the advent of computer technology and computational capabilities in the last few decades, the applications of heuristic algorithms are widespread.

The annealing process, used in metal working, involves heating the metal to a high temperature and then letting it gradually cools down to reach a minimum stable energy state. If the metal is cooled too fast, it won't reach the minimum energy state. Kirkpatrick and his colleagues [17] used this concept to develop a search algorithm called Simulated Annealing (SA). Among different heuristic algorithms, SA is one of the most powerful optimization methods that simulates the cooling process of a molten metal. The general stages of the SA algorithm for the job scheduling on parallel machines are as follows:

1. Initialization: determining the temperature parameter T_0 and the cooling schedule*:* r $(0 < r < 1)$ and also the termination criterion (in this study, number of iterations $k = 1\ldots K$). Generate and evaluate an initial candidate solution (perhaps at random); call this the current solution, *c*.
2. Generate a new neighboring solution, *m*, by making a small change in the current permutation of jobs and evaluate this new solution.
3. Accept this new solution as the current solution if:
   a. The objective value of new solution, *f (m)*, is better than of the current solution, *f (c)*.
   b. If *f (m)* is worse than *f (c)*, the value of acceptance is determined by probability function given by ($exp\ (f(m) - f(c)) / T_k$) which must be greater than a uniformly generated random number "*rand*"; where $0 < rand < 1$.
4. Check the stop criteria and update the temperature parameter (i.e., $T_k = r * T_{k-1}$) and return to Step 2. $(0<r<1)$

The algorithm is flexible, it needs fewer tuning parameters, and it can be adapted to a wide range of problems as well as ability to escape local optima. In addition, for any heuristic optimization procedure, the algorithm parameters should be tuned to enhance its performance. Therefore, the ease of tuning a given algorithm is an important feature in selecting a proper solution technique. In SA there are only two major tuning parameters: *the initial temperature* and *cooling schedule*. As a result, SA can easily be "tuned" with minimum trial runs.

Simulated annealing can avoid local optima by accepting a non-improving neighbor (see Step 3.b).Thus, at the start of SA most worsening moves are accepted, but at the end only improving ones are likely to be accepted. This, to a large extend, helps the algorithm jump out of local optima.

IV. Numerical Example and Results

In this study, we consider a hypothesis problem with 80 jobs and 5 unrelated parallel machines. The ranges of data related to the costs and times of performing the operations are as follows: processing times, 1–30(time unit); due dates, 12–240(time unit); tardiness penalties, 1.3–2.8($/time unit), earliness penalties, 0.1–1($/time unit); machining costs, 1.5–18($/time unit).The algorithm was coded in MATLAB 7.0 software and was executed on a Pentium 4 computer with 1.70 GHz CPU and 512 MB of RAM. The best set of search parameters, found through several trial runs, is as follow: initial temperature (C_0) = 20000; cooling rate (α) =0.99; and termination criteria = 20000 iterations or temperature less than 0.01.

Computational time, the time taken to reach final solution, is an important criterion in the performance of any solution techniques; especially when we need a day-to-day scheduling. The convergence curve of cost function for SA algorithm, using best parameters is plotted in Figure 1. This curve shows that the algorithm converges towards final solution very quickly. The fluctuation of the curve in the beginning of the search due to high initial temperature is great. However, as the temperature decreases by cooling rate α, algorithm would select only the improving solutions.

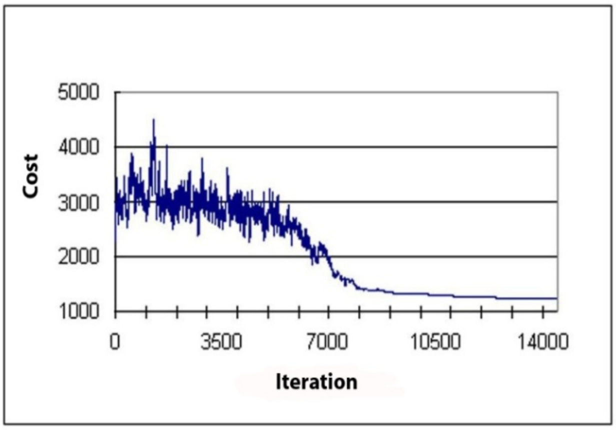

Fig. 1 The convergence curve of the SA algorithm (total cost)

As it shows, at first SA algorithm accepts some non-improving solutions, but by increasing number of iterations and reducing temperature probability of transition to better solutions increases, therefore, after some iterations algorithm converges to the optimal solution.

Diagrams of machining costs, earliness and tardiness penalties are shown in Figure 2 and 3.

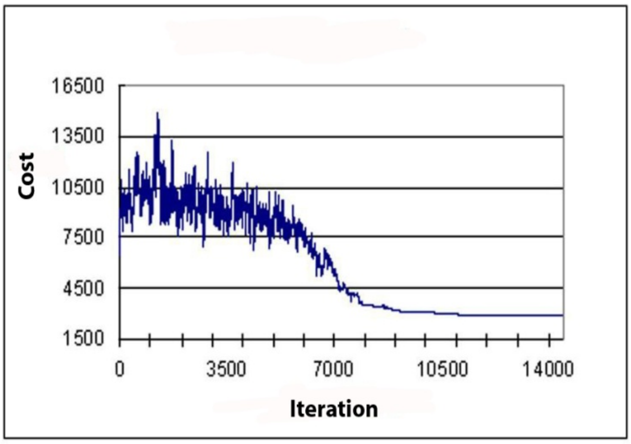

Fig. 2 The convergence diagram of the SA algorithm (earliness and tardiness penalties)

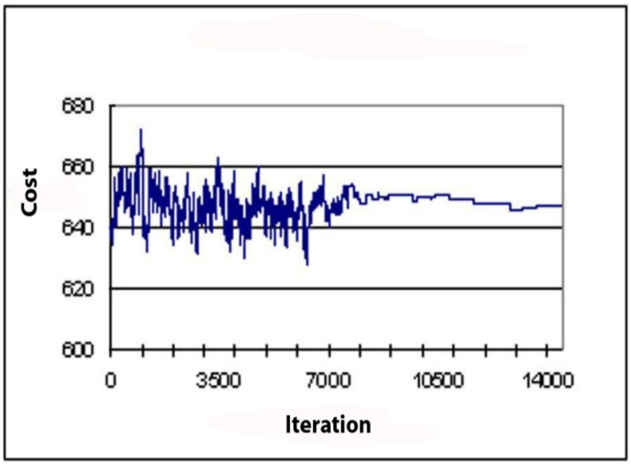

Fig. 3 The convergence diagram for SA algorithm (machining costs)

As they show, by decreasing makespan and Just-In-Time production costs, machining costs slightly increase which shows the importance of combinatorial optimization. The computational results for this case are summarized in Tables 1. As shown, sum of the weighted costs is improved by more than 46% from 2295 to 1227 unit cost. In this problem, lateness cost is improved by more than 56% and the improvement of makespan cost is 34% while machining cost is decreased about 1.5%.

TABLE I
THE RESULT OBTAINED BY SA ALGORITHM FOR THE SCHEDULING PROBLEM

	Initial production plan	Final production plan	Percent of improvement
Machining cost ($/time unit)	637.5	647.2	-%1.5
Lateness penalties ($/time unit)	6485	2816	%56.5
Makespan (time unit)	376	247	%34
Sum of the weighted costs ($)	2295	1227	%46.5

The weighted penalties are assigned to different cost components to show their relative importance's in the objective function. Since there are no exact solution procedures for such complex problems in reasonable computational times, a SA algorithm is employed. Computational experiments demonstrate that the SA method, in terms of both convergence speed and solution quality is an effectiveness method towards solving large-size and multi-criteria scheduling problems. The results and high convergence rate of objective function along with very short computational time, shows the capability of suggested algorithm for solving the combinatorial optimization of sequencing and scheduling problem.

V. Conclusion

In this study, a manufacturing scheduling problem for unrelated parallel machines has been investigated and solved by Simulated Annealing (SA) algorithm. The objective of the model was to simultaneously minimize the total machining costs, JIT related costs (earliness and tardiness penalties) and the cost associated with system utilization; i.e. total makespan. The objective function is the linear weighted combination of the above mentioned costs. The assigned weights to each parameter is determined according to the type of industry, products, amount of supply and demand, time limitations due to deterioration; such as food products. The multi-machine scheduling problems are combinatorial in nature whose solution space, for a problem with *n* independent jobs and *m* unrelated machines, may reach $(n!)^m$. Using the proposed solution procedure, the cost function has been minimized with respect to manufacturing resources and system specifications. Computational results showed that the proposed algorithm is quite capable of providing fast and high-quality solutions for such complicated problems.

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