A New Approach to Solve Blasius Equation using Parameter Identification of Nonlinear Functions based on the Bees Algorithm (BA)

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Abstract—In this paper, a new approach is introduced to solve Blasius equation using parameter identification of a nonlinear function which is used as approximation function. Bees Algorithm (BA) is applied in order to find the adjustable parameters of approximation function regarding minimizing a fitness function including these parameters (i.e. adjustable parameters). These parameters are determined how the approximation function has to satisfy the boundary conditions. In order to demonstrate the presented method, the obtained results are compared with another numerical method. Present method can be easily extended to solve a wide range of problems.

Keywords—Bees Algorithm (BA); Approximate Solutions; Blasius Differential Equation.

I. INTRODUCTION

IN applied mathematics, where a real-world problem is simulated with a differential equation, numerical methods of predictor-corrector, Runge-Kutta, finite difference, finite element, finite volume, boundary element, spectral and collocation represent a strategy by which scientists can attack many problems, subject to some initial or boundary conditions [1]. Artificial intelligence techniques are used in order to solve different differential equation.

Lee and Kang [2] used parallel processor computers in order to solve a first order differential equation using Hopfield neural network models. Meade and Fernandez [3] used feed forward neural networks architecture and B1 splines to solve linear and nonlinear ordinary differential equations. Lagaris et al. represented a new method to solve First order linear ordinary and partita differential equations using artificial neural networks [4]. Malek and Shekari Beidokhti used a hybrid artificial neural network- Nelder-Mead method to solve

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high order linear differential equations [1]. A hybrid artificial neural network- swarm intelligence method was used by Khan et al. to solve first order nonlinear ODEs [5]. Lee [6] introduced a new bilaterally approach to find the upper and lower bounds of Blasius equation. In that work (i.e. Lee [6]) the presented method could not satisfy the boundary conditions. In contrast with Lee [6], present study introduces a new method to solve Blasius differential equation which satisfies the boundary conditions. Finally, in order to demonstrate the presented method, a comparison is made between present method and the method which is used in [6]. The paper is organized as follows. A brief review of Bees Algorithm (BA) is brought in Section 2. Section 3 gives details of problem formulation. Numerical results are discussed in Section 4. Finally, conclusions and directions for future research are presented in section 5.

II. BEES ALGORITHM (BA)

The Bees Algorithm (BA) was first proposed by Pham et al. [7], inspired by the bees foraging behavior in nature. Bees foraging behavior in nature has been discussed in [8-11]. This section summarizes the main steps of the Bees Algorithm. Fig. 2 shows the flowchart of the Bees Algorithm. The algorithm requires a number of parameters to be set, namely: number of scout bees (s), number of sites selected for neighborhood search (out of s visited sites) (m), number of top-rated (elite) sites among m selected (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other (m-e) selected sites (nsp), the initial size of each patch (ngh) (a patch is a region in the search space that includes the visited site and its neighborhood), and the stopping criterion. These parameters are considered as important factors in BA. The algorithm starts with s scout bees randomly distributed in the search space. The fitness of the sites (i.e. the performance of the candidate solutions) visited by the scout bees are evaluated in step 2.

In step 4, the m sites with the highest fitnesses are designated as "selected sites" and chosen for neighborhood search. In steps 5 and 6, the algorithm searches around the selected sites, assigning more bees to search in the vicinity of the best e sites. Selection of the best sites is made according to the fitness associated with them. Search in the neighborhood of the best e sites – those which represent the most promising solutions – are made more detailed. As already mentioned,

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this is done by recruiting more bees for the best e sites than for the other selected sites. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm. In step 6, for each patch only the bee of highest fitness value is selected to form the next bee population. In nature, there is no such restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are placed randomly around the search space to scout for new potential solutions. At the end of each iteration, the colony has two parts to its new population: representatives from the selected patches, and scout bees assigned to conduct random searches. These steps are repeated until a stopping criterion is met. For more details about intelligent optimization techniques, the reader is referred to [7, 12, and 13-18]. In this study, BA algorithm is coded with MATLAB 2007.

III. PROBLEM FORMULATION

A Blasius boundary layer, in physics and fluid mechanics, describes the steady two-dimensional boundary layer that forms on a semi-infinite plate which is held parallel to a constant unidirectional flow. The differential equation governing the free oscillation of the mathematical Blasius equation, when friction is neglected, is written as follow:

$$f''' + \frac{1}{2} f f'' = 0 \quad D : x \in [0, \infty] \tag{1-a}$$

subject to the following boundary conditions:

$$f(0) = 0, \; f'(0) = 0 \text{ and } f'(\infty) = 1 \tag{1-b}$$

In order to solve Eq.(1), assume a discretization of the domain D with m arbitrary points. Now, the problem can be transformed to the following set of equations:

$$f'''(x_i) + \frac{1}{2} f(x_i) f''(x_i) = 0 \quad \forall x_i \in D \quad , i = 1, 2, ..., m \tag{2}$$

subject to given boundary conditions.

The following nonlinear trial function is assumed as approximate solution:

$$f_T = x - a_1 + a_2 e^{-x} + a_3 x e^{-x} + a_4 e^{-2x} \tag{3}$$

where f_T is trial function and $\vec{a}$ (i.e. a_1 to a_4) are adjustable parameters.

These parameters (i.e. adjustable parameters) should be determined regarding to minimize the following sum of squared errors, subject to given boundary conditions.

$$Error(\vec{a}) = \sum_{i=1}^{m} \left(f_T'''(x_i) + \frac{1}{2} f_T(x_i) f_T''(x_i) \right)^2 \tag{4}$$

According to given boundary conditions in eq. (1-b), following equations are obtained between a_1 to a_4.

$$a_3 = 2a_1 - a_2 - 1 \tag{5-a}$$

$$a_4 = a_1 - a_2 \tag{5-b}$$

In order to calculate $Error(\vec{a})$ function, trail function derivations respect to independent variable x are needed. Using Eqs (5-a), (5-b), required terms in Eq.4 are written as follows:

$$f_T(x_i) = x_i - a_1 + a_2 e^{-x_i} + (2a_1 - a_2 - 1)x_i e^{-x_i} + (a_1 - a_2)e^{-2x_i} \tag{6-a}$$

$$f_T''(x_i) = a_2 e^{-x_i} - (4a_1 - 2a_2 - 2)e^{-x_i} + 4(a_1 - a_2)e^{-2x_i} + (2a_1 - a_2 - 1)x_i e^{-x_i} \tag{6-b}$$

$$f_T'''(x_i) = (6a_1 - 3a_2 - 3)e^{-x_i} - a_2 e^{-x_i} - 8(a_1 - a_2)e^{-2x_i} - (2a_1 - a_2 - 1)x_i e^{-x_i} \tag{6-c}$$

Now, the Bees Algorithm can be applied in order to determine optimal values of a_1 and a_2 regarding to minimize $Error(\vec{p})$ function. At the end, a_3 and a_4 will be calculated from Eqs (5-a), (5-b).

IV. NUMERICAL RESULTS

In order to demonstrate the presented method, a comparison is made between present method here and in [6]. The collocation point locations at x_i were used in interval [0, 10] with step size of 0.1.

Following combination of user-specified parameters of BA were used for this problem:

Initial number of scout bees (s): 50
Selected patches (m): 8
Elite patches (e):3
Number of bees recruited for the best sites (nep):10
Number of bees recruited for the other sites (nsp):4
Neighborhood radius (ngh):0.2

For the best solution, $Error(\vec{p})$ was 2.80×10^{-4} . The corresponding adjustable parameters were

a_1= 1.9091, a_2=1.5665, a_3=1.2516, and a_4=0.3425.

Comparison of results obtained using proposed method in this study and another numerical method (i.e. [6]) are shown in Table 1. Fig.1 shows the results of this study and the numerical solution in [6] for example. The results show that our proposed method is in good agreement with obtained solution in [6].

V. CONCLUSION

A new approach was introduced to solve Blasius equation using parameter identification of a nonlinear function which was used as approximation function. Bees Algorithm (BA) was applied in order to find the adjustable parameters of approximation function regarding to minimize a fitness function including these parameters (i.e. adjustable parameters). These parameters were determined so that the approximation function has to satisfy the boundary conditions. In order to demonstrate the presented method, a comparison was made between present method here and in the literature. The results showed that our proposed method is in good agreement with obtained solution in the literature. Future work is focused on comparing the effect of changing optimization technique on minimizing the error function.

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TABLE I COMPARISON BETWEEN RESULTS OBTAINED USING
PROPOSED METHOD IN THIS STUDY AND [6]

| x_i	Present method	Lower bound [6]	Upper bound [6]
0	0	-0.00053	0.050001
0.5	0.046660	0.055298	0.07159
1	0.174020	0.161791	0.209158
1.5	0.376426	0.334666	0.449752
2	0.647977	0.586389	0.76134
2.5	0.978663	0.904743	1.127819
3	1.356703	1.275431	1.535018
3.5	1.770820	1.685283	1.971511
4	2.211421	2.123457	2.428593
4.5	2.670931	2.581608	2.899886
5	3.143655	3.053582	3.380849
5.5	3.625458	3.534977	3.868316
6	4.113417	4.022718	4.360115
6.5	4.605504	4.514691	4.854778
7	5.100335	5.009464	5.35132
7.5	5.596976	5.506076	5.849089
8	6.094802	6.003889	6.347655
8.5	6.593401	6.502482	6.846736
9	7.092501	7.001581	7.346148
9.5	7.591925	7.501004	7.845774
10	8.091557	8.000637	8.345536
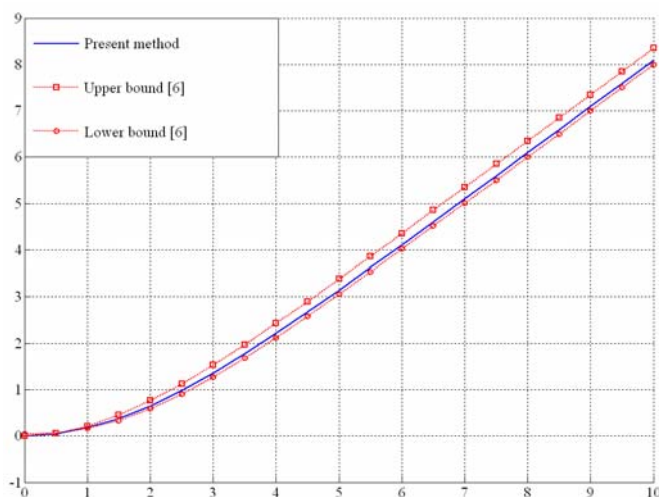
Fig. 1 Comparison between obtained results using present method
and [6].

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