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The Contraction Point for Phan-Thien/Tanner Model of Tube-Tooling Wire-Coating Flow

V. Ngamaramvaranggul, S. Thenissara

***Abstract*—**The simulation of extrusion process is studied widely in order to both increase products and improve quality, with broad application in wire coating. The annular tube-tooling extrusion was set up by a model that is termed as Navier-Stokes equation in addition to a rheological model of differential form based on single-mode exponential Phan-Thien/Tanner constitutive equation in a two-dimensional cylindrical coordinate system for predicting the contraction point of the polymer melt beyond the die. Numerical solutions are sought through semi-implicit Taylor-Galerkin pressure-correction finite element scheme. The investigation was focused on incompressible creeping flow with long relaxation time in terms of Weissenberg numbers up to 200. The isothermal case was considered with surface tension effect on free surface in extrudate flow and no slip at die wall. The Stream Line Upwind Petrov-Galerkin has been proposed to stabilize solution. The structure of mesh after die exit was adjusted following prediction of both top and bottom free surfaces so as to keep the location of contraction point around one unit length which is close to experimental results. The simulation of extrusion process is studied widely in order to both increase products and improve quality, with broad application in wire coating. The annular tube-tooling extrusion was set up by a model that is termed as Navier-Stokes equation in addition to a rheological model of differential form based on single-mode exponential Phan-Thien/Tanner constitutive equation in a two-dimensional cylindrical coordinate system for predicting the contraction point of the polymer melt beyond the die. Numerical solutions are sought through semi-implicit Taylor-Galerkin pressure-correction finite element scheme. The investigation was focused on incompressible creeping flow with long relaxation time in terms of Weissenberg numbers up to 200. The isothermal case was considered with surface tension effect on free surface in extrudate flow and no slip at die wall. The Stream Line Upwind Petrov-Galerkin has been proposed to stabilize solution. The structure of mesh after die exit was adjusted following prediction of both top and bottom free surfaces so as to keep the location of contraction point around one unit length which is close to experimental results.

***Keywords*—**wire coating, free surface, tube-tooling, extrudate swell, surface tension, finite element method.

I. INTRODUCTION

SIMULATION of wire coating problem is a way to deal with real problems especially for difficulties that might be encountered experimentally in extrusion processes of polymeric solutions. The technique can have vast applications in the industry of wires, cables, fiber optics and numerous types and sizes of containers that are widely used in houses, factories and vehicles around the world.

In general, wire coating process modeling consists of two particular dies: pressure tooling within which the wire coating process begins coating the die cast, and tube tooling in which wire is coated by polymer melt outside the die. For the second die, the location where the polymer melt flows to contact the wire at the beginning of coating is called the contraction point. The factors influential to the contraction point are pressure, velocity, viscosity, surface tension of polymer melt, and wire speed. These are considered under the following assumptions: incompressible, laminar, isothermal flow and no gravity. In addition, surface tension has been considered in extrudate swell with no slip condition at die wall.

Computational studies for wire coating flows are abound in literature with industry-related concerns. For two dimensional axisymmetric incompressible fluid employing finite element method (FEM) under isothermal condition, Caswell and Tanner[1] have designed wire coating die for low speed non-Newtonian fluid through power law model. Han and Rao[2] studied wire coating extrusion in theory and experiment for pressure-tooling die using the materials of low density polyethylene (LDPE) and high density polyethylene (HDPE) via applying the same model. In 1986, Mitsoulis[3] simulated the creeping flow of Newtonian and Power law fluid for wire coating problem in axisymmetric system. Binding et al.[4] studied high speed wire coating process for inelastic constitutive model. They varied viscosity models and commented on the limitation of modeling approach. Then, Mutlu et al.[5] employed tube-tooling die for coating problem. In their work, viscoelastic coating flows were simulated and solved by FEM technique for the PTT model due to the past work of Ngamaramaramvaranggul and Webster [6,7] made us know that PTT model can be fit well for viscoelastic fluid better than other models because it can be predict the properties of high elastics for comparison curve shown in Tanner's book [8]. Stability was attained by mean of coupled and decoupled schemes for single mode. Recently, Matallah et al.[9] considered with tube-tooling wire coating flow for HDPE applying FEM technique for the multi-mode Phan-Thien/Tanner (PTT) constitutive model. In another research, Ngamaramvaranggul and Webster[7] have focused on wire coating problem for LDPE. They publish a paper of two dimensional annular pressure-tooling wire coating flow using FEM to solve an isothermal and free surface flow for single-mode PTT model.

V. Ngamaramvaranggul, Department of Mathematics, Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand. Tel: +668 1314420, Fax: +662 2552287, (Email: vimolrat.n@chula.ac.th)

S. Thenissara, Department of Mathematics, Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan, Nakornratchasima 30000 , Thailand , Tel: +66818558511, (Email : saitharn@gmail.com).

In the present article, tube-tooling wire coating flow has been studied under the influence of surface tension to adjust free surface shape according to the study of Anastasiadis[10] about effect of surface tension on polymer melts so in 1998 he used the sessile drop method to predict a free surface curve. Later on a numerical method how to solve his work has been shown by Neumann and Spelt [11]. An exponential PTT constitutive and momentum equations have been solved by semi-implicit Taylor-Galerkin scheme under treatment of streamline upwind Petrov-Galerkin (SUPG), which was used by Hughes and Brooks [12] for its strong consistent stabilization nature. the past work of Ngamaramaramvaranggul and Webster [6,7] made us know that PTT model can be fit well for viscoelastic fluid better than other models because it can be predict the properties of high elastics for comparison curve shown in Tanner's book [8]. Stability was attained by mean of coupled and decoupled schemes for single mode. Recently, Matallah et al.[9] considered with tube-tooling wire coating flow for HDPE applying FEM technique for the multi-mode Phan-Thien/Tanner (PTT) constitutive model. In another research, Ngamaramvaranggul and Webster[7] have focused on wire coating problem for LDPE. They publish a paper of two dimensional annular pressure-tooling wire coating flow using FEM to solve an isothermal and free surface flow for single-mode PTT model.

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II. Governing Equations

For incompressible isothermal fluid with no gravity, the continuity equation is obtained from the conservation of mass in terms of velocity gradient. The Navier-Stokes equations from the conservation of momentum contain viscous term, convective acceleration and pressure gradient. Both non-dimensional equations are expressed in the forms:

$$\nabla \cdot \boldsymbol{U} = 0 \tag{1}$$

$$\mathrm{Re}(\partial \boldsymbol{U}/\partial t) = \nabla \cdot \boldsymbol{T} - \mathrm{Re}\,\boldsymbol{U} \cdot \nabla \boldsymbol{U} - \nabla p \tag{2}$$

Here, $\boldsymbol{U}$ is fluid velocity vector, $\boldsymbol{T}$ is stress tensor, p is isotropic fluid pressure, ∇ is differential operator and Re is non-dimensional Reynolds number

$$\mathrm{Re} = \frac{\rho U R}{\mu_0}$$

In this problem, ρ is fluid density, U is characteristic velocity in term of wire speed, R is characteristic length in term of die radius and μ_0 is the zero-shear viscosity which combines a polymeric solute viscosity μ_1 and a solvent μ_2 as $\mu_0 = \mu_1 + \mu_2$. Further information regarding non-dimensionalization is available in Ngamaramaramvaranggul and Webster [6].

The equation of viscoelastic fluid for exponential Phan-Thien/Tanner (EPTT) model [7] has considered below.

$$We\,\boldsymbol{\tau}_t = 2\mu_1 \boldsymbol{D} - \mathrm{f}\boldsymbol{\tau} + \mathrm{We}\{\boldsymbol{\tau} \cdot \nabla \boldsymbol{U} + (\nabla \boldsymbol{U})^\dagger \cdot \boldsymbol{\tau} - \boldsymbol{U} \cdot \nabla \boldsymbol{\tau} + \xi[\boldsymbol{D} \cdot \nabla \boldsymbol{\tau} + (\boldsymbol{D} \cdot \boldsymbol{\tau})^\dagger\,]\} \tag{3}$$

Where

a nondimensional Weissenberg number is $We = \dfrac{\lambda_1 U}{L}$,

a derivative of extra stress tensor with respect to time is $\boldsymbol{\tau}_{t}$

the deformation rate tensor $\boldsymbol{D}$ is defined as $\boldsymbol{D} = \frac{1}{2}(\nabla \boldsymbol{U} + \nabla \boldsymbol{U}^\dagger)$,

a constant value ξ,

the extra stress tensor $\boldsymbol{\tau}$ is then

defined by $f\boldsymbol{\tau} + \lambda_1 \overset{\nabla}{\boldsymbol{\tau}} = 2\mu_1 \boldsymbol{D}$,

the exponential Phan-Thien/Tanner function is

$$f = \exp\left[\frac{\varepsilon \lambda_1}{\mu_1} trace(\boldsymbol{\tau})\right],$$

Where $\overset{\nabla}{\boldsymbol{\tau}} = (\partial \boldsymbol{\tau}/\partial t) + \boldsymbol{U} \cdot \nabla \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \nabla \boldsymbol{U} - (\boldsymbol{\tau} \cdot \nabla \boldsymbol{U})^\dagger$

and $\boldsymbol{T} = \boldsymbol{\tau} + 2\mu_2 \boldsymbol{D}$

III. Numerical Discretization and Problem Specification

Numerical method is used to solve differential forms of equations (1), (2) and (3) by transforming the continuous form of differential equations to discrete set of linear equations as following scheme.

a. Fractional step

In this paper, the fractional step is used to solve non-linear partial differential equations (2) and (3) with semi-implicit time step Taylor expansion termed as Taylor-Galerkin algorithm [7]. The discretization stages are as follows,

Stage 1a:

This stage is related to updating both stress and non-solenoidal velocity fields. The half time step of velocities and stresses can be derived from the equations below:

$$\frac{2\,\mathrm{Re}}{\Delta t}(\boldsymbol{U}^{n+\frac{1}{2}} - \boldsymbol{U}^n) = [\nabla \cdot (2\mu_2 \boldsymbol{D} + \boldsymbol{\tau}) - \mathrm{Re}\,\boldsymbol{U} \cdot \nabla \boldsymbol{U} - \nabla p]^n + \nabla \cdot \mu_2(\boldsymbol{D}^{n+\frac{1}{2}} - \boldsymbol{D}^n)$$

$$\frac{2We}{\Delta t}(\boldsymbol{\tau}^{n+\frac{1}{2}}-\boldsymbol{\tau}^{n})=\{2\mu_{1}\boldsymbol{D}-f\boldsymbol{\tau}-We[\boldsymbol{U}\cdot\nabla\boldsymbol{\tau}-\nabla\boldsymbol{U}\cdot\boldsymbol{\tau}-(\nabla\boldsymbol{U}\cdot\boldsymbol{\tau})^{\dagger}]\}^{n}$$

This stage is used for solving half time step of velocity and stress by a method of Jacobi iterative solver. Solutions of this stage are the input for stage 1b as below.

Stage 1b:

The transient stage of intermediate velocities and a full time step of stresses are updated as in the following equations:

$$\frac{\mathrm{Re}}{\Delta t}(\boldsymbol{U}^{*}-\boldsymbol{U}^{n})=[\nabla\cdot(2\mu_{2}\boldsymbol{D}-\nabla p]^{n}+[\nabla\cdot\boldsymbol{\tau}-\mathrm{Re}\boldsymbol{U}\cdot\nabla\boldsymbol{U}]^{n+\frac{1}{2}}+\nabla\cdot\mu_{2}(\boldsymbol{D}^{*}-\boldsymbol{D}^{n})$$

$$\frac{We}{\Delta t}(\boldsymbol{\tau}^{n+1}-\boldsymbol{\tau}^{n})=\{2\mu_{1}\boldsymbol{D}-f\boldsymbol{\tau}-We[\boldsymbol{U}\cdot\nabla\boldsymbol{\tau}-\nabla\boldsymbol{U}\cdot\boldsymbol{\tau}-(\nabla\boldsymbol{U}\cdot\boldsymbol{\tau})^{\dagger}]\}^{n+\frac{1}{2}}$$

Having obtained the results from previous stage, the intermediate velocities and a full time step of stresses are calculated by the same method of stage 1a; namely, Jacobi iteration. When stresses have converged at this stage the velocities yet have not; therefore, velocity quantities at this stage are applied to compute pressure in stage 2 and then full time step of velocities at the final stage.

Stage 2:

Full time step of pressure is related to velocity according to the equation:

$$\frac{\Delta t}{2\,\mathrm{Re}}\nabla^{2}(p^{n+1}-p^{n})=\nabla\cdot\boldsymbol{U}^{*}$$

Pressure is computed through Cholesky decomposition after intermediate velocity values from stage 1b are computed. Hence, the full time step pressure is conducted to correct the full time step velocity in next stage.

Stage 3:

$$\frac{2\,\mathrm{Re}}{\Delta t}(\boldsymbol{U}^{n+1}-\boldsymbol{U}^{*})=-\nabla(p^{n+1}-p^{n})$$

Solve full time step velocities by Jacobi iterative method.

After time expansion by finite difference method, the weight residual of Galerkin method has been used to discretise space in order to set up the equations of stages 1-3 as the system of algebraic linear equations therefore the complex non-linear differential equations become to simple linear equations.

b. Surface tension

In 1998, Anastasiadis[10] has studied the effect of surface tension on two types of polymer melts, linear low-density polyethylene (LLDPE) and high-density polyethylene (HDPE) by applying the sessile drop method to find a free surface curve as shown in figure 1. Further details have been provided by Neumann and Spelt [11].

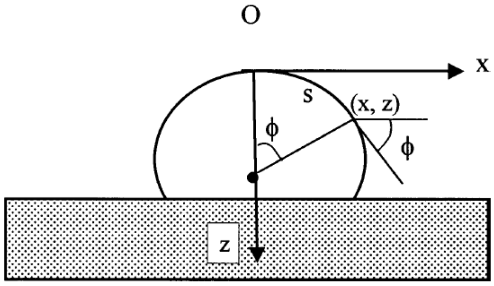


Fig. 1. Schema of the sessile drop

The relationship between coordinates x and z in dimensionless form is given by the Bashforth–Adams in 1882 [13]. The free surface shape of a pendant/sessile drop as shown in figure 1 can be constructed based on the following equations:

$$\frac{d\phi}{dS}=\frac{2}{B}+z-\frac{\sin\phi}{x} \tag{4}$$

$$\frac{dX}{dS}=\cos\phi \tag{5}$$

$$\frac{dZ}{dS}=\sin\phi \tag{6}$$

$$X(0)=Z(0)=\phi(0)=0$$

$$B=a\sqrt{\frac{g\delta\rho}{\gamma_{LV}}}$$

where the dimensionless variables, *X*, *Z*, and *S* are defined as $X=x\sqrt{c}$,

ϕ is angle between the tangent and the profile at point (x,z)

(x,z) is a coordinate of point in drop profile

S is distance from the drop apex to coordinate (x,z)

a is radius of curvature at the drop apex

g is gravitational acceleration constant (m/s^2)(LT^{-2})

ρ is polymer density (kg.m^{-3})(ML^{-3})

γ_{LV} is the interfacial tension between the liquid and its vapor

Anastasiadis [10] has calculated all parameters and used them to predict the shape of pendant/sessile drop for polymer melt at temperatures up to 300°C. Free surface shape has been calculated by varying *B* values at *B* ={ -2.429, -1.5539, -0.989, -0.779, -0.680, -0.649, -0.570, -0.440} and an optimum *B* value for HDPE of -0.680 has been used to modify streamline path for free surface location, which has already been explained by Ngamaramvaranggul and Webster [14,15]. After the calculation of free surface path without surface tension, the approximation of first position is a bit higher so we have obtained condition of surface tension to adjust the free surface path. The coordinate (x,z) for free surface shape of sessile drop that appearing in equations (4)-(6) has been solved by predictor-corrector method of Runge-Kutta up to four order [16]. The approximation of second free surface curve for coordinate (x,z) is a bit lower than the first curve at the beginning and growth up to near the position of first curve for a while then dropped immediately so the second position has been determined from the beginning position until the highest position and cut the last part when it dropped. The average path from both locations has been calculated to be proper position.

c. Flowchart

The easy way to depict the schema for solving a numerical finite element method through 3 fractional steps explained above can be outlined schematically in figure 2. The basis algorithm shows the simulation of isothermal flow for single-mode with couple scheme as following explanation. First, generate finite mesh for input file that is used for setting up shape functions and matrices in order to formulate the system of linear equations then solve the equations by fractional steps at the same time of applying the stream line upwind Petrov-Galerkin to maintain the stability and accuracy. After solution is computed, calculate free surface location and adding surface tension for die swell. Adjusting mesh according to new location and modify solution that is belong to new position. Check the final solution whether it is going to converge. Repeat the beginning step if the solution is too far from the acceptable result until it is less than the small amount that we set to 10^{-5}. At the end of the process, save the converge solution in file and analyse the result.

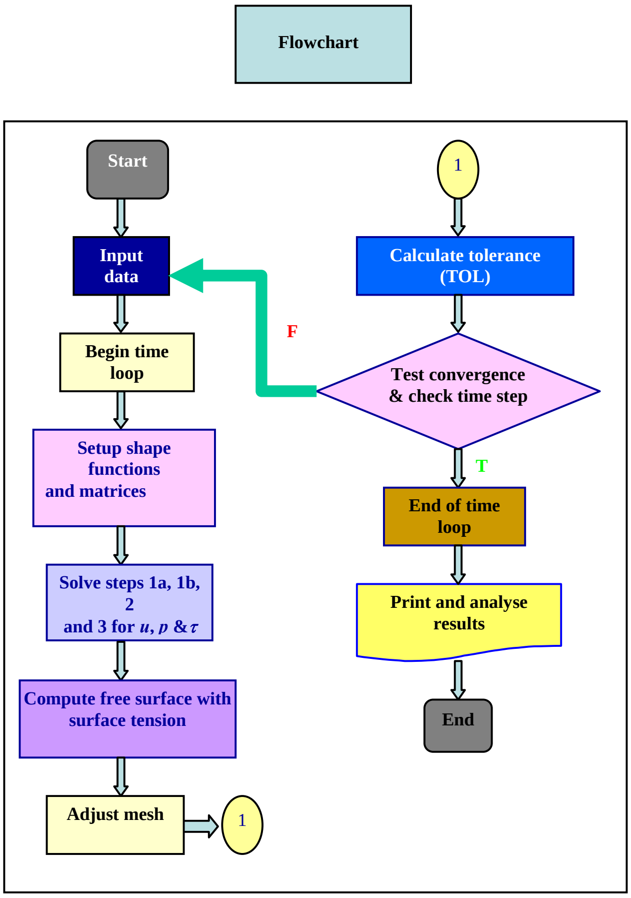


Fig. 2. Flowchart for wire coating flow program

IV. Problem Specification

A schema of tube-tooling die is shown in figure 3 and the considered domain is displayed as simple finite element mesh in figure 4. For this problem, the fine mesh has been generated with 4,714 elements; 9,755 nodes and 61,015 degrees of freedom (DOF).

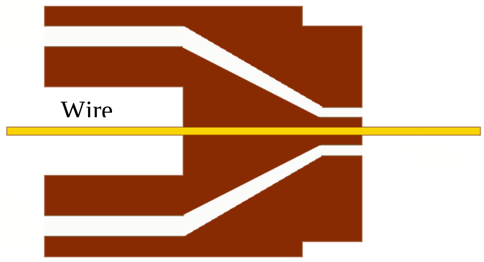


Fig. 3. Schema of tube-tooling die

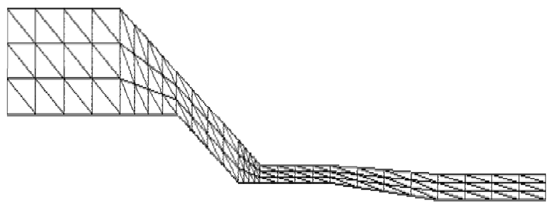


Fig. 4. Simple Mesh pattern

Schema of boundary domain is shown in figure 5. At inlet boundary (AA/), $u_r = \tau_{rr} = \tau_{\theta\theta} = 0$, $v_z = f(r)$, $\tau_{rz} = h(r)$ and $\tau_{zz} = g(r)$. At die walls (ABCD, A/ B/ C/D/), there is no slip so $u_r = v_z = 0$. For top and bottom free surfaces (D/ E/F/, DE), $p = 0$

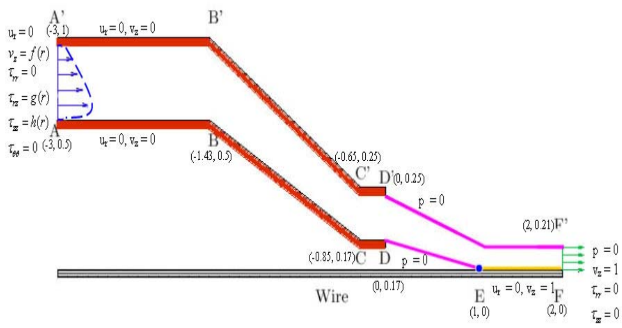


Fig. 5. Schema of boundary domain

V. Results and Discussion Results and Discussion

As shown in Figure 6, the velocity vector is in annular flow at the inlet and plug flow at the outlet. The color contours are exhibited for Weissenberg number equals 200 (*We*=200) in Figure 7a-g with the highest value of radial velocity at the die exit (Figure 7a) due to swell but no considerable change in the velocity value. Axial velocity almost vanishes at the inlet whilst it is maximized in section $C'D'DC$ as indicated in

Figure 7b according to conservation of flow rate the entrance has larger area and smaller velocity when compared against section $C'D'DC$.

Development of the annular flow at the inlet to the plug flow at the outlet is shown in Figure 6 with the maximum set at about 1.62 units. Pressure varies linearly in a descending manner from maximum to minimum with a gradient equivalent to seventeen units as shown in Figure 7c at die exit. The flow is deformed when passing a corner leading to a high shear rate as displayed in Figure 7d at every corner when direction changes especially in the corner of smaller section $C'D'DC$. It has been observed that extension rate of Figure 7(e) at small part of die geometry is a big value because the flow has been squeezed.

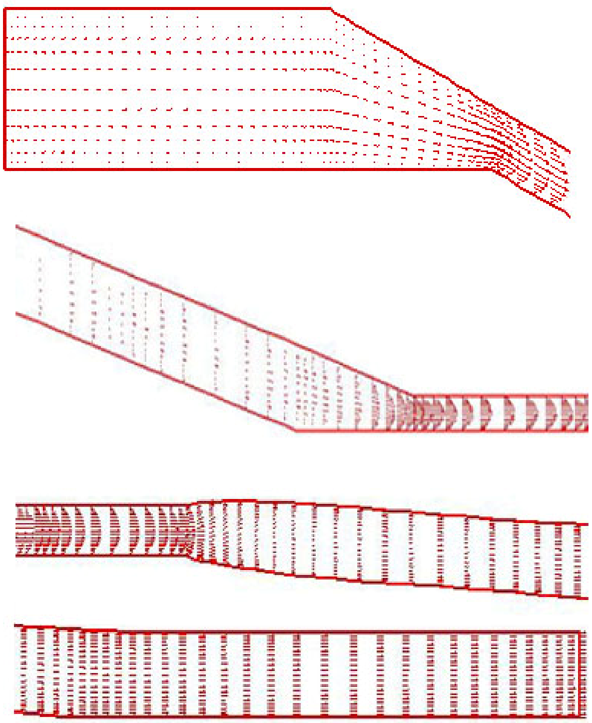
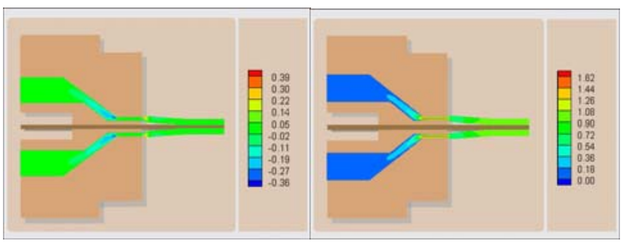
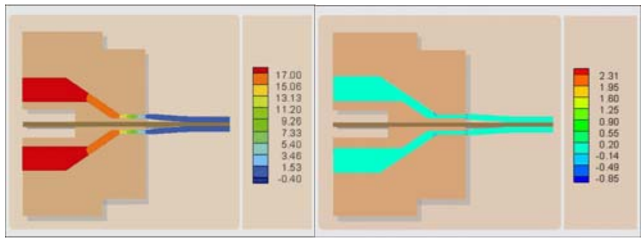


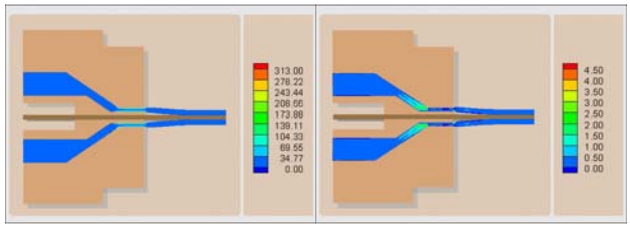
Fig. 6. Velocity vector



(a) Radial velocity u_r (b) Axial velocity v_z



(c) Pressure *P* (d) Shear stress T_{rz}

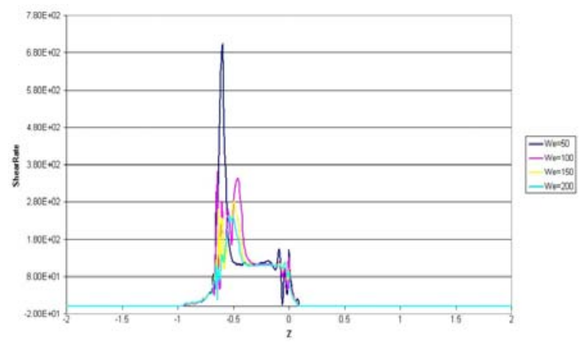


(e) shear rate, (f) extension rate

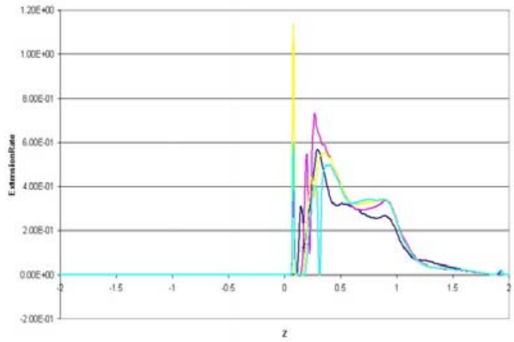
Fig. 7. Color contour of *We*=200 for
(a) radial velocity u_r, (b) axial velocity v_z,
(c) pressure *P*, (d) shear stress T_{rz}
(e) Shear rate (f) Extension rate

At various Weissenberg numbers, line contours for top surfaces are compared and nearly the same trend is detected for every *We*; therefore, every figure is displayed for the largest *We* of 200 in Figures 8(a)-(d).

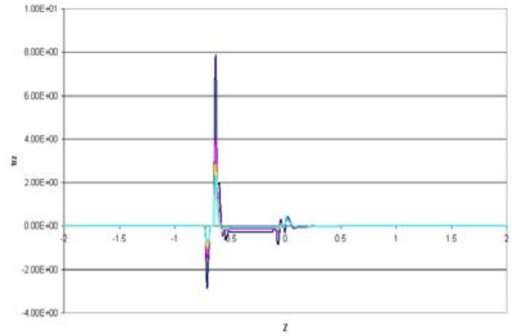
Fig. 8a displays the value of shear rate at top surface, which rises sharply at point C' and the die exit to 241.84 units because of sharp corner and swell. The extension rate is high at die exit and oscillated beyond die as shown in Fig. 8(b). The flow is deformed at corner C' and the die exit causing shear stress to increase to 2.29 units as can be seen in Figure 8c. Normal stress of figure 8d indicates a sharp rise of 3.71 at point C'.



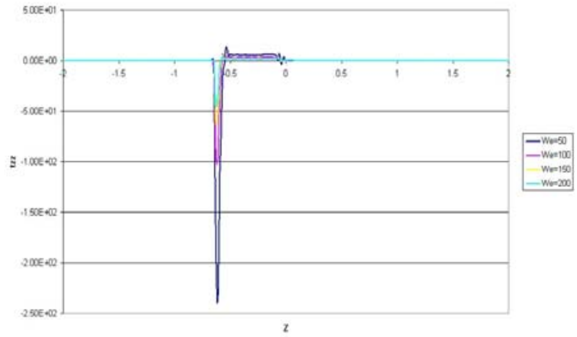
(a) Shear rate at top surface



(b) Extension rate at top surface



(c) Shear stress T_{rz} at top surface

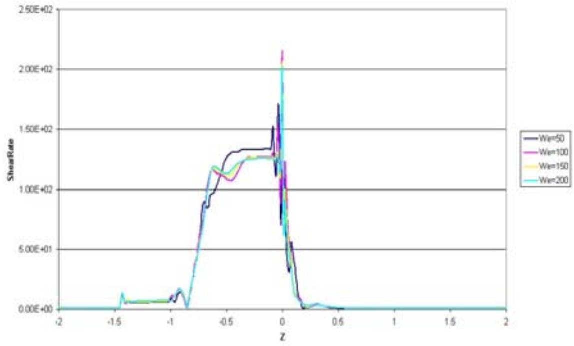


(d) Normal stress T_{zz} at top surface

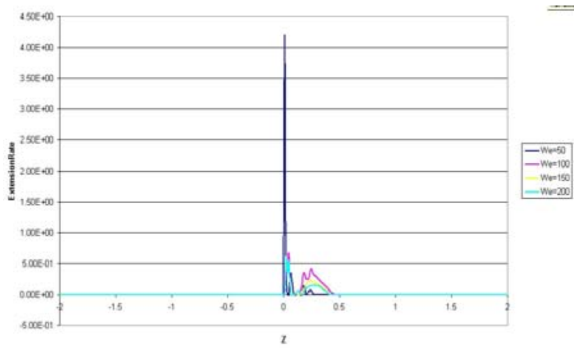
Fig. 8. Line contour for
(a) shear rate, (b) extension rate,
(c) shear stress T_{rz}, (d) normal stress T_{zz}
at top surface

Line contours for the bottom surfaces reflect similar behavior to their counterparts for the top surface so the explanation holds for Figures 9(a)-(d). Shear rate of Figure 9(a) rise suddenly at every corner especially at the die exit at the value around 200 units consistent with the swell after the die. Figure 9(b) shows extension rate with a high peak value of 0.6 at the die exit which corroborates Figure 7(e) with an oscillation range from 0 to 0.5.

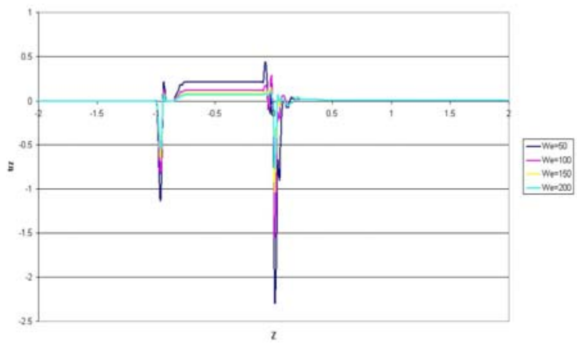
In Fig. 9(c), shear stress *Trz* demonstrates dual peaks at points $z = -1$ and $z = 0$. The curve is oscillated slightly before $z = -1$ and beyond die. Normal stress *Tzz* trend is displayed in Fig. 9(d) and the figure bears close resemblance to that of the shear rate with the value only one third of the shear rate.



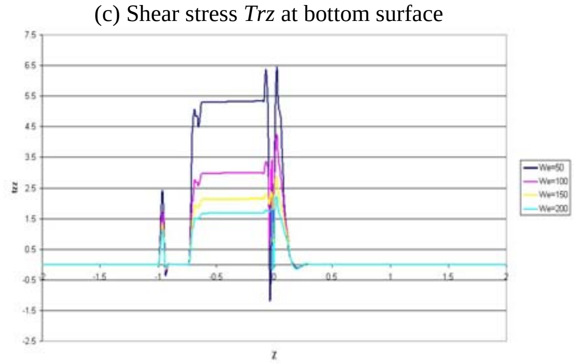
(a) Shear rate at bottom surface



(b) Extension rate at bottom surface



(c) Shear stress *Trz* at bottom surface

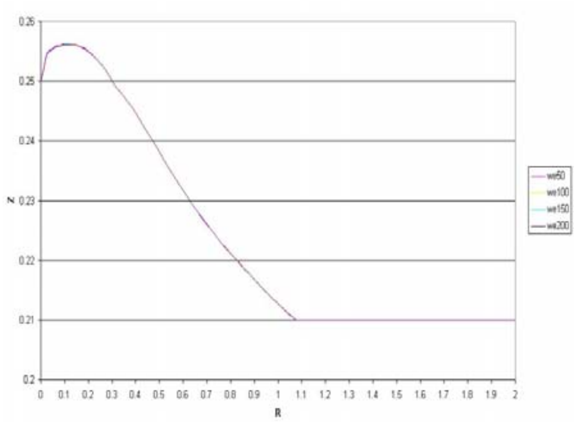


(d) Normal stress *Tzz* at bottom surface

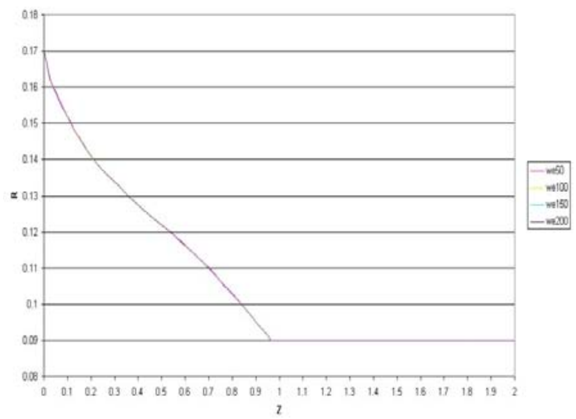
Fig. 9. Line contours for

(a) extension rate (b) shear rate,
(c) shear stress *Trz*, (d) normal stress *Tzz*
at bottom surface

The flow after die exit is swelled at top and bottom free surface as shown in Figure 10 and it draws down to coat the wire after die as is observable in Figure 7.



(a) Top surface



(b) Bottom surface

Fig. 10. Swell for (a) top surface and (b) bottom surface

VI. Conclusion

In case of large Weissenberg number *(We)* that presents the high elastic property, the curves from many figures are very oscillatory and it concerns to the ability of program computing. In the current work, the contraction point has been calculated for high *We* via imposition of surface tension effect on the whole process of computing. After the solution has converged, the contraction point shifts to the point (0.09, 0.98), which indicates close agreement to the value disclosed by the cable factory. Consideration of surface tension effect is useful for the die swell, which draws down whilst surface tension is absent for die swell problem along horizontal case.

Acknowledgment

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Dr. Vimolrat Ngamaramvaranggul is Assistant Professor of Mathematics at Department of Mathematics, Faculty of Science, Chulalongkorn University in Bangkok, Thailand, one of the world’s leading technology institutes. She’s a lecturer of both Mathematics and Computer Sciences. In 2000, she was awarded her PhD in Computer Sciences from the University of Wales Swansea. Her research includes the computing of free surface flows as well as Internet Web Management and Content Management Information. She had ever been a managing director of APAN (Asia-Pacific Advanced Network) Secretariat since 2007-2009. APAN is an Internet networking consortium. Founded in 1997 and it was designed to promote advanced research in networking technologies and the development of high-performance broadband applications.

Mr. Saitharn Thenissara who was born in 1974, is a lecturer at Department of Mathematics, Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan, Nakornratchasima, Thailand. He has finished degree of Master of Science in Computational Science from Chulalongkorn University since 2004. His research area is computational fluid dynamics.