Hysteresis Modulation Based Sliding Mode Control for Positive Output Elementary Super Lift Luo Converter

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Abstract—The Object of this paper is to design and analyze a Hysteresis modulation based sliding mode control (HMSMC) for positive output elementary super lift Luo converter (POESLLC), which is the start-of-the-art DC-DC converter. The positive output elementary super lift Luo converter performs the voltage conversion from positive source voltage to positive load voltage. This paper proposes a HMSMC capable of providing the good steady state and dynamic performance compared to conventional controllers. Dynamic equations describing the positive output elementary super lift Luo converter are derived by using state space average method. The simulation model of the positive output elementary super lift Luo converter with its control circuit is implemented in Matlab/Simulink. The HMSMC for positive output elementary super lift Luo converter is tested for line changes, load changes and also for components variations.

Keywords—DC-DC converter, Positive output elementary super lift Luo converter (POESLLC), Hysteresis modulation based sliding mode control (HMSMC).

I. INTRODUCTION

DC-DC conversion technology has been developing very rapidly, and DC-DC converters have been widely used in industrial applications such as dc motor drives, computer systems and communication equipments. The positive output elementary super lift Luo converter is a new series of DC-DC converters possessing high-voltage transfer gain, high power density; high efficiency, reduced ripple voltage and current [1]. These converters are widely used in computer peripheral equipment, industrial applications and switch mode power supply, especially for high voltage-voltage projects [1]-[2]. Control for them needs to be studied for the future application of these good topologies.

The super-lift technique considerably increases the voltage transfer stage-by-stage gain in geometric progression [3]-[4]. However, their circuits are complex. An approach, positive output elementary super lift Luo converters, that implements the output voltage increasing in geometric progression with a simple structured have been introduced. These converters also effectively enhance the voltage transfer gain in power-law terms [1].

Due to the time variations and switching nature of the power converters, their dynamic behavior becomes highly non-linear. The design of high performance control for them is a challenge for both the control-engineering engineers and power electronics engineers. In general, a good control for DC-DC converters always ensures stability in arbitrary operating condition. Moreover, good response in terms of rejection of load variations, input voltage variations and even parameter uncertainties is also required for a typical control scheme. The steady state and dynamic characteristics of these converters have been well discussed in the literature [5]. With different state-space averaging techniques, a small-signal state-space equation of the converter system could be derived.

The classical linear control methods have been applied on DC-DC converters with these equations. The variable structure control which is also as sliding mode control (SMC) is habitually appropriate to DC-DC converters with any level of complexity. The sliding mode control technique offers several advantages compared to traditional control methods: stability, even for large line and load variations, robustness, good dynamic response and simple implementation. Various studies in the application of SMC for DC-DC converters have been reported in the past several decades [6]-[7]. Different sliding surfaces are proposed in those papers. Many literatures have report the general design issues of sliding mode controller in dc-dc converters like buck, cuk, and buck–boost converters [8]. Intensive research in the area of DC-DC converter has resulted in novel circuit topologies [9]. These converters in general have complex non-linear models with parameter variation. The averaging approach has been one of the most widely adopted modeling strategies for switching converters that yields a simple model [10].

In this paper, state-space model for positive output elementary super lift Luo converter (POESLLC) are derived at first. A Hysteresis modulation based sliding mode controller (HMSMC) with zero steady state error and fast response is brought forward. The steady state and dynamic performance of Hysteresis modulation based sliding mode control (HMSMC) for positive output elementary super lift Luo converter is studied in Matlab/Simulink. The HMSMC are used to operate the positive output elementary super lift Luo converter at variable frequency gives the good steady state and dynamic performances compared to conventional controllers. Details on operation, analysis, control strategy and simulation results for positive output elementary super lift Luo converter (POESLLC) are presented in the subsequent sections.
II. CONVERTER OPERATION AND SMC

For the purpose of optimizing positive output elementary super lift Luo converter dynamics, while ensuring correct operation in any working condition, a sliding mode controller is a more feasible approach.

Sliding mode control has been presented as a good alternative to the control of switching power converters [6]–[8], [11]-[12]. The main advantage over the classical control schemes is its insusceptibility to plant/system parameter variations that leads to invariant dynamics and steady-state response in the ideal case.

A. Circuit Description and Operation

The positive output elementary super lift Luo converter is shown in Fig. 1. It includes dc supply voltage $V_{in}$, capacitors $C_1$ and $C_2$, inductor $L_1$, power switch (n-channel) $S$, freewheeling diodes $D_1$ and $D_2$ and load resistance $R$.

The principle of the sliding mode controller is to make the capacitor voltages $V_{C1}$ and $V_{C2}$ follow as faithfully as possible capacitor voltage references.

In the description of the converter operation, it is assumed that all the components are ideal and also the positive output elementary super lift Luo converter operates in a continuous conduction mode. Fig. 2 and Fig. 3 shows the modes of operation of the converter [1].

In Fig. 2 when the switch $S$ is closed, voltage across capacitor $C_1$ is charged to $V_{in}$. The current $i_{L1}$ flowing through inductor $L_1$ increases with voltage $V_{in}$.

In Fig. 3 when the switch $S$ is closed, decreases with voltage ($V_o - 2 V_{in}$). Therefore, the ripple of the inductor current $i_{L1}$ is

$$\Delta_{i_{L1}} = \frac{V_{in}}{L_1} dT = \frac{V_o - 2V_{in}}{L_1} dT$$

(1)

The voltage transfer gain is

$$G = \frac{V_o - 2 - d}{V_{in}} \frac{1 - d}{1 - d}$$

(3)

The variation ratio of inductor current $i_{L1}$ is

$$\xi = \frac{\Delta i_{L1}}{i_{L1}} = \frac{d(2 - d)TV_{in}}{2L_1 I_{in}} = \frac{d(1 - d)^2 R}{2(2 - d)d L_1}$$

(5)

The ripple voltage of output voltage $V_o$ is

$$\Delta_{V_o} = \frac{\Delta V_o}{C_2} = \frac{I_o (1 - d) T}{C_2} = \frac{(1 - d) V_o}{fC_2 R}$$

(6)

Therefore, the variation ratio of output voltage $V_o$ is

$$\xi = \frac{\Delta V_o}{V_o} = \frac{(1 - d)}{2 R fC_2}$$

(7)
\[ \frac{dL_1}{dt} = - \frac{1}{L_1} \frac{dL_1}{dt} - \frac{1}{R_{in}} + \frac{1}{L_1} \frac{dC_1}{dt} + \frac{1}{L_1} \frac{dC_2}{dt} \]
\[ \frac{dC_1}{dt} = - \frac{1}{C_1} \frac{dC_1}{dt} + \frac{1}{V_{C1}} \frac{dV_{C1}}{dt} \]
\[ \frac{dC_2}{dt} = - \frac{1}{C_2} \frac{dC_2}{dt} + \frac{1}{V_{C2}} \frac{dV_{C2}}{dt} \]

\[ V = Av + By + C \]

Where \( R_{in} \) is internal resistance source which is not shown in the circuit but it is very small value, \( \gamma \) is the status of the switches, \( u \) and \( \dot{u} \) are the vectors of the state variables (\( i_{L1}, V_{C1}, V_{C2} \)) and their derivatives respectively,

\[ \gamma = \begin{cases} 1 \rightarrow S \rightarrow ON \\ 0 \rightarrow S \rightarrow OFF \end{cases} \]

**B. Sliding Mode Controller**

When good transient response of the output voltage is needed, a sliding surface equation in the state space, expressed by a linear combination of state-variable errors \( \varepsilon \) (defined by difference to the references variables), can be given by [11]-[12]

\[ S = (i_{L1}, V_{C1}, V_{C2}) = K_1 \varepsilon_1 + K_2 \varepsilon_2 + K_3 \varepsilon_3 \]

Where coefficients \( K_1, K_2 \) and \( K_3 \) are proper gains, \( \varepsilon_1 \) is the feedback current error, \( \varepsilon_2 \) is the feedback voltage error and \( \varepsilon_3 \) is the feedback voltage error, or

\[ \varepsilon_1 = i_{L1} - i_{L1ref} \]
\[ \varepsilon_2 = V_{C1} - V_{C1ref} \]
\[ \varepsilon_3 = V_{C2} - V_{C2ref} \]

By substituting (11) in (10), one obtains

\[ S = (i_{L1}, V_{C1}, V_{C2}) = K_1(i_{L1} - i_{L1ref}) + K_2(V_{C1} - V_{C1ref}) + K_3(V_{C2} - V_{C2ref}) \]

The signal \( S (i_{L1}, V_{C1}, V_{C2}) \) obtained by the simulation implementation of (10) and applied to a simple circuit (hysteresis comparator), can generate the pulses to supply the power semiconductor drives. The corresponding control scheme is shown in Fig. 4. Status of the switch \( \gamma \) is controlled by hysteresis block \( H \), which maintains the variables \( S (i_{L1}, V_{C1}, V_{C2}) \) near zero.

In theory, the sliding mode control requires sensing of all state variables and generation of suitable references for each of them. However, the inductor current reference is difficult to evaluate since that generally depends on load power demand supply voltage, and load voltage. To overcome this problem in implementation, the state variable error for the inductor current \( (i_{L1} - i_{L1ref}) \) can be obtained from feedback variable \( i_{L1} \) by means of a high-pass filter in the assumption that their low-frequency component is automatically adapted to actual converter operation. Thus, only the high-frequency component of this variable is needed for the control. This high pass filter increases the system order and can heavily alter the converter dynamics. In order to avoid this problem, the cutoff frequency of the high-pass filter must be suitably lower than the switching frequency to pass the ripple at the switching frequency, but high enough to allow a fast converter response.

**III. DESIGN METHODOLOGY**

In the design of the converter, the following are assumed:

- ideal power switches.
- power supply free of dc ripple.
- converter operating at high-switching frequency.

**A. Selection of Control Parameters**

Once the positive output elementary super lift Luo converter parameters are selected, inductance \( L \) and are designed from specified input and output current ripples, capacitors \( C_1 \) and \( C_2 \) are designed so as to limit the output voltage ripple in the case of fast and large load variations, and maximum switching frequency is selected from the proposed converter ratings and switch type. The system behavior is completely determined by coefficients \( K_1, K_2 \) and \( K_3 \) which must be selected so as to satisfy existence and ensure stability and fast response, even for large supply and load variations.

According to the variable structure system theory, the converter equations must be written in the following form [11]-[12]:

\[ \dot{x} = Ax + Bu + D \]

Where \( x \) represents the vector of state-variables errors, given by

\[ x = \nu - V^* \]

Where \( V^* = [i_{L1ref}, V_{C1ref}, V_{C2ref}]^T \) is the vector of references. By substituting (14) in (8), one obtains

\[ D = AV^* + C \]
Substituting (14) in (12), the sliding function can be rewritten in the form

\[ S(x) = K_1x_1 + K_2x_2 + K_3x_3 = K^T x \]  

where \( K^T = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \) and \( x = \begin{bmatrix} x_1, x_2, x_3 \end{bmatrix}^T \).

The existence condition of the sliding mode requires that all state trajectories near the surface be directed toward the sliding plane. The controller can enforce the system state to remain near the sliding plane by proper operation of the converter switch.

To make the system state move toward the switching surface, it is necessary and sufficient that \( S(x) > 0 \) if \( S(x) > 0 \) (18)

Sliding mode control is obtained by means of the following feedback control strategy, which relates to the status of the switch with the value of \( S(x) \):

\[ \gamma = \begin{cases} 0, & \text{for } S(x) > 0 \\ 1, & \text{for } S(x) < 0 \end{cases} \]  

The existence condition (18) can be expressed in the form

\[ \dot{S}(x) = K^T A x + K^T D < 0, S(x) > 0 \]  

(20)

\[ S(x) = K^T A x + K^T B + K^T D > 0, S(x) < 0. \]  

(21)

From a simulation point of view, assuming that error variables \( x \) are suitably smaller than references \( V^* \), (20) and (21) can be rewritten in the form

\[ K^T D < 0, S(x) > 0 \]  

(22)

\[ K^T B + K^T D > 0, S(x) < 0. \]  

(23)

By substituting matrices B and D in (22) and (23), one obtains

\[ D = \begin{bmatrix} \frac{1}{R_m L_1} & -\frac{1}{L_1} & -\frac{1}{L_1} \\ \frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L1\text{ref}} \\ V_{C1\text{ref}} \\ V_{C2\text{ref}} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} i_{L1\text{ref}} \\ V_{C1\text{ref}} \\ V_{C2\text{ref}} \end{bmatrix} \]

(16a)

\[ D = \begin{bmatrix} \frac{1}{R_m L_1} & -\frac{1}{L_1} & -\frac{1}{L_1} \\ \frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L1\text{ref}} \\ V_{C1\text{ref}} \\ V_{C2\text{ref}} \end{bmatrix} \]

(16b)

The existence condition is satisfied if the inequalities (24) and (25) are true.

Finally, it is necessary to guarantee that the designed sliding plane is reached for all initial states. If the sliding mode exists, in the system defined by (13), it is a sufficient conditions that coefficients \( K_1, K_2 \) and \( K_3 \) be nonnegative.

**B. Switching Frequency**

In the ideal sliding mode at infinite switching frequency, state trajectories are directed toward the sliding surface and move exactly along it. A practical system cannot switch at infinite frequency. Therefore, a typical control circuit features a practical relay, as indicated in Fig. 5.

A practical relay always exhibits hysteresis modeled by

\[ \gamma(s) = \begin{cases} 0, & \text{when } S > +\delta \text{ or } \dot{S} < 0 \\ \dot{S} < 0 \text{ and } |S| < \delta \end{cases} \]  

(26)

\[ \text{when } S < -\delta \text{ or } \dot{S} > 0 \]

Where \( \delta \) is an arbitrarily small positive quantity and \( 2\delta \) is the amount of hysteresis in \( S(x) \). The hysteresis characteristic makes it impossible to switch the control on the surface \( S(x) = 0 \). As a consequence, switching occurs on the lines \( S = \pm \delta \), with a frequency depending on the slopes of \( i_{L1} \). This hysteresis causes phase plane trajectory oscillations of width \( 2\delta \), around the surface \( S(x) = 0 \) as shown in Fig. 6.
Note that Fig. 6 simply confirms that in \( \Delta t_1 \), the function \( S(x) \) must increase from \((-\delta \rightarrow \delta) \) ( \( S > 0 \) ), while in \( \Delta t_2 \), it must decrease from \(+\delta \rightarrow -\delta \) ( \( S < 0 \) ). The switching frequency equation is obtained from Fig. 6 by considering that the state trajectory is invariable, near to the sliding surface \( S(x) = 0 \) and is given by
\[
\frac{1}{f_s} = \frac{1}{\Delta t_1 + \Delta t_2} \tag{26}
\]
Where, \( \Delta t_1 \) is conduction time of the switch \( S \) and \( \Delta t_2 \) is the off time of the switch \( S \). The conduction time \( \Delta t_1 \) is derived from (25) and it is given by
\[
\Delta t_1 = \frac{K_1}{R_n L_1}\left[ I_{L1} + R_n V_{in} \right] + \frac{K_2}{C_n R_n} \left[ V_{C1} - V_{in} \right] - \frac{K_1 V_{C2r} - R L_1}{R C_2} \tag{27}
\]
The off time \( \Delta t_2 \) is derived from (24), and it is given by
\[
\Delta t_2 = \frac{K_1}{R_n L_1}\left[ I_{L1} - R_n V_{C1r} - R_n V_{in} + V_{in} \right] + \frac{K_1 V_{C1r} - R L_1}{R C_2} \tag{28}
\]
The maximum value of switching frequency is obtained substituting (27) and (28) in (26) with the assumption that \( \delta \rightarrow 0.5 \). The adopted value of the output voltage is 36 V which is in Table I, and a variation of the duty cycle between \( \delta_{min} = 0.3 \) and \( \delta_{max} = 0.56 \) is expected. Finally \( V_{C2max} = 36.5V \).

A. Calculation of \( V_{C2} \)

From (31) and a simulation point of view, the output voltage is chosen to produce a variation of the duty cycle close to 0.5. The adopted value of the output voltage is 36 V which is in Table I, and a variation of the duty cycle between \( \delta_{min} = 0.3 \) and \( \delta_{max} = 0.56 \) is expected. Finally \( V_{C2max} = 36.5V \).

B. Determination of Ratio \( K_1 / L_1 \)

Substituting \( V = V_{C2r} = V_{C1r} = V_{in} \) and \( \delta = 0.3 \) in (29) one obtains
\[
K_1 / L_1 = 6666.67.
\]

C. Determination of Ratio \( K_3 / C_1 \) and \( K_3 / C_2 \)

From (24) and (25) and taking \( i_{L1r} = I_{L1r} = 2.35A \), one obtains 1208 < \( K_2 / C_1 < 248433 \) and 1208 < \( K_3 / C_2 < 248433 \).

There are some degrees of freedom in choosing the ratio \( K_2 / C_1 \) and \( K_3 / C_2 \). In this controller, the ratio \( K_2 / C_1 \) and \( K_3 / C_2 \) is a tuning parameter. It is recommendable to choose the ratio \( K_2 / C_1 \) and \( K_3 / C_2 \) to begin with some levels of stability and response speed. The ratio \( K_2 / C_1 \) and \( K_3 / C_2 \) is chosen by iterative procedure (i.e the ratio is modified until the transient response is satisfactory), and it is verified by simulation. The final adopted value is, \( K_2 / C_1 \) and \( K_3 / C_2 = 7248 \).

D. Calculation of \( L_1 \)

The maximum inductor current ripple is chosen to be equal to 15 % maximum inductor current, and \( L_1 = 100 \mu H \) which is obtained from (30).

E. Calculation of \( C_1 \) and \( C_2 \)

The controller operates over the switch to make the voltage \( V_{C2} \) follow a low-frequency reference. Over \( V_{C2} (t) \), a high-frequency ripple (switching) is imposed, which is given by
\[
\Delta V_{C2}(t) = \frac{V_{C2r}(t)}{RC_2} \tag{33}
\]
The maximum capacitor ripple voltage $\Delta V_{C1\text{max}}$ and $\Delta V_{C2\text{max}}$ is chosen to be equal to 0.5% maximum capacitors voltage, and $C_1 = C_2 = 30\ \mu F$ which is obtained from (33).

\textbf{F. Values of the coefficients $K_1$, $K_2$ and $K_3$}

Having decided on the values of the ratio $K_1/L_1$ and inductor, the value of $K_1$ is unswervingly obtained ($K_1 = 0.667$). Similarly the $K_2 = K_3 = 0.217$ is computed using the ratio $K_2/C_1$ and $K_3/C_2$ and the $C_1$, $C_2$.

\begin{table}[h]
\centering
\caption{Parameters of Positive Output Super Lift Luo Converter}
\begin{tabular}{|l|l|l|}
\hline
Parameters name & Symbol & Value \\
\hline
Input Voltage & $V_{in}$ & 12V \\
Output Voltage & $V_o$ & 36V \\
Inductor & $L_1$ & 100$\mu$H \\
Capacitors & $C_1$, $C_2$ & 30 $\mu$F \\
Nominal switching frequency & $F_s$ & 100kHz \\
Load resistance & $R$ & 50$\Omega$ \\
Output power & $P_o$ & 25.92W \\
Input power & $P_{in}$ & 28.236W \\
Input current & $I_{in}$ & 2.353 A \\
Efficiency & $\eta$ & 91.79\% \\
\hline
\end{tabular}
\end{table}

The steady state and dynamic performance of HMSMC for the positive output elementary super lift Luo converter is evaluated in Matlab/Simulink. The Matlab/Simulink simulation model is depicted in Fig. 7. The signal $S(i_{L1}, V_{C1}, V_{C2})$, obtained by the simulation model, which guided by (12) and applied to the hysteresis modulator to generate the gating of switch. Status of the switch, $\gamma$ is controlled by hysteresis block $H$, and thus maintains the variable error of $i_{L1}$ and $V_{C1}$, $V_{C2}$ near zero. The system response is determined by the circuit parameters and coefficients $K_1$, $K_2$ and $K_3$. With a proper selection of these coefficients in any operating condition, high control robustness, stability, and fast response can be achieved.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{simulation_model.png}
\caption{Simulation model of HMSMC with positive output elementary super lift Luo converter.}
\end{figure}

In Fig. 8 shows the variation of average output voltage of SMC with positive output elementary super lift Luo converter for the input step change (12 V to 15 V). It can be found that converter output voltage has a maximum overshoot of 4.9 V and 0.01 s settling time with designed SMC. In Fig. 9 shows the output voltage variation for another input step change (12 V to 9 V). It can be seen that converter output voltage has a maximum overshoot of 4.7 V and 0.01 s settling time.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{output_voltages.png}
\caption{Output voltage when input takes a step change from 12V to 15V.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{output_voltages_2.png}
\caption{Output voltage when input takes a step change from 12V to 9V.}
\end{figure}

Fig. 10 shows the variation of output voltage with the step change in load from 50 $\Omega$ to 60 $\Omega$. It could be seen that there is a small overshoot of 1 volt and steady state is reached with a very less time 0.016 s. In Fig. 11 shows the another variation of output voltage with step change in load from 50 $\Omega$ to 40 $\Omega$. It could be seen that there is a small overshoot of 1 volt and steady state is reached with a very small time 0.016 s.
Fig. 12 shows the instantaneous output voltage and current of the inductor current in the steady state. It is evident from the figure that the output voltage ripple is very small about 0.2V and the peak to peak inductor current is 0.72A while the switching frequency is 100 kHz.

An interesting result has been illustrated in Fig. 13, which shows response for the variation in capacitor values 30 µF to 100 µF. The HMSMC is very successful in suppressing effect of capacitance variation effect that a minute output ripple voltage. The capacitor change has no severe effect on the value of inductor current. In Fig. 14 shows the output voltage for inductor variation from 100 µH to 500 µH and the change has no severe effect on the converter behavior due to the efficient HMSMC.

Fig. 12 shows the instantaneous output voltage and current of the inductor current in the steady state condition. It is evident from the figure that the output voltage ripple is very small about 0.2V and the peak to peak inductor current is 0.72A while the switching frequency is 100 kHz.
Using simulation analysis computes that the input and output power values are 28.236W and 25.92W respectively, which is closer to the calculated theoretical value listed in Table I.

V. CONCLUSIONS

The positive output elementary super lift Luo converter (POESLLCC) performs the voltage conversion from positive source voltage to positive load voltage. Due to the time variations and switching nature of the power converters, their dynamic behavior becomes highly non-linear. This paper has successfully demonstrated the design, analysis, and suitability of Hysteresis modulation based sliding mode controlled positive output elementary super lift Luo converter. The simulation based performance analysis of a Hysteresis modulation based sliding mode controlled positive output elementary super lift Luo converter circuit has been presented along with its state space averaged model. The Hysteresis modulation based SMC scheme has proved to be robust and its triumph has been validated with load and line regulations and also with circuit components variations. The positive output elementary super lift Luo converter with HMSMC thus claims its use in applications such as computer peripheral equipment, switch mode power supply and industrial applications, especially for high voltage projects etc.

REFERENCES