World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:5, No:10, 2011

Mechanical Buckling of Functionally Graded Engesser-Timoshenko Beams Located on a Continuous Elastic Foundation

M. Karami Khorramabadi, A. R. Nezamabadi

Open Science Index, Mechanical and Mechatronics Engineering Vol:5, No:10, 2011 publications.waset.org/13934.pdf

Abstract**—**This paper studies mechanical buckling of functionally graded beams subjected to axial compressive load that is simply supported at both ends lies on a continuous elastic foundation. The displacement field of beam is assumed based on Engesser-Timoshenko beam theory. Applying the Hamilton's principle, the equilibrium equation is established. The influences of dimensionless geometrical parameter, functionally graded index and foundation coefficient on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

Keywords**—**Mechanical Buckling, Functionally graded beam-Engesser-Timoshenko beam theory

I. Introduction

THE concept of functionally graded materials (FGMs) was first suggested by a group of Japanese scientists in 1984 to address the needs of aggressive environment of thermal shock [1]. Nowadays, FGMs have been widely explored in various engineering applications including electronics, chemistry, optics, biomedicine and the like [2]. More recently, Ichinose et al. [3] succeeded in fabricating ultrasonic transducers with functionally graded piezoelectric ceramics. On the macroscopic scale, FGMs are anisotropic, inhomogeneous and possess spatially continuous mechanical properties. Because discernible internal seams or boundaries do not exist in FGM, no internal stress peaks are caused when external load is applied and thus failure from interfacial debonding or from stress concentration can be avoided. In this respect, FGMs are more superior to the conventional laminated materials [4–6]. Piezoelectric materials have coupled effects between electric field and elastic deformation and have been widely integrated with structures to control deformation, vibration, acoustics, etc. These new structures including FGM members bonded with piezoelectric actuators and sensors are smart in response to environmental changes [7–14]. Ootao and Tanigawa [15] investigated the three-dimensional transient piezothermoelastic problem of an FGM rectangular plate bonded to a piezoelectric plate due to partial heat supply. They modeled the FGM plate as a laminate and adopted a solution methodology similar to Pagano [16]. Through numerical examination, they showed that the maximum transient states of transverse normal stress and transverse shear stress in the plate can be reduced by functional grading.

M. Karami Khorramabadi is with the Department of Mechanical Engineering, Khorramabad Branch, Islamic Azad University, Khorramabad, Iran. (e-mail: mehdi_karami2001@yahoo.com)

A. R. Nezamabadi, is with the Department of Mechanical Engineering, Arak Branch, Islamic Azad University, Arak, Iran (e-mail: alireza.nezamabadi@gmail.com)

A comprehensive study was conducted for the shape and vibration control of FGM plates and shells with integrated piezoelectric sensors and actuators by Liew and his associates using finite element method [17–18].To the author's knowledge, there is no analytical solution available in the open literatures for mechanical buckling of functionally graded Engesser-Timoshenko beams located on a continuous elastic foundation. In the present work, the mechanical buckling of a functionally graded Engesser-Timoshenko beam subjected to axial compressive loads lies on a continuous elastic foundation is studied. Appling the Hamilton's principle, the equilibrium equations of beam are derived and solved. The effects of the foundation coefficient, dimensionless geometrical parameter and functionally graded index on the critical buckling load of beam are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

II. Formulation

The formulation that is presented here is based on the assumptions of Engesser-Timoshenko beam theory. Based on this theory, the displacement field can be written as [20]:

$$\begin{aligned} u(x,z) &= z\phi(x) \\ w(x,z) &= w_0(x,z) \end{aligned} \tag{1}$$

In view of the displacement field given in Eqs (1), the strain displacement relations are given by [20]:

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} = z\frac{d\phi}{dx} \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi + \frac{dw}{dx} \end{aligned} \tag{2}$$

Consider a functionally graded beam with rectangular cross-section. The thickness, length, and width of the beam are denoted, respectively, by $h, L,$ and $b.$ The $x-y$ plane coincides with the midplane of the beam and the $z-$axis located along the thickness direction. The Young's modulus E is assumed to vary as a power form of the thickness coordinate variable z $(-h/2 \le z \le h/2)$ as follow [19]:

$$E(z) = (E_c - E_m)V + E_m, \qquad V = \left(\frac{2z+h}{2h}\right)^k \tag{3}$$

Open Science Index, Mechanical and Mechatronics Engineering Vol:5, No:10, 2011 publications.waset.org/13934.pdf

where k is the power law index and the subscripts m and c refer to the metal and ceramic constituents, respectively. The constitutive relations for functionally graded Engesser-Timoshenko beam are given by [21]:

$$\begin{aligned}\sigma_{xx} &= Q_{11}(z)\varepsilon_{xx} \\ \sigma_{xz} &= Q_{55}(z)\gamma_{xz}\end{aligned} \quad (4)$$

where σ_{xx}, σ_{xz}, Q_{11} and Q_{55} are the normal ,shear stresses and plane stress-reduced stiffnesses respectively. Also, u and w are the displacement components in the $x-$ and $z-$ directions, respectively.

The potential energy can be expressed as [20]:

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\,\mathrm{d}v \quad (6)$$

Substituting Eqs. (2)-(4) into Eq. (6) and neglecting the higher-order terms lead to

$$\begin{aligned}U = \frac{1}{2}\int_v [(Q_{11}\left(z\frac{d\phi}{dx}\right))(z\frac{d\phi}{dx}) \\ + (Q_{55}(\phi + \frac{dw}{dx}))(\phi + \frac{dw}{dx})]dv\end{aligned} \quad (7)$$

The width of beam is assumed to be constant, which is obtained by integrating along y over v. Then Eq. (7) becomes

$$\begin{aligned}U = \frac{b}{2}\int_0^L [D\left(\frac{d\phi}{dx}\right)^2 + \frac{A}{2(1+\nu)}(\phi^2 + (\frac{dw}{dx})^2 \\ + 2\phi\frac{dw}{dx})]dx\end{aligned} \quad (8)$$

where

$$A = \int_{-\frac{h}{2}}^{+\frac{h}{2}} Q_{55}(\mathrm{z})\mathrm{dz}$$

$$D = \int_{-\frac{h}{2}}^{+\frac{h}{2}} z^2 Q_{11}(\mathrm{z})\mathrm{dz} \quad (9)$$

where A and D are the shear rigidity and flexural rigidity respectively. Note that, the extensional displacement is neglected. Thus, the potential energy can be written as

$$U = \frac{b}{2}\int_0^L [D\ (\frac{d\phi}{dx})^2 + A(\phi^2 + (\frac{dw}{dx})^2 + 2\phi\frac{dw}{dx}]dx \quad (10)$$

The beam is subjected to the axial compressive loads, P as shown in Fig. 1.

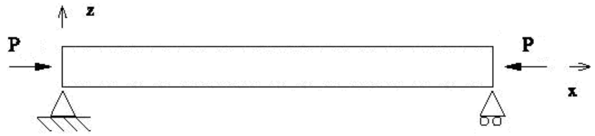

Fig. 1 Simply supported beam under compressive loads.

The work done by the axial compressive load can be expressed as [20]:

$$W = \frac{1}{2}\int_0^L P\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \quad (11)$$

We apply the Hamilton's principle to derive the equilibrium equations of beam, that is [21]:

$$\delta\int_0^t (T - U + W)\,\mathrm{d}t = 0 \quad (12)$$

Substitution from Eqs. (10) and (11) into Eq. (12) leads to the following equilibrium equations of the the functionally graded beam based on first order shear deformation theory. Assume that a functionally graded beam that is simply supported at both ends lies on a continuous elastic foundation, whose reaction at every point is proportional to the deflection (Winkler foundation). The equilibrium equation of the functionally graded beam based on first order shear deformation theory located on a continuous elastic foundation subjected to a axial compressive load is obtained from equilibrium equations by the addition of ηw for the foundation reaction as

$$\begin{aligned}(P - bA)\frac{d^2 w}{dx^2} + bA(\frac{d\phi}{dx}) = 0 \\ A(\phi + \frac{dw}{dx}) + 2D(\frac{d^2\phi}{dx^2}) = 0\end{aligned} \quad (13)$$

where η is the foundation coefficient.

III. Stability Analysis

The boundary conditions for the pin-ended Engesser-Timoshenko beam are given by:

$$w = \frac{d^2 w}{dx^2} = \frac{d\phi}{dx}, \qquad at \quad x = 0 \quad and \quad x = L \quad (14)$$

Substituting Eq. (14) into (13) and by equating power-law index to zero and neglecting the foundation coefficient, the critical buckling load of a functionally graded Engesser-Timoshenko beam will be derived, that is:

$$p_{cr} = \frac{(\frac{\pi}{L})^2 \frac{bh^3 Q_{11}}{12}}{1 + (\frac{L}{\pi})^2 \frac{12 Q_{55}}{bh^2 Q_{11}}} \tag{15}$$

The above equation has been reported by Wang and Reddy [20].

IV. Numerical Results

The mechanical buckling behaviors of simply supported functionally graded Engesser-Timoshenko beams lies on a continuous elastic foundation are studied in this paper. The material properties of the beam are listed in Table 1.

TABLE I
MATERIAL PROPERTIES

Property	FGM layer	
	Stainless steel	Nickel
Young's modulus E (GPa)	221.04	223.95
Poisson's ratio ν	0.3	0.3
Length L (m)	0.3	0.3
Thickness h (m)	0.01	0.01
Density ρ (Kgm^{-3})	8166	8900

The Poisson's ratio is chosen to be 0.3 for both materials. The variation of the critical buckling loads for functionally graded Engesser-Timoshenko evaluated considering of $b/h=1$, $L=1$ and several values of foundation coefficient are shown in Table 2. It is seen that the critical buckling loads for FG Engesser-Timoshenko beam increased with an increase of the foundation coefficient η. Fig. 2. demonstrates the critical buckling loads for functionally graded Engesser-Timoshenko beam. It is seen that the critical buckling loads for Engesser-Timoshenko beam increased with an increase of the ratio h/L and decreased with an increase of power-law index of constituent volume fraction.

TABLE II
VARIATION OF THE CRITICAL BUCKLING LOAD OF FG BEAM WITH PIEZOELECTRIC ACTUATORS VERSUS η

Foundation Coefficient (η)	Critical Buckling Load (P_{cr})
1000	43000N
2000	48620N
3000	50872N
4000	55468N

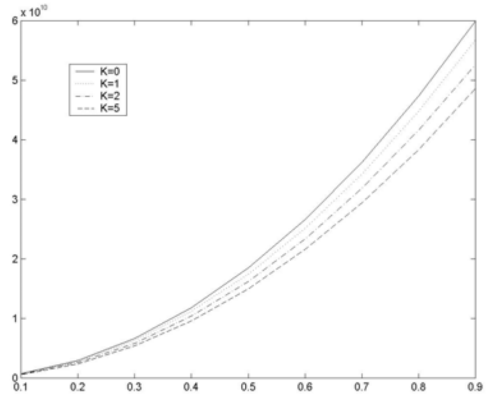

Fig. 2 Critical Buckling Load of FG Beam Versus h/L

V. Conclusion

The mechanical buckling of a functionally graded Engesser-Timoshenko beam located on a continuous elastic foundation subjected to axial compressive loads is studied. It is conclude that:

1- The critical buckling loads of FG Engesser-Timoshenko beam are generally lower than corresponding values for the homogeneous Engesser-Timoshenko beam.
2- The critical buckling loads of FG Engesser-Timoshenko beam under axial compressive load generally increases with the increase of relative thickness h/L.
3- The critical buckling loads of FG Engesser-Timoshenko beam under axial compressive load generally increase with the increase of foundation coefficient η.
4- The accuracy of Engesser-Timoshenko beam theory is more than Bernoulli-Euler beam theory.

References

[1] O. Rabin BH, Shiota I. Functionally gradient materials. Mater Res Soc Bull 1995;20:14–8.
[2] Koizumi M. FGM activities in Japan. Compos Part B: Eng 1997;28:1–4.
[3] Ichinose N, Miyamoto N, Takahashi S. Ultrasonic transducers with functionally graded piezoelectric ceramics. J Eur Ceram Soc 2004;24:1681–5
[4] Wu CCM, Kahn M, Moy W. Piezoelectric ceramics with functional gradients: a new application in material design. J Am Ceram Soc 1996;79:809–12.
[5] Li CY, Weng GJ. Antiplane crack problem in functionally graded piezoelectric materials. J Appl Mech 2002;69:481–8.
[6] Chen WQ, Ye GR, Cai JB. Thermoelastic stresses in a uniformly heated functionally graded isotropic hollow cylinder. J Zhejiang Univ (Science) 2002;3:1–5.
[7] Cheng ZQ, Lim CW, Kitipornchai S. Three-dimensional asymptotic approach to inhomogeneous and laminated piezoelectric plates. Int J Solids Struct 2000;37:3153–75.
[8] Lim CW, He LH. Exact solution of a compositionally graded piezoelectric layer under uniform stretch, bending and twisting.Int J Mech Sci 2001;43:2479–92.
[9] Lim CW, He LH, Soh AK. Three dimensional electromechanical responses of a parallel piezoelectric bimorph. Int J Solids Struct 2001;38:2833–49.
[10] Chen WQ, Ding HJ, Hou PF. Exact solution of an external circular crack in a piezoelectric solid subjected to shear loading. J Zhejiang Univ (Science) 2001;2:9–14.

Open Science Index, Mechanical and Mechatronics Engineering Vol:5, No:10, 2011 publications.waset.org/13934.pdf

[11] Hou PF, Ding HJ, Guan FL. A penny-shaped crack in an infinite piezoelectric body under antisymmetric point loads. J Zhejiang Univ (Science) 2001;2:146–51.
[12] Zhang W, Hong T. Adaptive Lagrange finite element methods for high precision vibrations and piezoelectric acoustic wave computations in SMT structures and plates with nano interfaces. J Zhejiang Univ (Science) 2002;3:6–12.
[13] Wang Y, Xu RQ, Ding HJ. Free vibration of piezoelectric annular plate. J Zhejiang Univ (Science) 2003;4:379–87.
[14] Lim CW, Lau CWH. A new two-dimensional model for electromechanical response of thick laminated piezoelectric actuator. Int J Solids Struct, in press.
[15] Ootao Y, Tanigawa Y. Three-dimensional transient piezothermoelasticity in functionally graded rectangular plate bonded to a piezoelectric plate. Int J Solids Struct 2000;37:4377–401.
[16] Pagano NJ. Exact solutions for composite laminates in cylindrical bending. J Compos Mater 1969;3:398–411.
[17] He XQ, Ng TY, Sivashanker S, Liew KM. Active control of FGM plates with integrated piezoelectric sensors and actuators. Int J Solids Struct 2001;38:1641–55.
[18] Liew KM, He XQ, Ng TY, Sivashanker S. Active control of FGM plates subjected to a temperature gradient: modelling via finite element method based on FSDT. Int J Numer Meth Eng 2001;52:1253–71.
[19] Reddy J.N. and Praveen G.N., Nonlinear Transient Thermoelastic Analysis of Functionally Graded Ceramic-metal Plates, International Journal of. Solids and Structures, Vol. 35, 1998, pp. 4467-4476
[20] Wang C.M., Reddy J.N., 2000, "Shear Deformable Beams and Plates", Oxford, Elsevier.
[21] Reddy J.N., 2004, " Mechanics of Laminated Composite Plates and Shells Theory and Analysis", New York, CRC.

Open Science Index, Mechanical and Mechatronics Engineering Vol:5, No:10, 2011 publications.waset.org/13934.pdf