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Finite Element Solution of Navier-Stokes Equations for Steam Flow and Heat Transfer

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Abstract**—**Computational simulation of steam flow and heat transfer in power plant condensers on the basis of the three-dimensional mathematical model for the flow through porous media is presented. In order to solve the mathematical model of steam flow and heat transfer in power plant condensers, the Streamline Upwind Petrov-Galerkin finite element method is applied. By comparison of the results of simulation with experimental results about an experimental condenser, it is confirmed that SUPG finite element method can be successfully applied for solving the three-dimensional mathematical model of steam flow and heat transfer in power plant condensers.

Keywords**—**Navier-Stokes, FEM, condensers, steam.

I. INTRODUCTION

THERE are numerous papers and reports concerning numerical simulation of steam flow and heat transfer in power plant condensers. In most of the papers the simulations are mainly based on the mathematical model which defines steam flow in the tube bundle as flow through porous media.

The common characteristic of the most of the current works on this subject is solving the mathematical model on the basis of finite volume method with use a rectangular grid for discretization of the condenser area.

Although this methods so far have shown good results in solving the mathematical model of steam flow and heat transfer in power plant condensers, the difficulties still remain in applying the rectangular grid for discretization of the condensers flow area, especially in modern condensers with complex irregular geometry.

In this paper in order to solve the mathematical model of steam flow and heat transfer in power plant condensers, the finite element method is applied.

II. MATHEMATICAL MODEL

The steam flow and heat transfer are modeled, under assumption that condensate has negligible momentum and occupies negligible volume, by three-dimensional mathematical model for flow of steam-air mixture in the porous media [1].

Mass conservation equation for the mixture:

$$\frac{\partial(\varepsilon\rho u)}{\partial x}+\frac{\partial(\varepsilon\rho v)}{\partial y}+\frac{\partial(\varepsilon\rho w)}{\partial z}=-\dot{m} \tag{1}$$

Momentum conservation equations for the mixture:

$$\frac{\partial(\varepsilon\rho u^2)}{\partial x}+\frac{\partial(\varepsilon\rho uv)}{\partial y}+\frac{\partial(\varepsilon\rho uw)}{\partial z}=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial x}\right)+ +\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial x}-\dot{m}u-F_x \tag{2}$$

$$\frac{\partial(\varepsilon\rho uv)}{\partial x}+\frac{\partial(\varepsilon\rho v^2)}{\partial y}+\frac{\partial(\varepsilon\rho vw)}{\partial z}=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial x}\right)+ +\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial y}-\dot{m}v-F_y \tag{3}$$

$$\frac{\partial(\varepsilon\rho uw)}{\partial x}+\frac{\partial(\varepsilon\rho vw)}{\partial y}+\frac{\partial(\varepsilon\rho w^2)}{\partial z}=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial x}\right)+ +\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial z}-\dot{m}w-F_z \tag{4}$$

Equation for conservation of air mass fraction:

$$\frac{\partial(\varepsilon\rho uc_g)}{\partial x}+\frac{\partial(\varepsilon\rho vc_g)}{\partial y}+\frac{\partial(\varepsilon\rho wc_g)}{\partial z}= \frac{\partial}{\partial x}\left(\varepsilon\rho D\frac{\partial c_g}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon\rho D\frac{\partial c_g}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\rho D\frac{\partial c_g}{\partial z}\right) \tag{5}$$

The dependent variables are mixture velocity component u, v and w, mixture pressure p, and air mass fraction c_g. The steam is assumed saturated.

The constitutive equations related to other parameters, which are present in the mathematical model, are defined by the following relations:

Local volume porosity ε:

- for staggered distribution of the tubes in the bundle

$$\varepsilon=1-\frac{\pi}{2\sqrt{3}}\left(\frac{d_n}{s}\right)^2 \tag{6}$$

- for in-line distribution of the tubes in the bundle

$$\varepsilon=1-\frac{\pi}{4}\left(\frac{d_n}{s}\right)^2 \tag{7}$$

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Mixture density is determined on the basis of the equation of state, where steam and air are treated as perfect:

$$\rho = \frac{p}{RT} \quad (8)$$

Effective viscosity μ_{eff} is defined as sum of molecular and turbulent viscosity:

$$\mu_{eff} = \mu + \mu_t \quad (9)$$

Molecular viscosity μ is defined as [2]:

$$\mu = \frac{\left(1 - E_g\right)\mu_p + 1.61 E_g \mu_g}{1 + 0.61 E_g} \quad (10)$$

Turbulent viscosity μ_t is defined as:

$$\mu_t = 20 \cdot \mu \quad (11)$$

according to the recommendation of Zhang et al. [3].

The flow resistance forces are determined by the linear Darcy law, for the flow through porous media.

$$F_x = \mu R_x u \quad (12)$$

$$F_y = \mu R_y v \quad (13)$$

$$F_z = 0 \quad (14)$$

The flow resistances R_x and R_y are determined by the adequate empirical relations about local flow resistance of a two-phase flow across the tube bundle [4].

$$R_x = d_n^{-2} G \cdot f \cdot \Theta_x \quad (15)$$

$$R_y = d_n^{-2} G \cdot f \cdot \Theta_y \quad (16)$$

Here G is the coefficient that takes into account the influence of tube bundle geometry

$$G = -1{,}017 + 0{,}3325/\varepsilon + 0{,}3574/\varepsilon^2 + 0{,}01348/\varepsilon^3 \quad (17)$$

f is the fraction factor

$$f = 350\,\mathrm{Re}^{0{,}0446} \text{ for } \mathrm{Re} < 20 \quad (18)$$

$$f = 103\,\mathrm{Re}^{0{,}338} \quad \text{for } 20 < \mathrm{Re} < 300 \quad (19)$$

$$f = 6{,}64\,\mathrm{Re}^{0{,}880} \quad \text{for } 300 < \mathrm{Re} \quad (20)$$

Θ_x and Θ_y are correction factors, which take into account the influence of the rate of condensation, i.e., two-phase flow, on the flow resistances. These factors are functions of the condensation rate m, Re-number and the direction of the flow.

The rate of steam condensation per unit volume is determined by the equation:

$$\dot{m} = a \frac{k\left(T - T_v\right)}{r} \quad (21)$$

where a is the heat transfer area per unit fluid volume.

$$a = \frac{4}{d_n} \cdot \left(1 - \varepsilon\right) \quad (22)$$

The overall heat transfer coefficient is determined as the sum of individual heat transfer coefficients:

$$k = \left(\frac{1}{\alpha_v} \frac{d_n}{d_v} + \frac{1}{\alpha_c} + \frac{1}{\alpha_k} + \frac{1}{\alpha_g} \right)^{-1} \quad (23)$$

α_v is determined by the well-known McAdams equation of forced convection in the circular tube:

$$\alpha_v = 0{,}023 \frac{\lambda_v}{d_v} Re_v^{0{,}8} Pr_v^{0{,}4} \quad (24)$$

α_c is equivalent heat transfer coefficient across the tube wall:

$$\alpha_c = \frac{2\lambda_c}{d_n \ln\left(d_n / d_v\right)} \quad (25)$$

α_k is determined on the basis of the equation of Honda et al. [5], for the condensation on the horizontal uninundated tube in the tube bundle, while the influence of the condensate inundation is taken via the relation of Cippolone et al. [6].

$$Nu = \left(Nu_N^4 + Nu_F^4 \right)^{0{,}25} \quad (26)$$

$$Nu_N = 0{,}728 \cdot F^{0{,}25} \left(1 + Z + 0{,}57 \cdot Z^2\right)^{0{,}25} Re_k^{0{,}5} \quad (27)$$

$$Nu_F = 0{,}11 \cdot Re_q^{0{,}8} Pr_k^{0{,}4} \quad (28)$$

$$Nu = Nu_0 \left[n^{7/8} - \left(n - 1\right)^{7/8} \right] \quad (29)$$

α_g is evaluated via Berman and Fuks [7], empirical equation for the mass transfer coefficient in the tube bundle during the downward flow of the steam-air mixture:

$$\alpha_g = a \frac{D_p}{d_n} Re^{1/2} E_g^{-0.6} p^{1/3} \left(\frac{\rho_p r}{T} \right)^{2/3} \frac{r}{\left(T - T_{pk}\right)^{1/3}} \quad (30)$$

The coefficient for the steam diffusion in the air D_p is determined as:

$$D_p = D / R_p T \quad (31)$$

where D is determined on the basis of well known empirical equation of Fuller at al. as:

$$D = 0{,}00011756552 \cdot \frac{T^{1{,}75}}{p} \quad (32)$$

Boundary conditions are specified for the inlet, walls and outlet of condenser:

1. At the inlet of the condenser, the boundary condition is the steam velocity determined on the basis of steam flow at the turbine exhaust and the cross sectional area of the inlet of the condenser.
2. On the walls of condenser, support plates and plates for the direction of steam flow and drainage of condensate, the boundary condition is the normal component of mixture velocity to be equal to zero.
3. At the outlet of the condenser, the boundary condition is the outlet velocity of the steam-air mixture, which is calculated on the basis of the characteristics of venting apparatus.

III. Mathematical Model Solving

The mathematical model of steam flow and heat transfer in power plant condensers is solved in two step procedure. In first step the system of PDE equations (1)-(4) is solved. As the result of this step the values of mixture velocity and pressure are obtained. In the second step, with the obtained results about mixture velocity and pressure, the equation for conservation of air mass fraction (5) is solved. As the result of this step the values of the air mass fraction in the mixture are obtained. The whole procedure is repeated until the satisfactory accuracy is achieved.

In the both steps, for the solving system equation (1)-(4) and equation (5) the Streamline Upwind Petrov-Galerkin (SUPG) finite element method is applied.

In order to not be violated the convergence conditions of SUPG method, the choice of the applied elements for the discretization of condenser area is in agreement with recommended combinations for interpolating functions for velocity and pressure [8]. Because condenser's area is usually highly irregular, hexahedral elements are used for discretization. For approximation of the mixture velocity the second order interpolating functions are applied (N_u=N_v=N_w). For approximation of the pressure and air mass fraction a linear interpolating functions are applied.

A. Discretization of the system of PDE

The system of partial differential equations (1)-(4) with evaluating partial derivatives in convective terms and inserting the continuity equation (1) in the momentum equations (2)-(4) can be transformed in following form:

$$\frac{\partial(\varepsilon\rho u)}{\partial x}+\frac{\partial(\varepsilon\rho v)}{\partial y}+\frac{\partial(\varepsilon\rho w)}{\partial z}=-\dot{m} \tag{33}$$

$$\begin{aligned}\varepsilon\rho u\frac{\partial u}{\partial x}+\varepsilon\rho v\frac{\partial u}{\partial y}+\varepsilon\rho w\frac{\partial u}{\partial z}&=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial x}\right)+\\&+\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial u}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial x}-F_x\end{aligned} \tag{34}$$

$$\begin{aligned}\varepsilon\rho u\frac{\partial v}{\partial x}+\varepsilon\rho v\frac{\partial v}{\partial y}+\varepsilon\rho w\frac{\partial v}{\partial z}&=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial x}\right)+\\&+\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial v}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial y}-F_y\end{aligned} \tag{35}$$

$$\begin{aligned}\varepsilon\rho u\frac{\partial w}{\partial x}+\varepsilon\rho v\frac{\partial w}{\partial y}+\varepsilon\rho w\frac{\partial w}{\partial z}&=\frac{\partial}{\partial x}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial x}\right)+\\&\frac{\partial}{\partial y}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon\mu_{eff}\frac{\partial w}{\partial z}\right)-\varepsilon\frac{\partial p}{\partial z}-F_z\end{aligned} \tag{36}$$

Main problem was how to apply some of the known finite element methods for solving this system of partial differential equations.

In order to overcome this problem it was adopted that steam-air mixture in the area of every single finite element has constant physical parameters (density ρ and effective viscosity μ_{eff}), which can be determined on the basis of mean pressure (temperature) in the element.

With such defined mixture physical parameters, the system of PDE (33)-(36) defined for single finite element can be formulated as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=-\frac{\dot{m}}{\varepsilon} \tag{37}$$

$$\begin{aligned}&\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}-\mu_{eff}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\&+\frac{\partial p}{\partial x}+\frac{F_x}{\varepsilon}=0\end{aligned} \tag{38}$$

$$\begin{aligned}&\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}-\mu_{eff}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\&+\frac{\partial p}{\partial y}+\frac{F_y}{\varepsilon}=0\end{aligned} \tag{39}$$

$$\begin{aligned}&\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}-\mu_{eff}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\\&+\frac{\partial p}{\partial z}+\frac{F_z}{\varepsilon}=0\end{aligned} \tag{40}$$

This system of PDE is very similar with the Navier-Stokes equations for incompressible flow. The difference is only in existence additional term about condensation rate (m) in continuity equation.

Therefore, on such defined system of PDE the Petrov-Galerkin procedure for solving Navier-Stokes equations for incompressible flow can be applied [8], [9].

With application of this procedure, the system PDE (37)-(40) in the area of single element can be discretized in the following form

$$\begin{bmatrix}\mathbf{K}+\widetilde{\mathbf{K}} & \mathbf{Q}\\ \mathbf{Q}^{\mathrm{T}} & 0\end{bmatrix}\begin{Bmatrix}\hat{\mathbf{u}}\\ \hat{\mathbf{p}}\end{Bmatrix}=\begin{Bmatrix}\mathbf{f}\\ \mathbf{m}\end{Bmatrix} \tag{41}$$

where:

$$\mathbf{K}=-\mu_{eff}\int_V \mathbf{B}^{\mathrm{T}}\mathbf{B}\,dV \tag{42}$$

$$\widetilde{\mathbf{K}}=\rho\int_V \mathbf{W}_u^{\mathrm{T}}(\mathbf{B}_c\hat{\mathbf{u}})\mathbf{N}_u\,dV \tag{43}$$

$$\mathbf{Q}=\int_V \mathbf{B}_q^T\mathbf{N}_p\,dV \tag{44}$$

$$\mathbf{f}=\int_V \mathbf{W}_u^T\frac{\mathbf{F}}{\varepsilon}\,dV \tag{45}$$

$$\mathbf{m}=\varepsilon^{-1}\int_V \mathbf{N}_p^T\dot{m}\,dV \tag{46}$$

$$\mathbf{B}=\mathbf{S}\mathbf{N}_u \tag{47}$$

$$\mathbf{B}_c\hat{\mathbf{u}}=\begin{bmatrix}\frac{\partial\mathbf{u}}{\partial x} & \frac{\partial\mathbf{u}}{\partial y} & \frac{\partial\mathbf{u}}{\partial z}\\ \frac{\partial\mathbf{v}}{\partial x} & \frac{\partial\mathbf{v}}{\partial y} & \frac{\partial\mathbf{v}}{\partial z}\\ \frac{\partial\mathbf{w}}{\partial x} & \frac{\partial\mathbf{w}}{\partial y} & \frac{\partial\mathbf{w}}{\partial z}\end{bmatrix} \tag{48}$$

$$\mathbf{B}_q=\mathbf{q}\mathbf{B} \tag{49}$$

$$\mathbf{q}=\begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix} \tag{50}$$

$$\mathbf{S}^{\mathrm{T}}=\begin{bmatrix}\partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z\\ 0 & \partial/\partial y & 0 & \partial/\partial x & \partial/\partial z & 0\\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & \partial/\partial x\end{bmatrix} \tag{51}$$

$$\hat{\mathbf{u}}^{\mathrm{T}}=\begin{bmatrix}\mathbf{u} & \mathbf{v} & \mathbf{w}\end{bmatrix} \tag{52}$$

$$\mathbf{u}^{\mathrm{T}} = \begin{bmatrix} \hat{u}_1 & \cdots & \hat{u}_{15} \end{bmatrix} \quad (53)$$

$$\mathbf{v}^{\mathrm{T}} = \begin{bmatrix} \hat{v}_1 & \cdots & \hat{v}_{15} \end{bmatrix} \quad (54)$$

$$\mathbf{w}^{\mathrm{T}} = \begin{bmatrix} \hat{w}_1 & \cdots & \hat{w}_{15} \end{bmatrix} \quad (55)$$

$$\hat{\mathbf{u}}^{\mathrm{T}} = \begin{bmatrix} \hat{u}_1 & \cdots & \hat{u}_{15} & \hat{v}_1 & \cdots & \hat{v}_{15} & \hat{w}_1 & \cdots & \hat{w}_{15} \end{bmatrix} \quad (56)$$

$$\hat{\mathbf{p}}^{\mathrm{T}} = \begin{bmatrix} \hat{p}_1 & \hat{p}_2 & \hat{p}_3 & \hat{p}_4 & \hat{p}_5 & \hat{p}_6 \end{bmatrix} \quad (57)$$

$$\mathbf{N_u} = \begin{bmatrix} N_{u1} & \dots & N_{u15} & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & N_{v1} & \dots & N_{v15} & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & N_{w1} & \dots & N_{w15} \end{bmatrix} \quad (58)$$

$$\mathbf{N_p} = \begin{bmatrix} N_{p1} & N_{p2} & N_{p3} & N_{p4} & N_{p5} & N_{p6} \end{bmatrix} \quad (59)$$

$$W_u = N_u + \frac{\alpha h}{2} \frac{u(\partial \mathbf{N_u}/\partial x) + u(\partial \mathbf{N_u}/\partial y) + u(\partial \mathbf{N_u}/\partial z)}{|\mathbf{u}|} \quad (60)$$

$$|\mathbf{u}| = \left(u^2 + v^2 + w^2\right)^{1/2} \quad (61)$$

Volume integration is performed with Gauss numerical qubature formula.

Because the Peclet number of the flow of steam-air mixture in power plant condenser is usually between 50 and 5000 (Pe>>1) the value of the upwind parameter α in Petrov-Galerkin weighting function is taken equal to 1.

Special characteristic of the discretization of system PDE (37)-(40) is that Petrov-Galerkin weighting is applied, beside in the discretization of convective term also in the discretization of resistance forces. This is done because the resistance forces are highly dependent from mixture velocity and therefore have convective nature.

The global discretized system of equations for the whole condenser area is expressed by summing system of equations (41) for every finite element.

$$\mathbf{A}\begin{Bmatrix} \hat{\mathbf{u}} \\ \hat{\mathbf{p}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{m} \end{Bmatrix} \quad (62)$$

This system of algebraic equation, because of non-linearity of the terms **K** and thereby also coefficients matrix **A,** is solved by Newton iteration method.

B. *Discretization of the equation of air mass fraction*

The equation for conservation of the air mass fraction (5) with evaluating partial derivatives in convective terms and inserting the continuity equation (1) can be transformed in following form:

$$\varepsilon\rho u \frac{\partial c_g}{\partial x} + \varepsilon\rho v \frac{\partial c_g}{\partial y} + \varepsilon\rho w \frac{\partial c_g}{\partial z} = = \frac{\partial}{\partial x}\left(\varepsilon\rho D \frac{\partial c_g}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon\rho D \frac{\partial c_g}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon\rho D \frac{\partial c_g}{\partial z}\right) \quad (63)$$

This equation applied in the area of single finite element, where (as was stated above) is adopted that steam-air mixture physical parameters (in this case density ρ and molecular diffusivity D) are constant, can be formulated as follows

$$u\frac{\partial c_g}{\partial x} + v\frac{\partial c_g}{\partial y} + w\frac{\partial c_g}{\partial z} = D\left(\frac{\partial^2 c_g}{\partial x^2} + \frac{\partial^2 c_g}{\partial y^2} + \frac{\partial^2 c_g}{\partial z^2}\right) \quad (64)$$

This equation is equal in the form to the classic convection-diffusion equation.

Therefore, for solving equation (64) the Petrov-Galerkin procedure for solving standard convection-diffusion equation [8] can be applied.

With application of this procedure, the equation (64) in the area of single element can be discretized in the following form

$$\left[\mathbf{K} + \widetilde{\mathbf{K}}\right]\left[\mathbf{c_g}\right] = 0 \quad (65)$$

where in this case

$$\mathbf{K} = \int_V D(\nabla \mathbf{W_c})^{\mathrm{T}} \nabla \mathbf{N_c} dV \quad (66)$$

$$\widetilde{\mathbf{K}} = \int_V \mathbf{W_c}^{\mathrm{T}} C \nabla \mathbf{N_c} dV \quad (67)$$

$$\mathbf{c_g}^{\mathbf{T}} = \begin{bmatrix} \hat{c}_{g1} & \hat{c}_{g2} & \hat{c}_{g2} \end{bmatrix} \quad (68)$$

$$\mathbf{N_c} = \begin{bmatrix} N_{c1} & N_{c2} & N_{c3} & N_{c4} & N_{c5} & N_{c6} \end{bmatrix} \quad (69)$$

$$W_u = N_u + \frac{\alpha h}{2} \frac{u(\partial \mathbf{N}_c/\partial x) + u(\partial \mathbf{N}_c/\partial y) + u(\partial \mathbf{N}_c/\partial z)}{|\mathbf{u}|} \quad (70)$$

while other nomenclature is identical with previously defined.

Volume integration is also performed with Gauss numerical qubature formula.

The global discretized system of equations for the whole condenser area is obtained by summing system of equations (64) for every finite element.

$$\mathbf{A}\,\mathbf{c_g} = 0$$

This system of algebraic equations, because of non-linearity of the terms **K** and thereby also coefficient matrix **A,** is also solved by Newton iteration method..

IV. Aplication of Proposed Procedure

The described procedure is applied to analyze the steam flow and heat transfer in two configurations of the experimental condenser (with external and internal vent) of the NEI-Parsons, Ltd., England [10].

Specific characteristic of the proposed model and its predictive capability in comparison to the other presented models of steam flow and heat transfer in power plant condensers are discussed elsewhere [1], [11], [12].

With comparison of the calculated results and experimental results about experimental condenser at NEI published in reference [10], is confirmed [1], [11], [12] that Streamline Upwind Petrov-Galerkin finite element method can be successfully applied for solving the three-dimensional mathematical model of steam flow and heat transfer in power plant condensers.

V. Conclusion

In this paper computational simulation of steam flow and heat transfer in power plant condensers on the basis of the three-dimensional mathematical model for the flow through porous media has been performed. For solving the model an iterative approach based on the Streamline Upwind Petrov-Galerkin finite element method was applied.

Calculations of steam flow and heat transfer performed for an experimental condenser has shown that the obtained numerical results significantly approximate adequate experimental results about this condenser.

NOMENCLATURE

a - heat transfer area per unit volume, m^2/m^3
c_g - air mass fraction $=\rho_g/\rho$
d_v - inner diameter of tube, m
d_n - outer diameter of tube, m
D - molecular diffusion coefficient, m^2/s
D_p - coefficient of vapour diffusion in air, kg/(Pa m s)
F_x, F_y, F_z - flow resistance forces, N/m^3
f - friction factor
k - overall heat transfer coefficient, $W/(m^2K)$
r - latent heat of condensation, J/kg
m - condensation rate per unit volume, $kg/(m^3s)$
n - tube number in a vertical row
p - pressure, Pa
s - tube pitch, m
q - heat flux per unit area, W/m^2
R - gas constant, J/(kg K)
R_x, R_y - flow resistance, $1/m^2$
T - temperature, K
u - velocity component in the x direction, m/s
v - velocity component in the y direction, m/s
w - velocity component in the z direction, m/s
α - heat transfer coefficient, $W/(m^2\ K)$
ε - local volume porosity
λ - thermal conductivity, W/(m K)
μ - molecular viscosity, kg/(m s)
μ_{eff} - effective viscosity, kg/(m s)
μ_t - turbulent viscosity, kg/(m s)
Θ - correction factors
ρ - density, kg/m^3

Dimensionless criteria

$E_g = p_g/p$
Fr - Froud number
$F = Pr_k/(Fr\ H)$
$G = R_p\ H/Pr_k$
$H = c_{pk}\ (T_{pk}-T_c)/r$
Nu - Nusselt number
Pr - Prandtl number
$R_\rho = (\rho_k\mu_k/\rho_p\mu_p)^{0.5}$
Re - Reynolds number
$Re_f = 2\pi d_n q/(\mu_k r)$
$Re_q = Re_f + Re_k(\rho_p/\rho_k)^{0.5}$
$Z = (1+1/G)^{2/3}/F^{0.5}$

Subscripts

k - condensate
g - gas
p - steam
pk - interphase (steam-condensate) surface
c – tube
v - cooling water

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