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Cubic Trigonometric B-Spline Applied to Linear Two-Point Boundary Value Problems of Order Two

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Abstract**—**Linear two-point boundary value problems of order two are solved using cubic trigonometric B-spline interpolation method (CTBIM). Cubic trigonometric B-spline is a piecewise function consisting of trigonometric equations. This method is tested on some problems and the results are compared with cubic B-spline interpolation method (CBIM) from the literature. CTBIM is found to approximate the solution slightly more accurately than CBIM if the problems are trigonometric.

Keywords**—**trigonometric B-spline, two-point boundary value problem, spline interpolation, cubic spline

I. Introduction

BOUNDARY value problems are abundant in the field of physics, chemistry and engineering. This paper considers the simplest form of boundary value problems which is linear two-point boundary value problems of order two, as in (1).

$$u''(x)+p(x)u'(x)+q(x)u(x)=r(x),\quad x\in[a,b],\quad u(a)=\alpha,\quad u(b)=\beta, \tag{1}$$

$$p,q,r\in C^0[a,b] \text{ and } q(x)<0 \text{ on } [a,b].$$

Moreover, another simplified form of the problems can be written as in (2). This form can also be transformed into (1).

$$-(p(x)u'(x))'=r(x),\quad x\in[a,b],\quad u(a)=u(b)=0, \tag{2}$$

$$p,r\in C^1[a,b] \text{ and } p(x)>0 \text{ on } [a,b].$$

The continuity, negativity and positivity conditions are necessary for the existence and uniqueness of the problems [1-3]. These problems are considered because the methods proposed in this paper are prototypes. Hence, it is wise to test them on the most direct form of the problems.

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The use of monomial cubic spline in solving these problems was first explored by Bickley in 1968 [4]. His work was further explored in [5, 6] in the following year. Following these development, many other analysis and improvements were made throughout the years as in [7, 8] and the references therein. However, only in 2006, the use of cubic B-spline, a better representation than the monomial cubic splines, was suggested by Caglar *et al.* This method was called cubic B-spline interpolation method (CBIM) [3]. Continuing with this work, we applied the same procedure using another type of splines, called trigonometric B-spline.

Trigonometric spline function was first introduced by I. J. Schoenberg in 1964. In this work, he proved the existence of trigonometric B-spline function, a locally supported trigonometric spline function. As the name suggests, trigonometric B-spline is constructed from trigonometric functions, as opposed to polynomial functions in the case of B-spline. The derivation and properties of this function are discussed in [9-11].

II. Cubic Trigonometric B-Spline Basis

The basis of trigonometric B-spline of order 1 is shown in the following formula,

$$T_i^1(x)=\begin{cases}1, & x\in[x_i,x_{i+1}),\\ 0, & \text{otherwise.}\end{cases}$$

From there, trigonometric B-spline basis of order $k=2,3,\ldots$ can be calculated using the recursive equation in (3). Calculating up to degree 3, where $k=4,$ the resulting basis, $T_i^4(x)$ is shown in (4) [11].

$$T_i^k(x)=\frac{\sin\left(\frac{x-x_i}{2}\right)}{\sin\left(\frac{x_{i+k-1}-x_i}{2}\right)}T_i^{k-1}(x)+\frac{\sin\left(\frac{x_{i+k}-x}{2}\right)}{\sin\left(\frac{x_{i+k}-x_{i+1}}{2}\right)}T_{i+1}^{k-1}(x). \tag{3}$$

$$T_i^4(x) = \frac{1}{\theta}\begin{cases} \sigma^3(x_i), & x\in[x_i, x_{i+1}], \\ \sigma(x_i)\begin{bmatrix}\sigma(x_i)\varsigma(x_{i+2}) \\ +\varsigma(x_{i+3})\sigma(x_{i+1})\end{bmatrix} & x\in[x_{i+1}, x_{i+2}], \\ +\varsigma(x_{i+4})\sigma^2(x_{i+1}), & \\ \sigma(x_i)\varsigma^2(x_{i+3}) & \\ +\varsigma(x_{i+4})\begin{bmatrix}\sigma(x_{i+1})\varsigma(x_{i+3}) \\ +\varsigma(x_{i+4})\sigma(x_{i+2})\end{bmatrix}, & x\in[x_{i+2}, x_{i+3}], \\ \varsigma^3(x_{i+4}), & x\in[x_{i+3}, x_{i+4}], \end{cases} \quad (4)$$

where

$$\sigma(x_i) = \sin\left(\frac{x - x_i}{2}\right), \qquad \varsigma(x_i) = \sin\left(\frac{x_i - x}{2}\right),$$

$$\theta = \sin\left(\frac{h}{2}\right)\sin(h)\sin\left(\frac{3h}{2}\right).$$

$T_i^4(x)$ is a piecewise trigonometric funtion of degree 3 with C^2 continuity. Plots of cubic trigonometric B-spline basis together with cubic B-spline basis is shown in Fig. 1.

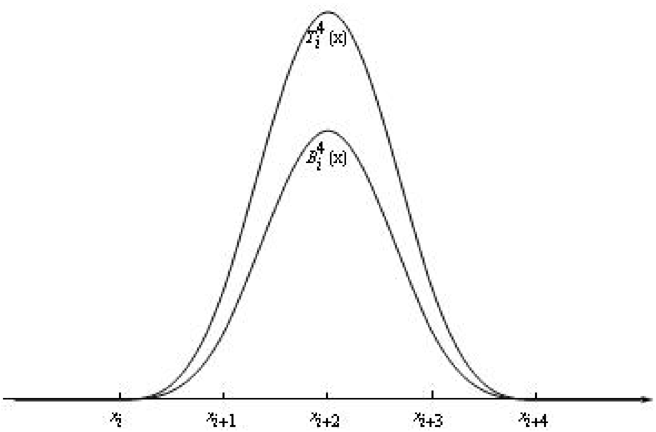

Fig. 1: Cubic trigonometric B-spline basis, $T_i^4(x)$ and cubic B-spline basis, $B_i^4(x)$

Some properties of cubic trigonometric B-spline basis arelisted below:

(i) $T_i^4(x)$ is nonnegative,

(ii) $T_i^4(x) \geq 0$ when $x\in[x_i, x_{i+4}]$, and zero otherwise

(iii) $\sum_i T_i^4(x) = 1.$

III. Cubic Trigonometric B-Spline

Cubic trigonometric B-spline basis can be manipulated to generate a piecewise function, called cubic trigonometric B-spline. Cubic trigonometric B-spline, $S_T(x)$ is a linear combination of the cubic trigonometric B-spline basis, as shown in (5).

$$S_T(x) = \sum_{i=-3}^{n-1} C_i T_i^4(x), \quad x\in[x_0, x_n], \quad C_i \in \mathbb{R}, \quad n \geq 1. \quad (5)$$

Therefore, similar to $T_i^4(x)$, $S_T(x)$ is also a piecewise trigonometric function of degree 3 with C^2 continuity. Here, C_i can be any real number and hence will be manipulated in solving the boundary value problems.

From property (ii) of the basis function, it can be observed that within interval $[x_i, x_{i+1}]$, there are only four nonzero basis functions, namely, $T_{i-3}(x)$, $T_{i-2}(x)$, $T_{i-1}(x)$, and $T_i(x)$. Thus, for $x^* \in [x_i, x_{i+1}]$,

$$\begin{aligned} S_T(x^*) &= C_{i-3}T_{i-3}^4(x^*) + C_{i-2}T_{i-2}^4(x^*) + C_{i-1}T_{i-1}^4(x^*) \\ &\quad + C_i T_i^4(x^*) \\ &= \frac{1}{\theta}\begin{pmatrix} C_{i-3}\left[\varsigma^3(x_{i+1})\right] + \\ C_{i-2}\begin{bmatrix}\sigma(x_{i-2})\varsigma^2(x_{i+1}) + \\ \varsigma(x_{i+2})\begin{bmatrix}\sigma(x_{i-1})\varsigma(x_{i+1}) \\ +\varsigma(x_{i+2})\sigma(x_i)\end{bmatrix}\end{bmatrix} + \\ C_{i-1}\begin{bmatrix}\sigma(x_{i-1})\begin{bmatrix}\sigma(x_{i-1})\varsigma(x_{i+1}) \\ +\varsigma(x_{i+2})\sigma(x_i)\end{bmatrix} \\ +\varsigma(x_{i+3})\sigma^2(x_i)\end{bmatrix} + \\ C_i\left[\sigma^3(x_i)\right] \end{pmatrix}. \end{aligned}$$

Since $T_i^4(x_i) = 0$,

$$\begin{aligned} S_T(x_i) &= C_{i-3}T_{i-3}^4(x_i) + C_{i-2}T_{i-2}^4(x_i) + C_{i-1}T_{i-1}^4(x_i) \\ &= C_{i-3}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right] \\ &\quad + C_{i-2}\left[\frac{2}{1+2\cos(h)}\right] \\ &\quad + C_{i-1}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right], \end{aligned} \quad (6)$$

By taking the first and second derivative of (5) and evaluating at x_i, we have (7) and (8).

$$S_T'(x_i) = C_{i-3}\left[-\frac{3}{4}\csc\left(\frac{3h}{2}\right)\right] + C_{i-2}[0] + C_{i-1}\left[\frac{3}{4}\csc\left(\frac{3h}{2}\right)\right], \quad (7)$$

$$S_T''(x_i) = C_{i-3}\left[\frac{3(1+3\cos(h))\csc^2\left(\frac{h}{2}\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}\right] + C_{i-2}\left[-\frac{3\cot^2\left(\frac{h}{2}\right)}{2+4\cos(h)}\right] + C_{i-1}\left[\frac{3(1+3\cos(h))\csc^2\left(\frac{h}{2}\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}\right]. \quad (8)$$

The simplifications of $S_T(x)$ and its derivatives at x_i are very useful in solving the problems.

IV. Cubic Trigonometric B-Spline Interpolation Method (CBIM)

In order to solve the problem, cubic trigonometric B-spline, $S_T(x)$ on $[a,b]$ is first presumed to be the solution of (1). Substituting $S_T(x)$ into (1), we have

$$S_T''(x) + p(x)S_T'(x) + q(x)S_T(x) = r(x), \quad x\in[a,b], \quad S_T(a)=\alpha, \quad S_T(b)=\beta. \quad (9)$$

Then, evaluating (9) at x_i results

$$S_T''(x_i) + p(x_i)S_T'(x_i) + q(x_i)S_T(x_i) = r(x_i), \quad i=0,1,...,n. \quad (10)$$

(10) comprises of $S_T(x_i)$, $S_T'(x_i)$, and $S_T''(x_i)$, which are already simplified in previous section. Therefore, substituting (6), (7) and (8) into (10), we have

$$\begin{pmatrix} C_{i-3}\left[\frac{3(1+3\cos(h))\csc^2\left(\frac{h}{2}\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}\right] \\ +C_{i-2}\left[\frac{-3\cot^2\left(\frac{h}{2}\right)}{2+4\cos(h)}\right] \\ +C_{i-1}\left[\frac{3(1+3\cos(h))\csc^2\left(\frac{h}{2}\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}\right] \end{pmatrix} + p(x_i)\begin{pmatrix} C_{i-3}\left[-\frac{3}{4}\csc\left(\frac{3h}{2}\right)\right]+C_{i-2}[0] \\ +C_{i-1}\left[\frac{3}{4}\csc\left(\frac{3h}{2}\right)\right] \end{pmatrix} + q(x_i)\begin{pmatrix} C_{i-3}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right] \\ +C_{i-2}\left[\frac{2}{1+2\cos(h)}\right] \\ +C_{i-1}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right] \end{pmatrix} = r(x_i).$$

Thus, (10) can be simplified into an expression consisting only of C_{i-3}, C_{i-2} and C_{i-1}. Collecting these terms, we have

$$C_{i-3}\left[\frac{3\left(1+3\cos(h)\csc^2\left(\frac{h}{2}\right)\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}-\frac{3}{4}p(x_i)\csc\left(\frac{3h}{2}\right)+q(x_i)\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]+$$
$$C_{i-2}\left[-\frac{3\cot^2\left(\frac{h}{2}\right)}{2+4\cos(h)}+q(x_i)\frac{2}{1+2\cos(h)}\right]+$$
$$C_{i-1}\left[\frac{3\left(1+3\cos(h)\csc^2\left(\frac{h}{2}\right)\right)}{16\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)}+\frac{3}{4}p(x_i)\csc\left(\frac{3h}{2}\right)+q(x_i)\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]=r(x_i). \quad (11)$$

Similarly, the boundary conditions can be simplified into (12) and (13).

$$\begin{aligned}S_T(a)&=S_T(x_0)\\&=C_{-3}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]\\&\quad+C_{-2}\left[\frac{2}{1+2\cos(h)}\right]\\&\quad+C_{-1}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]\\&=\alpha,\end{aligned} \quad (12)$$

$$\begin{aligned}S_T(b)&=S_T(x_n)\\&=C_{n-3}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]\\&\quad+C_{n-2}\left[\frac{2}{1+2\cos(h)}\right]\\&\quad+C_{n-1}\left[\sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right)\right]\\&=\beta.\end{aligned} \quad (13)$$

(11), (12), and (13) can be arranged in a matrix equation of the form

$$[\mathbf{A}]_{(n+3)\times(n+3)}[\mathbf{C}]_{(n+3)\times 1}=[\mathbf{R}]_{1\times(n+3)}, \quad (14)$$

where the first and last lines of **A** are the boundary conditions from (12) and (13), respectively, whereas the rest are from (11). (14) is a linear system of order $(n+3)\times(n+3)$, where **C** is the unknown vector. Hence, **C** can be solved by taking

$$\mathbf{C}=\mathbf{A}^{-1}\mathbf{R}. \quad (15)$$

Lastly, the obtained values of C_i, for $i=-3,-2,...n-1$ are substituted in (5), which becomes the approximated analytical solution to (1).

V. Numerical Examples And Discussions

CTBIM was implemented on Problems 5.1, 5.2, 5.3 and 5.4 with $n=10$. Therefore, there are 13 coefficients that are needed to be solved, $C_{-3},C_{-2},...,C_{9}$. The problems and their respective exact solutions are as the following:

Problem 5.1 [3]

$$-\frac{d}{dx}\left[e^{1-x}\frac{du}{dx}\right]=1+e^{1-x},\quad x\in[0,1],\quad u(0)=0,\quad u(1)=0,$$
$$\Rightarrow u''(x)-u'(x)=-e^{x-1}-1,\quad x\in[0,1],\quad u(0)=0,\quad u(1)=0.$$

Exact solution: $u(x)=x\left(1-e^{x-1}\right)$.

Problem 5.2 [12]

$$u''(x)+(x+1)u'(x)-2u(x)=\left(1-x^2\right)e^{-x},$$
$$x\in[0,1],\quad u(0)=-1,\quad u(1)=0.$$

Exact solution: $u(x)=(x-1)e^{-x}$.

Problem 5.3 [1]

$$u''(x)-\pi^2 u(x)=-2\pi^2\sin(\pi x),$$

$$x\in[0,1],\quad u(0)=0,\quad u(1)=0.$$

Exact solution: $u(x)=\sin(\pi x)$.

Problem 5.4 [12]

$$u''(x)-u(x)=0,\quad x\in[0,1],\quad u(0)=0,\quad u(1)=\sinh(1).$$

Exact solution: $u(x)=\sinh(x)$.

Maximum-norm and L^2-norm are used to gauge the accuracy of the method. Suppose $u(x)$ and $S_T(x)$ are the exact and approximate solutions of (1), respectively. Maximum-norm or max-norm measures the upper bound of the absolute error and is calculated using the following formula,

$$\text{Max-Norm }=\left\|S(x_i)-u(x_i)\right\|_\infty=\max_i\left|S(x_i)-u(x_i)\right|.$$

Thus, max-norm reports the largest error of the approximated solution. L^2-norm measures the distribution of the absolute error and is calculated using the following formula,

$$L^2\text{-norm }=\left\|S(x_i)-u(x_i)\right\|_2=\sqrt{\sum_i\left[S(x_i)-u(x_i)\right]^2}.$$

Table 1 shows the max-norms and L^2-norms for each problem using CTBIM compared to CBIM.

TABLE 1 NORMS FOR PROBLEMS 5.1, 5.2, 5.3, AND 5.4 USING CBIM AND CTBIM

Problem	Method	Max-Norm	L^2-Norm
5.1	CBIM	2.8996E-04	6.6089E-04
	CTBIM	6.8895E-04	1.5679E-03
5.2	CBIM	2.3108E-04	5.2220E-04
	CTBIM	5.7190E-04	1.2898E-03
5.3	CBIM	4.1088E-03	9.1875E-03
	CTBIM	3.0962E-03	6.9233E-03
5.4	CBIM	5.2011E-05	1.1794E-04
	CTBIM	2.1178E-04	4.8023E-04

CTBIM did not approximate the solution better for Problems 5.1, 5.2 and 5.4, but did so for Problem 5.3. This might be due to the trigonometric equations appearing in the differential equation and exact solution. Therefore, three more problems of trigonometric nature were tested using CBIM and CTBIM. They are Problems 5.5, 5.6 and 5.7. The results are shown in Table 2.

Problem 5.5 [1]

$$u''(x)+u(x)=0,\quad x\in\left[0,\frac{\pi}{4}\right],\quad u(0)=1,\quad u\left(\frac{\pi}{4}\right)=1.$$

True solution: $u(x)=\cos(x)+\left(\sqrt{2}-1\right)\sin(x)$

Problem 5.6[1]

$$u''(x)-u'(x)-2u(x)=\cos(x),\quad x\in\left[0,\frac{\pi}{2}\right],$$

$$u(0)=-0.3,\quad u\left(\frac{\pi}{2}\right)=-0.1.$$

True solution: $u(x)=-\frac{1}{10}\left[\sin(x)+3\cos(x)\right]$

Problem 5.7 [1]

$$u''(x)+4u(x)=\cos x,\quad x\in\left[0,\frac{\pi}{4}\right],\quad u(0)=0,\quad u\left(\frac{\pi}{4}\right)=0.$$

True solution:

$$u(x)=-\frac{1}{3}\cos(2x)-\frac{\sqrt{2}}{6}\sin(2x)+\frac{1}{3}\cos x$$

TABLE II
MAX-NORM AND L^2-NORM VALUES FOR PROBLEMS 5.5, 5.6, AND 5.7 USING CBIM AND CTBIM

Problem	Method	Max-Norm	L^2-Norm
4.5	CBIM	4.5213E-05	1.0404E-04
	CTBIM	4.2456E-05	9.7692E-05
4.6	CBIM	1.1188E-04	2.5955E-04
	CTBIM	1.0532E-04	2.4435E-04
4.7	CBIM	3.0748E-04	7.0064E-04
	CTBIM	1.4814E-04	3.3778E-04

For all the problems, CTBIM gave slightly better approximations than that of CBIM. Hence, it is safe to say that CTBIM approximates linear two-point boundary value problems better for problems involving trigonometric expressions.

VI. CONCLUSIONS

Referring to our previous work in [13], we used extended cubic B-spline in place of B-spline. Extended cubic B-spline is an improved version of B-spline, where one free parameter is added to the basis function [14]. The results of CBIM, CTBIM and extended cubic B-spline interpolation method (ECBIM) are shown in Table 3.

From the table, it is obvious that ECBIM approximates the solution a lot better than CBIM and CTBIM. But CTBIM produced more accurate results compared to CBIM if the problems were trigonometric. Thus, if cubic trigonometric B-spline with a free parameter is available, further tests can be done on the problems and our hypothesis would be such spline would approximate the solution to Problem 5.2 better than ECBIM

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TABLE III
ERROR FOR PROBLEMS 5.1, 5.2, 5.3, AND 5.4 USING CBIM, CTBIM AND ECBIM

Problem	Method	Max-Norm	L^2-Norm
5.1	CBIM	2.8996E-04	6.6089E-04
	CTBIM	6.8895E-04	1.5679E-03
	ECBIM	3.2452E-06	7.2555E-06
5.2	CBIM	2.3108E-04	5.2220E-04
	CTBIM	5.7190E-04	1.2898E-03
	ECBIM	3.0130E-06	6.8241E-06
5.3	CBIM	4.1088E-03	9.1875E-03
	CTBIM	3.0962E-03	6.9233E-03
	ECBIM	4.0147E-09	8.9771E-09
5.4	CBIM	5.2011E-05	1.1794E-04
	CTBIM	2.1178E-04	4.8023E-04
	ECBIM	3.5072E-10	7.9531E-10

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