

The Minimum PAPR Code for OFDM Systems

Inderjeet Kaur, M. Kulkarni, Daya Gupta, Kamal Thakur, and Janki Arora

Abstract—In this paper, a block code to minimize the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals is proposed. It is shown that cyclic shift and codeword inversion cause not change to peak envelope power. The encoding rule for the proposed code comprises of searching for a seed codeword, shifting the register elements, and determining codeword inversion, eliminating the look-up table for one-to-one correspondence between the source and the coded data. Simulation results show that OFDM systems with the proposed code always have the minimum PAPR.

Keywords—Wireless communications, OFDM, peak-to average power ratio, peak envelope power, block codes.

I. INTRODUCTION

WITH the rapid growth of digital wireless communication in recent years, the need for high speed mobile data transmission has increased. Many wireless communication systems being developed use OFDM to achieve high data rates. OFDM is a strong candidate and has been suggested or standardized in high speed communication Systems like WLAN [1], [2]. A high PAPR is a main drawback of OFDM. There are many methods for reducing the PAPR with an ultimate goal of reducing the PAPR as much as possible [3]-[8]. In block coding schemes [3]-[5], source information is transmitted through selected code words having relatively low PAPR. When combined with an error control code [3], the scheme is appropriate for high-quality wireless communications.

Most of the block coding schemes [3], [4] have a limitation in PAPR reduction. The scheme introduced in [5] can achieve the minimized PAPR, but has no generalized encoding rule. Hence, a huge look-up table may be required for one-to-one mapping between the source and the coded data when the codeword length is increased [8].

In this paper, the proposed block code guarantees the minimized PAPR in OFDM signals. It is also verified that the

peak envelope power (PEP) is invariant to the cyclic shift of the register elements and codeword inversion. Based on such properties, a systematic encoding rule for the proposed minimum PAPR code (MPC) is presented. This is achieved in three steps: a search for an appropriate seed codeword, cyclic shift of the register elements, and codeword inversion. This rule enables the MPC encoder to generate all code words using only a few seed code words so that a look-up table is not necessary.

II. DESIGN OF THE MINIMUM PAPR

A. Definition of Seed Codeword

The peak envelop power of OFDM signals is computed as

$$\max_{0 \leq n \leq N-1} |x|^2 = \max_{0 \leq n \leq N-1} \left\{ \frac{1}{N^2} \sum_k \sum_m X_k X'_m e^{j2\pi m(k-m)/N} \right\} \quad (1)$$

A min-max algorithm gives the solution for the minimum PEP as

$$\min \left\{ \max_{0 \leq n \leq N-1} \left\{ \frac{1}{N^2} \sum_r |X_r|^2 \right\} \right\} \quad (2)$$

A set of code words of length N , having minimum PEP is obtained, however the exhaustive search has very high computational complexity.

Let $G = \{G_k \mid 0 \leq k \leq K-1\}$ be the set of entire code words having minimum PEP are obtained. G can be divided into L disjoint subsets of code words, that is, a code word $C_k \in C_j$ cannot be the element of C_l , $0 \leq j, l \leq L-1, j \neq l$. Then, there are L seed code words taken from the L disjoint subsets. A seed codeword is, therefore, the representative of each subset.

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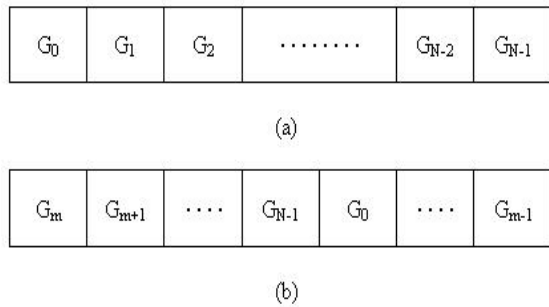


Fig. 1 The codeword stored in a shift register: (a) initial state and after m cyclic shifts

B. Invariance of PEP to Cyclic Shift and Codeword Inversion

Let a binary word $G_s = [G_0 G_1 \dots G_{N-1}]$ be a seed codeword of length N , and fed as input to a N -bit shift register. Actually, the accurate notation of the seed codeword is $C_{j,s} \in C_j$. The index is shortened in rest of the paper. Here, a seed codeword is a codeword whose binary phase shift keying (BPSK) mapped real sequence $X_s = [X_1 X_2 \dots X_{N-1}]$ has the minimum PAPR after an inverse discrete Fourier transforms (IDFT). Consider a new codeword G^m obtained by m cyclic shifts on G_s as shown in Fig. 1(b). The corresponding real sequence is $X_s^m = [X_m X_{m+1} \dots X_{N-1} X_0 X_1 \dots X_{m-1}]$. Then, the transmitted signal of the new sequence is

$$x_n^m = \frac{1}{N} X_s^m W_n \tag{3}$$

where $W_n = [1 e^{j2\pi/N} e^{j4\pi/N} \dots e^{j2\pi(N-1)/N}]^T$ is an $N \times 1$ column vector of the $N \times N$ IDFT matrix W_N . The superscript T denotes the transpose of the matrix. The vector W_N can be represented as

$$W_N = e^{-j2\pi m/N} W_n^m$$

where

$$W_n^m = [e^{j2\pi m/N} e^{j2\pi(m+1)/N} \dots e^{j2\pi(N-1)/N} 1 \dots e^{j2\pi(m-1)/N}]^T \tag{4}$$

Hence, the signal in (3) is modified as

$$x_n^m = \frac{1}{N} e^{-j2\pi m/N} X_s^m W_n^m = e^{-j2\pi m/N} x_n \tag{5}$$

where x_n is the transmitted signal of the seed. Note that a cyclic shift of the register elements results in a phase shift after IDFT. As the phase shift does not alter the PEP of the OFDM signals, both the new and seed code words have the same PEP. Thus, $N-1$ code words having the minimum PAPR exist for every seed.

Consider a codeword $\overline{G_s} = [\overline{G_0} \overline{G_1} \dots \overline{G_{N-1}}]$ is obtained by inverting codeword seed G_s . The modulated signal after BPSK mapping of $\overline{G_s}$ is

$$\overline{x_n} = \frac{1}{N} \overline{X_s} W_n \tag{6}$$

Since $\overline{X_k} = -X_k$ with the BPSK signal mapper, the transmitted signal is computed as

$$\overline{x_n} = -\frac{1}{N} X_s W_n = e^{-j\pi} x_n \tag{7}$$

As inverting the sign of a codeword is the same as a rotation by $e^{j\pi}$, it does not change the power distribution of the transmitted signals. Thus, the PEP is invariant to the codeword inversion. It is clear that we have an additional set of N code words with the inversion. Thus, a seed codeword is the representative of a disjoint subset consisting of a total of $2N$ code words of length N . This implies that a set of seed code words can be used to find all the code words of an MPC. In addition, the invariance properties are still valid although the signal mapper generates a complex sequence.

C. A Systematic Encoding Rule for the MPC

Assume that there exist N_s seed code words. Then, it is possible to produce an MPC consisting of $2NN_s$ code words. This results in a $(N, \log_2(2NN_s))$ block coding scheme as shown in Fig. 2(a). The source binary information of length $\log_2(2NN_s)$ bits is classified into three parts by their functions. First $\log_2 N_s$ least significant bits (LSB) are exploited to select one of the N_s seed code words. If only one seed is available, that is, $\log_2 N_s = 0$ no seed selection procedure is required. It can be shown that such a special case occurs in a (4, 3) MPC. The next $\log_2 N$ bits determine the number of cyclic shifts of the register elements. Finally, the register elements are inverted to generate a coded bit sequence if the most significant bit (MSB) of the source data is '1'. Otherwise, the elements are the output codeword. Such an encoding rule is illustrated in Fig. 2 (b).

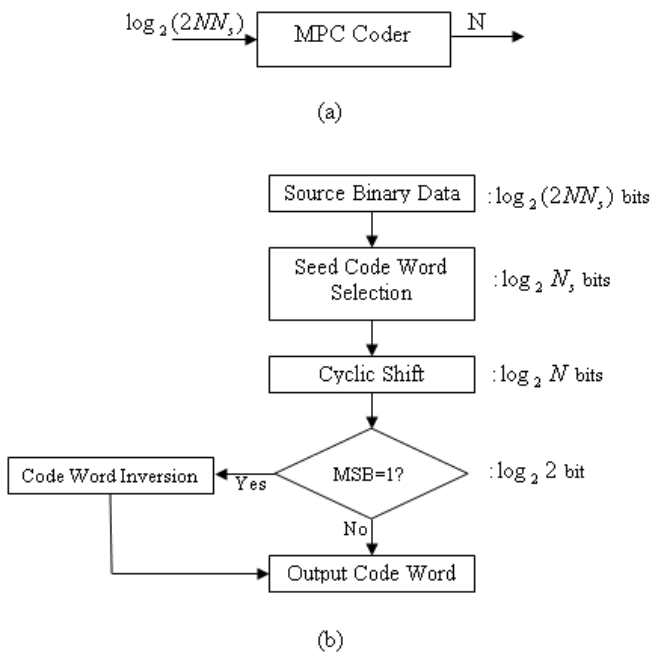


Fig. 2 (a) The MPC encoder (b) The systematic encoding rule for stored in a shift register

III. NUMERICAL ANALYSIS

To verify if the proposed code really provides the minimum PAPR characteristic, computer simulation for 8- and 16-subcarrier OFDM systems with BPSK signal mapper is carried out. As a performance measure, a complementary cumulative distribution function (CCDF) of the PAPR defined as

$$CCDF(P_{ref}) = 1 - \Pr(PAPR > P_{ref}) \quad [6]$$

is used. Here, P_{ref} is a reference value of the PAPR. $\Pr(PAPR > P_{ref})$ is the probability that the PAPR of the OFDM signals exceeds the reference value.

Example 1: An 8-subcarrier OFDM There are two seed code words of length 8: one is '00001011' and the other is '00001101'. Thus, $2N_s = 32$ code words constituting an (8, 5) MPC exist. That is, 5-bit source information is encoded to generate a codeword of length 8. Consider that a 5-bit source word is '11010' as an example. The LSB chooses one of two possible seeds. Assume that '00001011' is selected as the seed codeword. Since the next $\log_2 N$ bits are '101', we have '01100001' by five cyclic shifts of the register elements. As the MSB of the source word is '1', the register elements are inverted to '10011110' and mapped with BPSK to produce a real sequence '1 -1 -1 1 1 1 -1'. CCDFs of an uncoded 8-subcarrier and the (8, 5) MPC coded OFDM are plotted in Fig. 3. It is observed that the uncoded 8-subcarrier OFDM system has eight levels of PAPR in the transmitted signals.

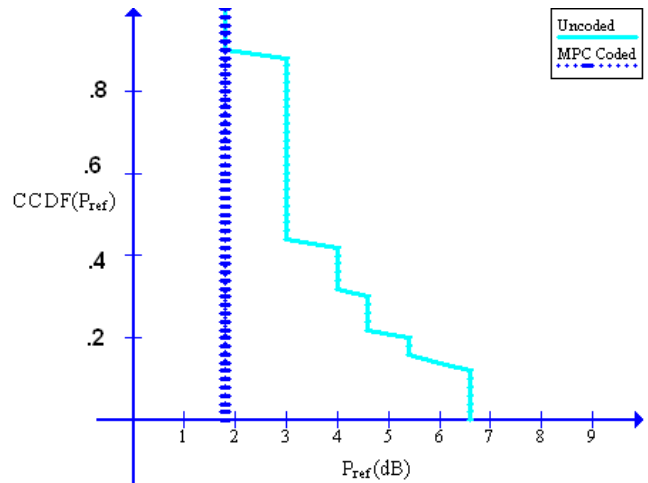


Fig. 3 CCDFs of an uncoded 8-subcarrier and the (8, 5) MPC coded OFDM

The theoretical maximum and minimum values are 9.03 and 1.76 dB, respectively. Signals having the PAPR of 3.01 dB are produced most frequently. Unlike the uncoded scheme, the MPC coded OFDM system has a unique PEP so that it shows only one PAPR level. The PAPR is 1.76 dB, which is the minimum value achievable in the uncoded system.

Example 2: A 16-subcarrier OFDM

In the case of a 16-subcarrier OFDM, eight 16-bit seed codewords are found by an exhaustive search. The seed words are presented in Table 1. By the cyclic shift and codeword inversion, the set of seeds can be expanded to 256 codewords to form a (16, 8) MPC. CCDFs of an uncoded 16-subcarrier and the (16, 8) MPC coded OFDM are presented in Fig. 4.

It can be observed that the possible range of the PAPR in the uncoded OFDM is from 1.76 to 12.04 dB. As can be expected, with an increase in the codeword length by a factor of 2, the maximum PAPR value is increased by 3.01 dB. While such minimum and maximum values rarely occur, the signals having a PAPR of 6.02 dB are produced most frequently. The probability that the PAPR exceeds 6.0 dB is around 0.25.

TABLE I
 SEED CODEWORDS FOR THE (16, 8) MPC CODE

No.	Seed Code Word	No.	Seed Code Word
0	0000011010110111	4	0000101111001101
1	0000011101101011	5	0000110100111011
2	0000101100111101	6	0000110111001011
3	0000101101100111	7	0000111001101101

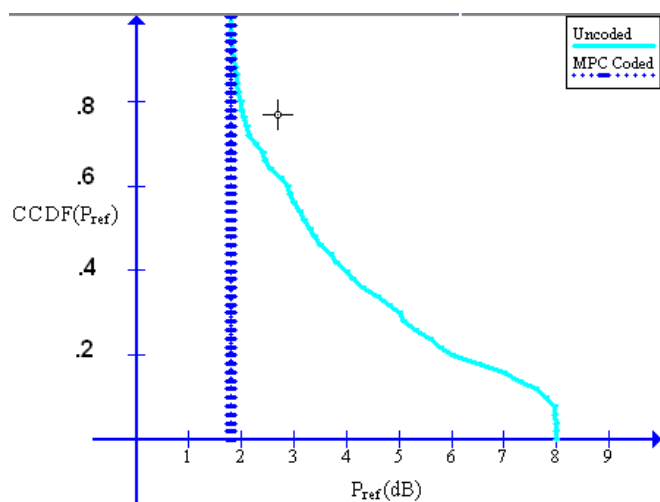


Fig. 4 (a) The MPC encoder (b) The systematic encoding rule for stored in a shift register

Thus, a quarter of the OFDM signals have a PAPR of 6.0 dB or more. Compared to that, the coded OFDM has a constant PEP in the transmitted signals, and the PAPR is only 1.76 dB. The PAPR can be reduced by 10.2 dB at maximum. It is, therefore, clear that the MPC guarantees the minimum PAPR.

IV. CONCLUSION AND FURTHER STUDIES

In this work, a block code that has an extreme PAPR performance is introduced. It is shown that the PEP of the OFDM signals is invariant to cyclic shift and codeword inversion. Based on such properties, a systematic encoding rule that does not depend on the codeword length is also presented. Due to the encoding rule, the look-up table to map the source data can be eliminated in the MPC encoder. Thus, the required memory size can be reduced significantly. Simulation results confirm that OFDM systems with the proposed code always have the minimum PAPR. Since the sets of seed code words provided in this letter are found by an exhaustive search, it must be the same for a fixed codeword length. Hence, no further investigations to find the set of seed code words other than that presented in this letter are needed. However, a further study on the way of finding the set of seed code words efficiently is needed.

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Inderjeet Kaur is a very active and dedicated research worker in her field and has published large number of research papers in the proceedings of International and National Conferences. She intends to build her career with leading corporate of committed people and willing to work in a challenging environment where she can exploit her potential to the maximum.