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# Characterization and Modeling of Packet Loss of a VoIP Communication

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***Abstract*—**In this work, a characterization and modeling of packet loss of a Voice over Internet Protocol (VoIP) communication is developed. The distributions of the number of consecutive received and lost packets (namely gap and burst) are modeled from the transition probabilities of two-state and four-state model. Measurements show that both models describe adequately the burst distribution, but the decay of gap distribution for non-homogeneous losses is better fit by the four-state model. The respective probabilities of transition between states for each model were estimated with a proposed algorithm from a set of monitored VoIP calls in order to obtain representative minimum, maximum and average values for both models.

***Keywords*—**Packet loss, gap and burst distribution, Markov chain, VoIP measurements.

## I. INTRODUCTION

IN this work, modeling of a *Voice over Internet Protocol* (VoIP) communication through a wide area network (WAN) is developed and simulation based on this model is performed. The effects of correlated delay and loss in the sequence of packets on a voice communication are studied. And parameters of the proposed models are obtained from measurements of these VoIP calls.

Consecutive packet receipts and losses are named gaps and bursts, respectively[1]. Due to the time-correlated occupancy of the network, packet losses commonly occur in bursts. At small time scales, i.e., a few seconds or minutes, bursts occur with approximately the same distribution, and a two-state Markov chain can reproduce this phenomenon. Non-homogeneous bursty behavior becomes noticeable at larger scales and in this case the two-state Markov chain is insufficient, thus a more general model is necessary. The four-state Markov chain is applied then in order to capture (or simulate) the widely known bursty, non-homogeneous behavior of the characteristics of network traffic. This approach allows us to represent and simulate those periods with low and high loss rate that alternate in sequence according to certain probability.

The knowledge of the packet loss rate (PLR) and burst length distribution is useful to quantify the quality of the communication in the sense of certain metrics, e.g., mean opinion score (MOS).

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Statistical description of the four-state Markov chain, assuming time-homogeneity, is presented. Theoretical packet receipt and loss rates are quantified. Respective gap length and burst length (measured in number of packets lost/received) distributions are also described and a comparison, by means of the *square root of the mean squared error* (SMSE) of gap and burst length distributions, of two-state and four-state models is also performed.

## II. MARKOV CHAINS

Let $S = S_1, S_2, \dots, S_m$ be the $m$ states of an m-state Markov chain and let $p_{ij}$ be the probability of the chain to pass from the state $S_i$ to the state $S_j$, i.e., $p_{ij} = P(X_i = x_i | X_{i-1} = x_{i-1})$. Having the Markov property means that, given the present state, future states are independent of the past states, i.e., $P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \dots) = P(X_{n+1} = x_{n+1} | X_n = x_n)$. The Markov chains used in this work also are time-homogeneous, which means that the probabilities of transition between states are constant over time,i.e., $P(X_{n+1} = x_{n+1} | X_n = x_n) = P(X_n = x_n | X_{n-1} = x_{n-1})$. All states communicate (are reachable from) each other, which makes the chain irreducible. Also, the chain is aperiodic, i.e., state $S_i$ can be reached from itself in any number of steps ($n = 1,2,3, \dots$).

The probabilities of transitions between states can be represented by a *transition matrix*. The elements of the one-step $m \times m$ transition matrix $\underline{T}$ are $T_{ij} = p_{ij}$. To obtain the $n$-step transition matrix it is necessary to multiply the matrix itself $n$ times[2], i.e.,

$$\underline{T_n} = \underline{T}^n. \tag{1}$$

As the number of steps ($n$) increases, the probability of transition to the state $S_i$ depends less of the initial state. i.e., as $n$ tends to $\infty$, the matrix $\underline{T_n}$ converges to a matrix with the next form:

$$\underline{T_\infty} = \lim_{n\to\infty} \underline{T_n} = \begin{bmatrix} s_1 & s_2 & \cdots & s_m \\ s_1 & s_2 & \cdots & s_m \\ \vdots & \vdots & \ddots & \vdots \\ s_1 & s_2 & \cdots & s_m \end{bmatrix} \tag{2}$$

such that

$$s_1 + s_2 + \cdots + s_m = 1 \tag{3}$$

In (3), $s_i$ represents the named *steady* probability of state $S_i$. The steady-state transition matrix $\underline{T}_\infty$ can be obtained then by solving (3) and (4) **Error! Reference source not found.**]:

$$[s_1 \quad s_2 \quad \cdots \quad s_m]\underline{T} = [s_1 \quad s_2 \quad \cdots \quad s_m] \tag{4}$$

Assuming that the chain is irreducible and aperiodic, the Matrix $\underline{T}_\infty$ is defined and unique.

## III. Two- and Four-state Models

The two-state Markov chain is shown in Fig. 1. State $S_1$ represents packet loss and $S_2$, packet receipt. Two substitutions ($p_{11} = 1 - p_{12}$ and $p_{22} = 1 - p_{21}$) are made in order to represent the chain with the lowest number of parameters. The steady-state probability of the chain to be in the state $S_1$, namely the packet loss rate, is given by (5) **Error! Reference source not found.**]:

$$s_1 = \frac{p_{21}}{p_{12} + p_{21}} \tag{5}$$

and clearly $s_2 = 1 - s_1$.

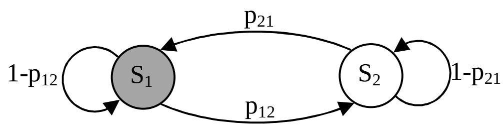


Fig. 1: Two-state Markov chain. White and shady circles represent correct and erroneous states, respectively.

The burst length and gap length distributions ($f_b(k)$ and $f_g(k)$, respectively) can be expressed in terms of $p_{12}$ and $p_{21}$, as expressed by (6) and (7):

$$f_b(k) = p_{12}(1 - p_{12})^{k-1} \tag{6}$$

$$f_g(k) = p_{21}(1 - p_{21})^{k-1} \tag{7}$$

which have also respective averages $E\{f_b(k)\} = 1/p_{12}$ and $E\{f_g(k)\} = 1/p_{21}$. It is easy to proof (6), as $\sum_{k=1}^{\infty} f_b(k) = 1$ and $f_b(k+1) = f_b(k) \cdot (1 - p_{21})$; and similarly for (7).

The four-state Markov chain is shown in Fig. 2. Missing arrows indicate zero probability. States $S_1$ and $S_3$ (shady circles) represent packet losses (erroneous); $S_2$ and $S_4$ (white circles), packet receipt (correct).

Six parameters ($p_{21}, p_{12}, p_{43}, p_{34}, p_{23}, p_{32} \in (0,1)$) are necessary to define all these probabilities. Without loss of generality, probabilities of transitions between correct states, as well as transitions between erroneous ones, have been assigned to zero.

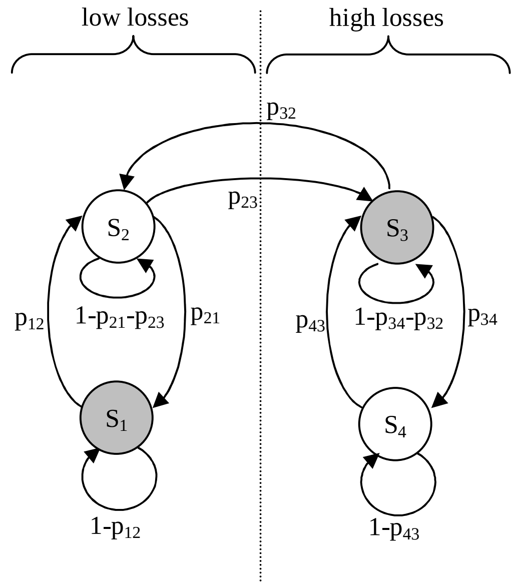


Fig. 2: Four-state Markov chain. White and shady circles represent correct and erroneous states, respectively.

The four steady-state probabilities of this chain are:

$$s_1 = \frac{1}{1 + \frac{p_{12}}{p_{21}} + \frac{p_{12} \cdot p_{23}}{p_{21} \cdot p_{32}} + \frac{p_{12} \cdot p_{23} \cdot p_{34}}{p_{21} \cdot p_{32} \cdot p_{43}}} \tag{8}$$

$$s_2 = \frac{1}{1 + \frac{p_{21}}{p_{12}} + \frac{p_{23}}{p_{32}} + \frac{p_{23} \cdot p_{34}}{p_{32} \cdot p_{43}}} \tag{9}$$

$$s_3 = \frac{1}{1 + \frac{p_{34}}{p_{43}} + \frac{p_{32}}{p_{23}} + \frac{p_{21} \cdot p_{32}}{p_{12} \cdot p_{23}}} \tag{10}$$

$$s_4 = \frac{1}{1 + \frac{p_{43}}{p_{34}} + \frac{p_{32} \cdot p_{43}}{p_{23} \cdot p_{34}} + \frac{p_{21} \cdot p_{32} \cdot p_{43}}{p_{12} \cdot p_{23} \cdot p_{34}}} \tag{11}$$

The packet loss rate, i.e., the probability of the chain to be either in $S_1$ or in $S_3$, is then:

$$r = s_1 + s_3 \tag{12}$$

The average burst length ($b$) is calculated as the quotient of the probability of loss and the probability of transition from a lossless state to a loss state or vice versa (13). Let $t_{c \to e}$ and $t_{e \to c}$ be the respective number of transitions from correct states to error states and from error states to correct states. Their absolute difference ($|t_{c \to e} - t_{e \to c}|$) is at most 1 and it can be considered 0 as $n$ tends to $\infty$, i.e., the transitions from error state to correct state and vice versa have equal probability ($s_2(p_{21} + p_{23}) + s_4(p_{43}) = s_1(p_{12}) + s_3(p_{34} + p_{32})$) and then the average burst length ($\bar{b}$) is:

$$\bar{b} = \frac{s_1 + s_3}{s_2(p_{21} + p_{23}) + s_4(p_{43})} \tag{13}$$

Similarly, the average gap length is:

$$\bar{g} = \frac{s_2 + s_4}{s_2(p_{21} + p_{23}) + s_4(p_{43})} \qquad (14)$$

The authors of **Error! Reference source not found.**] derived the gap length distribution for a two-state model, in which, as opposite to the two-state of Fig. 1, losses and receipts are allowed in the two states. The distribution of burst length of the four-state Markov chain of Fig. 2 is obtained, similarly as in **Error! Reference source not found.**], as follows:

Let $f_b(k)$ denote the probability that the burst length is $k$; $C_1(k)$, the probability that the burst length is $k$ or greater and the $k^{th}$ transmission is from state $S_1$ and $C_3(b)$, the probability that the burst length is $k$ or greater and $k^{th}$ transmission is from state $S_3$ and $C_b(k)$, the probability that the burst length is $k$ or greater such that $C_b(k) = C_1(k) + C_3(k)$ and $f_b(k) = C_b(k) - C_b(k+1)$. Clearly $C_b(k) = \sum_{i=k}^{\infty} f_b(i)$. Also, as transitions between states $S_1$ and $S_3$ have zero probability, $C_1(k+1) = C_1(k) \cdot (1 - p_{12}) = C_1(1) \cdot (1 - p_{12})^k$ and $C_3(k+1) = C_3(k) \cdot (1 - p_{34} - p_{32}) = C_3(1) \cdot (1 - p_{34} - p_{32})^k$. Then to calculate $f_b(k)$ it is necessary to obtain $C_1(1)$ and $C_3(1)$, whose respective values are $C_1(1) = s_2 \cdot p_{21}/[s_2(p_{21} + p_{23}) + s_4 \cdot p_{43}]$ and $C_3(1) = (s_2 \cdot p_{23} + s_4 \cdot p_{43})/[s_2(p_{21} + p_{23}) + s_4 \cdot p_{43}]$

As the minimum burst length is 1, $C_b(1) = C_1(1) + C_3(1) = 1$. Then, the distribution of the burst length is:

$$f_b(k) = C_1(1) \cdot Q_1(k) + C_3(1) \cdot Q_3(k) \qquad (15)$$

where $Q_1(k) = (1 - p_{12})^{k-1} - (1 - p_{12})^k = p_{12} \cdot (1 - p_{12})^{k-1}$ and $Q_3(k) = (1 - p_{34} - p_{32})^{k-1} - (1 - p_{34} - p_{32})^k = (p_{34} + p_{32}) \cdot (1 - p_{34} - p_{32})^{k-1}$. As expressed by (15), $f_b(k)$ is the sum of two geometric series with respective rates $1 - p_{12}$ and $1 - p_{34} - p_{32}$; this implies that $f_b(k)$ is a decreasing function of $k$, i.e., greater bursts have lower probabilities than shorter ones.

A similar procedure can be followed to obtain the gap length distribution ($f_g(k)$), which is:

$$f_g(k) = C_2(1) \cdot Q_2(k) + C_4(1) \cdot Q_4(k) \qquad (16)$$

Where $C_2(1) = (s_1 \cdot p_{12} + s_3 \cdot p_{32})/[s_1 \cdot p_{12} + s_3 \cdot (p_{32} + p_{34})]$, $C_4(1) = (s_3 \cdot p_{34})/[s_1 \cdot p_{12} + s_3 \cdot (p_{32} + p_{34})]$, $Q_2(k) = (1 - p_{21} - p_{23})^{k-1} - (1 - p_{21} - p_{23})^k = (p_{21} + p_{23}) \cdot (1 - p_{21} - p_{23})^{k-1}$ and $Q_4(k) = (1 - p_{43})^{k-1} - (1 - p_{43})^k = p_{43} \cdot (1 - p_{43})^{k-1}$. Also note that $C_2(1) + C_4(1) = 1$.

## IV. Numerical Approximation of the Matrix of Probabilities

Obtaining analytical expressions for the elements of $\underline{T_\infty}$ (i.e., $s_1, s_2 \ldots$) can be difficult when the number of states is large. In this case, a numerical approximation is more suitable, which is described as follows:

Let $\underline{T}$ be a $m \times m$ transition matrix, which has a unique steady-state solution, and let $\{(\lambda_i, \bar{v}_i); i = 1, \ldots, m\}$ be its pairs of eigenvalues and eigenvectors (i.e., $\underline{T}\bar{v}_i = \lambda_i \bar{v}_i$), such that $\lambda_i > \lambda_j$ for $i < j$. This matrix $\underline{T}$ can be decomposed into the special form

$$\underline{T} = \underline{PDP}^{-1} \qquad (17)$$

where $\underline{P}$ is a matrix composed of the eigenvectors of $\underline{T}$, $\underline{D}$ is the diagonal matrix constructed from the corresponding eigenvalues and $\underline{P}^{-1}$ is the inverse of $\underline{P}$. Then $\underline{T_n}$ can be calculated easily as

$$\underline{T_n} = \underline{PD}^n \underline{P}^{-1} \qquad (18)$$

As all elements of the diagonal of the matrix $\underline{D}$ are lower than 1 except $D_{1,1}$, then

$$\underline{T_\infty} = \underline{PD}^\infty \underline{P}^{-1} = \underline{PD'}\underline{P}^{-1} \qquad (19)$$

where the only non-zero element of $\underline{D}'$ is $D_{1,1} = 1$.

This method is also useful when obtaining short-term approximations, i.e., $\underline{T_n}$ for small $n$.

## V. Modeling Loss Sequences

The traces studied in this work are those corresponding to Sets 3 and 4, described in **Error! Reference source not found.**] and **Error! Reference source not found.**]. There are 48 traces in total. Each one represents the packet receipt and loss of an 1-hour VoIP call.

An empirical algorithm is used to estimate the parameters of the Markov chain. For the two state Markov model let $Y_t$ be the sequence that represent packet receipts and losses, i.e., $Y_t = 0$ if packet $t$ was received and $Y_t = 1$ if it was lost. Packets are sent with a constant rate, e.g., a packet is sent each $20ms$.

### A. Two-state Case

In this case the values of $p_{12}$ and $p_{21}$ are estimated as follows: $p_{12} = t_{c\to e}/n_1$ and $p_{21} = t_{e\to c}/n_0$. Where $t_{c\to e}$ and $t_{e\to c}$ are the respective number of transitions from correct states to error states and from error states to correct states, and $n_0$ and $n_1$ are the respective number of received and lost packets.

### B. Four-state Case:

In this case the values of the sequence $Y_t$ are divided into regions of two types: the first with lower loss rate (whose first and last values are zeros) and the second with higher loss rate (whose first and last values are ones) than certain threshold, e.g., 1%. Then, from the first region, $p_{12}$ and $p_{21}$ are estimated the same way than in a two-state model. Similarly, $p_{43}$ and $p_{34}$ are estimated from the second region. Finally, let $t_{1st\to 2nd}$ be the number of transitions from the first region to the second; $t_{2nd\to 1st}$, the number of transitions from the second to the first; $n_{1st}$, the number of received packets in the first region (zeros) and $n_{2nd}$, the number of lost packets in the second region (ones), then $p_{23} = t_{1st\to 2nd}/n_{1st}$ and $p_{32} = t_{2nd\to 1st}/n_{2nd}$. Also, the burst and gap histograms can be obtained from the sequence $Y_t$.

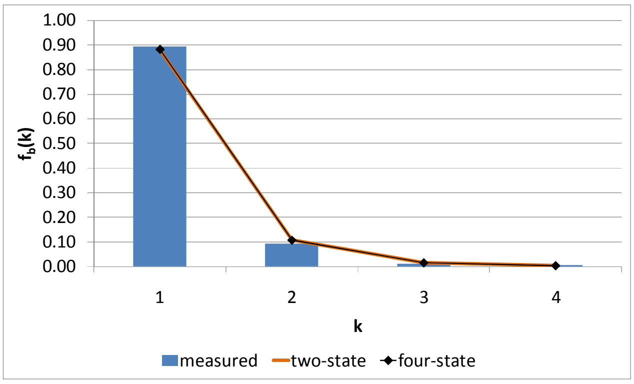


Fig. 3: Burst length distribution of a VoIP communication. Two-state and four-state models represent it adequately

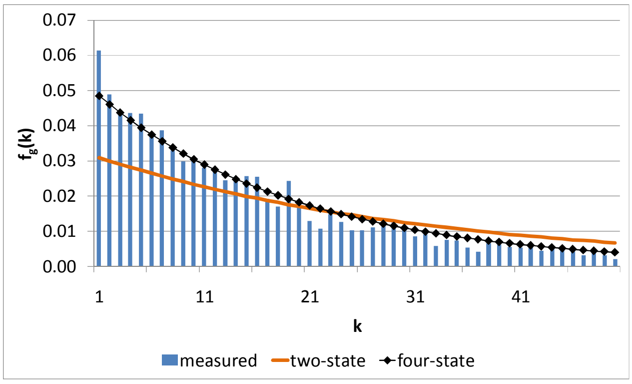


Fig. 4: Gap length distribution of a VoIP communication. Four-state model fits better this distribution than two-state model

## VI. Comparison of Two- and Four-state Models

An example with one of the traces measured, obtained from a VoIP communication with codec G.711 and sampling time of $20ms$, is shown in Fig. 3 and Fig. 4.

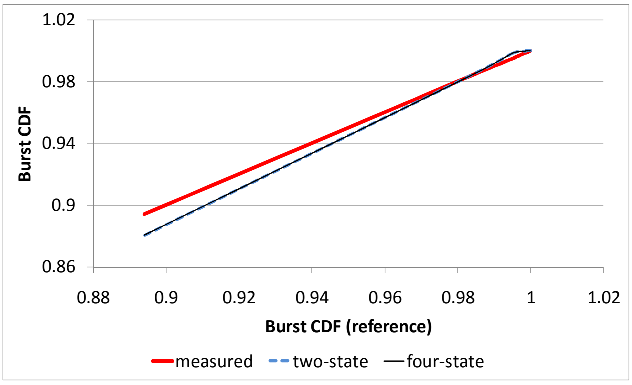


Fig. 5: PP-plot of the respective two-state and four-state burst histograms

In the traces under study it is found that, although the two-state model is adequate for the burst length distribution (see Fig. 5), the four-state model performs better when modeling the gap length distribution, as shown in Fig. 6. This is because the burst length distribution approaches rapidly to very low values (near zero), but in the case of the gap length distribution, which presents slower decay, the two-state model does not fit the measured distribution due to its non-flexible one-parameter formula (see (7)).

For each trace that represents a loss sequence, the parameters of both two-state and four-state models are obtained.

The estimated statistics of the two-state and four-state transition probabilities are shown, respectively, in Table I and Table II.

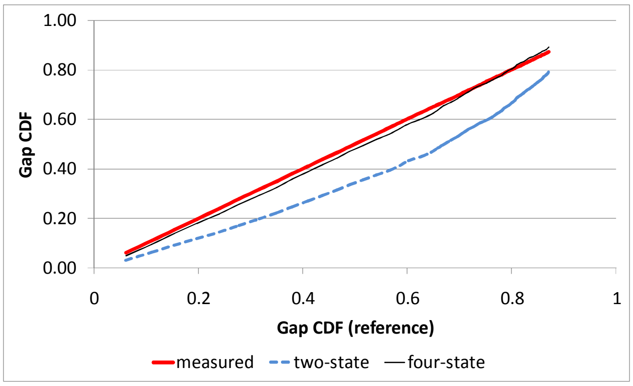


Fig. 6: PP-plot of the respective two-state and four-state gap histograms

TABLE I
Statistics For Two-State Model

| | $p_{21}$ | $p_{12}$ |
|---|---|---|
| **MIN** | 0.000322 | 0.595744 |
| **MAX** | 0.038605 | 0.934703 |
| **MEAN** | 0.013923 | 0.852838 |
| **STD. DEV.** | 0.012234 | 0.074697 |

TABLE II
Statistics For Four-State Model

| | $p_{21}$ | $p_{12}$ | $p_{43}$ | $p_{34}$ | $p_{23}$ | $p_{32}$ |
|---|---|---|---|---|---|---|
| **MIN** | 0.000188 | 0.963636 | 0.015503 | 0.096774 | 0.000016 | 0.000890 |
| **MAX** | 0.002053 | 1.000000 | 0.272727 | 0.931464 | 0.002551 | 0.361111 |
| **MEAN** | 0.000793 | 0.998997 | 0.054215 | 0.688747 | 0.000602 | 0.092536 |
| **STD. DEV.** | 0.000527 | 0.005424 | 0.037093 | 0.259845 | 0.000609 | 0.106315 |

Fig. 7 and Fig. 8 show the SMSE between the burst length and gap length distributions and their respective two-state and four-state models. The four-state model fits the distribution better than two-state model for most traces. In cases where SMSE is large (e.g., greater than 0.001 for the four-state gap length distribution of Fig. 8) the packet loss is low (e.g., lower than 0.23%), then the gap length distribution cannot be adequately sampled, a larger number of samples is necessary in these cases. Note that each one of these traces represents an 1-hour call. In shorter periods, where non-bursty losses are present, these can be modeled adequately by two-state model.

## VII. Conclusion

In this work, packet receipt and loss are modeled by discrete finite-state Markov chains. The packet loss rate, packet receipt rate and the distributions of burst length and gap length are described in terms of the Markov chain parameters, i.e., the probabilities of transition between states. Algorithms for estimating the probabilities of transition for both two-state and four-state models, which are shown in Fig. 1 and Fig. 2, are presented.

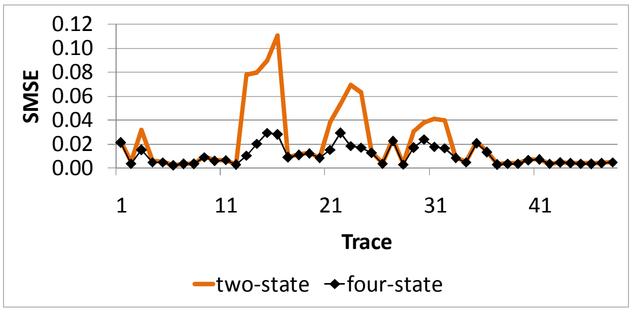

Fig. 7: SMSE of two-state and four-state burst histogram for all traces

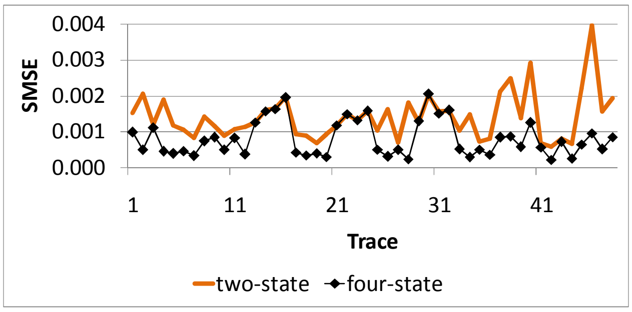

Fig. 8: SMSE of two-state and four-state gap histogram for all traces

For modeling bursts and gaps of short communications, e.g. calls with a duration of a few minutes, the two-state model is sufficient; but for longer periods, the four-states model fits better. The performance of both models was evaluated by means of the SMSE of the burst length and gap length distributions, showing that although the two-state model fits adequately the burst length distribution, it cannot follow the slow decay of the gap length distribution of measurements.

The traces for which the SMSE of the four-state model was higher (e.g., greater than 0.001) had very low loss rates (less than 0.23%), so the gap length distribution was not adequately sampled.

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