World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:2, No:8, 2008

Noise-Improved Signal Detection in Nonlinear Threshold Systems

Youguo Wang, and Lenan Wu

***Abstract*—**We discuss the signal detection through nonlinear threshold systems. The detection performance is assessed by the probability of error Per. We establish that: (1) when the signal is complete suprathreshold, noise always degrades the signal detection both in the single threshold system and in the parallel array of threshold devices. (2) When the signal is a little subthreshold, noise degrades signal detection in the single threshold system. But in the parallel array, noise can improve signal detection, i.e., stochastic resonance (SR) exists in the array. (3) When the signal is predominant subthreshold, noise always can improve signal detection and SR always exists not only in the single threshold system but also in the parallel array. (4) Array can improve signal detection by raising the number of threshold devices. These results extend further the applicability of SR in signal detection.

***Keywords*—**Probability of error, signal detection, stochastic resonance, threshold system.

I. INTRODUCTION

NOISE can bring beneficial help to signal processing or signal transmission in some nonlinear systems. This phenomenon is called stochastic resonance (SR) [1]. Most studies of SR involve a subthreshold (weak) signal through a single nonlinear system. The nonlinear system can produce a stronger beneficial response by the addition of noise [1]-[4]. Very recently, another new form of SR has been introduced under the name of suprathreshold stochastic resonance (SSR) through a parallel array [5]-[12], where the input signal is suprathreshold with respect to the common threshold. In a single threshold system, noise always degrades signal transmission for suprathreshold signal. However, in a parallel array, when different and independent noises are added on every device in the array, every device will in general produce a different response. When all these responses are summed, the output can be improved and SR may occur. SR and SSR are two distinct forms of improvement by noise [8]. The study of SR in signal detection has received some attentions [13]-[21]. Here, we study further SR in signal detection through the array of threshold devices. We use the probability of error Per to assess the performance. We establish that: (1) when the signal is complete suprathreshold, noise always degrades the signal detection both in the single threshold system and in the parallel array. (2) When the signal is a little subthreshold, noise degrades signal detection in the single threshold system. But noise can improve signal detection in the parallel array, i.e., SR exists in the parallel array. (3) When the signal is predominant subthreshold, SR always exists not only in the single threshold system but also in the parallel array. (4) Array can improve signal detection by raising the number of threshold devices. These results extend further the applicability of SR in signal detection.

Manuscript received April 3, 2005..

Youguo Wang is with Department of Applied Mathematics and Physics, Nanjing University of Posts and Telecommunications, Nanjing, 210003, and with Department of Radio Engineering, Southeast University, Nanjing 210096, People's Republic of China (e-mail: guowy2489@126.com).

Lenan Wu is with Department of Radio Engineering, Southeast University, Nanjing 210096, People's Republic of China (e-mail: wuln@seu.edu.cn).

II. OPTIMAL NONLINEAR DETECTION IN A PARALLEL ARRAY

A continuous signal x is with probability density function (PDF) $f_0(x)$ (hypothesis H_0) or with PDF $f_1(x)$ (hypothesis H_1) with prior probability P_0 and P_1 ($P_0 + P_1 = 1$). A noise η_i, independent of x, can be added to x before quantization by threshold device i, which is with a threshold level u_i, delivers the output $y_i = H[x+\eta_i - u_i], i = 1,2,...,N$, where $H(u) = 1$ if $u \succ 0$ and is zero otherwise. We consider here that N noises η_i are mutually independent, and identically distributed with PDF $f_\eta(x)$. The response of the parallel array is obtained by summing the outputs of all the threshold devices as $y = \sum_{i=1}^{N} y_i$. When N is one, the array is the single threshold system. Such conditions, with arrays of threshold devices, are relevant for existing and future multi-sensor networks having to cope with limited time and resources for data processing, storage, communication, and for energy supply [8]. The detection of signal x is based on the observation y, which assumes integer values between 0 and N ($y \in R = \{0,1,\cdots,N\}$). As discussed in [21], we can obtain the optimal detector, also known as the maximum a posterior probability (MAP) detector, which implements the test $L(y=n) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P_0}{P_1}$, by use of the likelihood ratio $L(y=n) = \frac{\Pr(y=n \mid H_1)}{\Pr(y=n \mid H_0)}$. The probability of error Per reached by the MAP detector is expressed as:

$$Per = \frac{1}{2} - \frac{1}{2}\sum_{n\in R}\left|P_0 \Pr(y=n \mid H_0) - P_1 \Pr(y=n \mid H_1)\right| \quad (1)$$

Where $\Pr(y=n \mid H_0)$ and $\Pr(y=n \mid H_1)$ are the conditional probabilities. Given a fixed value x_0 of input signal x, we have the conditional probability $\Pr(y_i = 0 \mid x_0) = \Pr(x_0 + \eta_i \le u_i) = q_i(x_0)$, and $\Pr(y_i = 1 \mid x_0) = 1 - q_i(x_0)$. Because the effect of threshold level distribution to the performance of the array is little, especially at larger noise intensity [12], we also can assume that all the thresholds share the same value u as in [8-10]. Then we have $q_i(x_0) = q(x_0)$ and the $\Pr(y=n \mid x_0)$ follows, according to the binomial distribution, as $\Pr(y=n \mid x_0) = \binom{N}{n}[1-q(x_0)]^n q(x_0)^{N-n}$, where $\binom{N}{n}$ is the binomial coefficient. We therefore obtain

$$\Pr(y=n \mid H_0) = \int_{-\infty}^{+\infty} \Pr(y=n \mid x_0) f_0(x_0) dx_0 . \quad (2)$$

$$\Pr(y=n \mid H_1) = \int_{-\infty}^{+\infty} \Pr(y=n \mid x_0) f_1(x_0) dx_0 . \quad (3)$$

The probability of error Per in (1) follows directly from (2) and (3), possibly through numerical integration, in broad conditions concerning the noises η_i and the input signal x.

III. Noise can Improve Signal Detection in the Array

For illustration of the possibility of SR and array improve signal detection, we choose signal x is with Gaussian PDF $f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp(-\frac{(x-\mu_0)^2}{2\sigma_x^2})$ or with Gaussian PDF $f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp(-\frac{(x-\mu_1)^2}{2\sigma_x^2})$, and noises η_i are with Gaussian PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. We adjust the threshold level from zero to a bigger value in steps of 0.10 for different threshold number in the parallel array. Figs. 1-4 show evolutions of the probability of error Per of (1), as a function of the noise root-mean-squared (rms) amplitude σ for some typical parameters. When threshold level $\mu = P_0\mu_0 + P_1\mu_1 = 0$ and the input signal is complete suprathreshold, i.e., the signal spends a half of its time above (or below) the threshold, Fig.1 shows that noise always degrades the signal detection both in the single threshold system and in the parallel array, and SSR does not occur. When the threshold level $u\ (> P_0\mu_0 + P_1\mu_1)$ is changed from 0.10 to 2 and there are a little subthreshold ingredients in the input signal, i.e., the signal spend more time below the threshold, Figs. 2-3 show that noise also degrades signal detection in the single threshold system. However, in the parallel array ($N > 1$), noise can improve signal detection and SR occurs. This condition shares the same mechanism with previous SSR [5]-[11]. When the threshold level $u\ (\succ\succ P_0\mu_0 + P_1\mu_1)$ is bigger than 2 and the input signal is predominant subthreshold, i.e., the signal spend many time below the threshold, Fig. 4 shows that noise always can improve signal detection and SR always exists not only in the single threshold system but also in the parallel array. In addition, Figs. 1-4 also show that when the number of threshold devices increases, the Per always decreases and the array can improve the signal detection for any fixed threshold level and noise intensity.

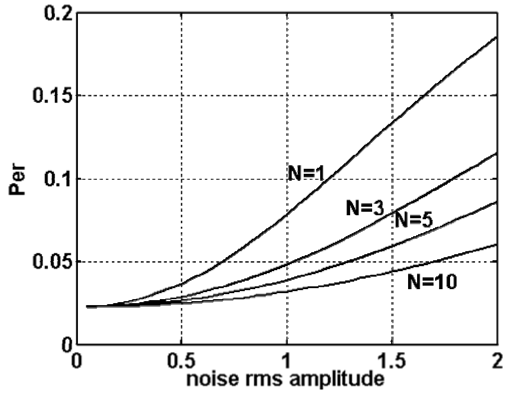

Fig. 1 Per is a function for different N. x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x-2)^2}{2}]$ or $f_1(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x+2)^2}{2}]$ and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 0$, $P_0 = P_1 = 0.5$.

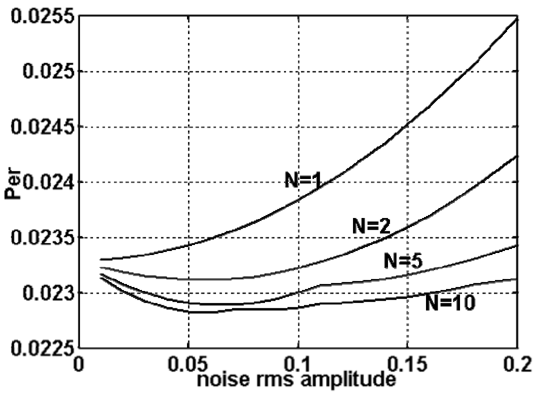

Fig. 2 Per is a function for different N. x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x-2)^2}{2}]$ or PDF $f_1(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x+2)^2}{2}]$, and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 0.1$, $P_0 = P_1 = 0.5$.

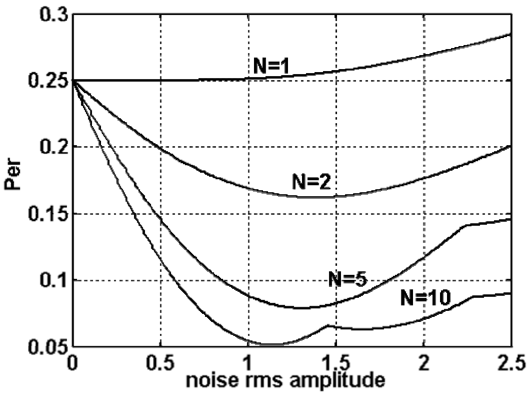

Fig. 3 Per is a function for different N . x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x-2)^2}{2}]$ or $f_1(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x+2)^2}{2}]$ and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 2$, $P_0 = P_1 = 0.5$.

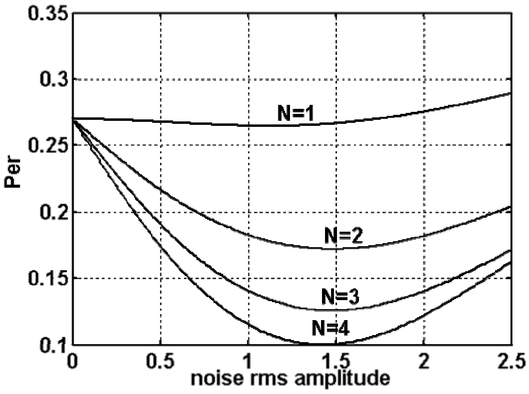

Fig. 4 Per is a function for different N . x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x-2)^2}{2}]$ or $f_1(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x+2)^2}{2}]$ and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 2.1$, $P_0 = P_1 = 0.5$.

IV. Conclusion

In this paper, we discuss further the signal detection through a parallel array of threshold devices. When the signal is complete suprathreshold, noise always degenerate signal detection and SSR does not occur. However, when the signal is a little subthreshold, although noise still degenerate signal detection in the single threshold system, noise can improve signal detection in the parallel array and SR occurs in the array, where the SR mechanism is similar with previous SSR. When the signal is predominant subthreshold, SR always occurs not only in the single threshold system but also in the parallel array. Other PDFs of signal and noise (for instance, uniform PDF or other PDF) may also have the similar effects on signal detection through nonlinear threshold system.

References

[1] B. McNamara, K. Wiesenfeld, "Theory of stochastic resonance," Physical Review A, vol. 39, pp. 4854-4869, 1989.
[2] A. Restrepo, L. F. Zuluaga, L. E. Pino, "Optimal noise levels for stochastic resonance," 1997 IEEE International Conference on Acoustics, Speech, and Signal Processing, vol. 3, pp. 1617-1620, 1997.
[3] B. Kosko, S. Mitaim, "Stochastic resonance in noisy threshold neurons," Neural Networks, vol. 16, pp. 755-761, 2003.
[4] A. Das, N. G. Stocks, A. Nikitin, and E. L. Hines, "Quantifying stochastic resonance in a single threshold detector for random aperiodic signals," Fluctuation and Noise Letters, vol. 4, no. 2, L247-L265, 2004.
[5] N. G. Stocks, "Supra-threshold stochastic resonance in multilevel threshold systems," Physical Review Letters, vol. 84, pp. 2310-2313, 2000.
[6] N. G. Stocks, "Information transmission in parallel threshold arrays: supra-threshold stochastic resonance," Physical Review E, vol. 63, pp. 041114, 2001.
[7] N. G. Stocks, "Supra-threshold stochastic resonance: an exact result for uniformly distributed signal and noise," Physics Letters A, vol. 279, pp. 308-312, 2001.
[8] D. Rousseau, F. Duan, and F. Chapeau-Blondeau, "Supra-threshold stochastic resonance and noise-enhanced Fisher information in an arrays of threshold devices," Physical Review E, vol. 68, pp. 031107, 2003.
[9] D. Rousseau, F. Chapeau-Blondeau, "Supra-threshold stochastic resonance and signal-to-noise ratio improvement in arrays of comparators," Physics Letters A, vol. 321, pp. 280-290, 2004.
[10] Mark D. Mcdonnell, Derek Abbott, "A characterization of supra-threshold stochastic resonance in an array of comparators by correlation coefficient," Fluctuation and Noise Letters, vol. 2, no. 3, pp. 213-228, 2002.
[11] N. G. Stocks, R. Mannella, "Generic noise-enhanced coding in neuronal arrays," Physical Review E, vol. 64, pp. 030902(R), 2001.
[12] N. G. Stocks, Optimizing information transmission in model neuronal ensembles: the role of internal noise, Stochastic Processes in Physics, Chemistry, and Biology, Edited by J. A.. Freund, Th, Pöschel, Lecture Notes in Physics, 557(2000)150.
[13] S. Zozor, P.-O.Amblard, "On the use of stochastic in sine detection," Signal Processing, vol. 82, no. 3, pp. 353-367, 2002
[14] S. Kay, "Can detectability be improved by adding noise?" IEEE Signal Processing Letters, vol. 7, pp. 8-10, 2000.
[15] Hu Gang, Gong De-chun, Wen Xiao-dong, Yang Chun-yun, Qing Guang-rong, and Li Rong, "Stochastic resonance in a nonlinear system driven by an a-periodic force," Physical Review A, vol. 46, pp. 3250-3254, 1992.
[16] M. E. Inchiosa, A. R. Bulsara, "Signal detection statistics of stochastic resonators," Physical Review E, vol. 53, pp. 2021-2024, 1996.
[17] F. Chapeau-Blondeau, "Stochastic resonance for an optimal detector with phase noise," Signal Processing, vol. 83, pp. 665-670, 2003.
[18] B. Kosko, S. Mitain; "Robust stochastic resonance: signal detection and adaptation in impulsive noise," Physical Review E, vol. 64, pp. 051110,1-11, 2001.
[19] V.Galdi, V. Pierro, I. M. Pinto, "Evaluation of stochastic-resonance-based detectors of weak harmonic signals in additive white Gaussian noise," Physical Review E, vol. 57, pp. 6470-6479, 1998.
[20] A. Saha, G.V. Anand, "Design of detectors based on stochastic resonance," Signal Processing, vol. 83, pp. 1193-1212, 2003
[21] Youguo Wang, Lenan Wu, "Stochastic resonance in nonlinear signal detection," to be published.