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# A new Cellular Automata Model of Cardiac Action Potential Propagation based on Summation of Excited Neighbors

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***Abstract*—**The heart tissue is an excitable media. A Cellular Automata is a type of model that can be used to model cardiac action potential propagation. One of the advantages of this approach against the methods based on differential equations is its high speed in large scale simulations. Recent cellular automata models are not able to avoid flat edges in the result patterns or have large neighborhoods. In this paper, we present a new model to eliminate flat edges by minimum number of neighbors.

***Keywords*—**Cellular Automata, Action Potential Simulation, Isotropic Pattern.

## I. Introduction

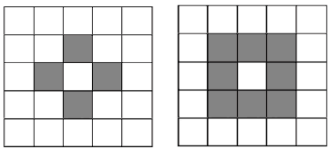
CARDIAC modeling and simulation have been the subject of important research during the last three decades [1] . Computational models are able to offer unique insights into both normal action potential conduction and arrhythmias[2] . Due to the large number of cells in cardiac tissue and the restrictions in the calculation of computer models, models with less computation are more considered. Cellular Automata (CA) model is one kind of cellular behavior models that has short computation in comparison with electrophysiological models. Many researchers have been used cellular automata for action potential propagation modeling.

CAs are discrete dynamic systems whose behaviors are completely based on local communications. They consist of a large number of relatively simple individual units, which is called cells. A network of these cells is represented the space. The state of a cell at each time is calculated from the states of some number of cells (called neighborhood) in previous time step. As time goes discretely, each of the cells can be in one of several finite numbers of states. All cells in CA are usually governed by the same rules. So, the state of neighbors and the rules of the CA determine how the states of a cell change.

There are two common and well-known neighborhoods in CA models. The Moore neighborhood comprises the eight cells surrounding a central cell. (See fig. 1 (a) ) the other one, a diamond-shaped neighborhood contains four cells. The cell above and below, right and left from each cell are called the von Neumann neighborhood of this cell. In this paper, both Moore and Von Neumann are studied.

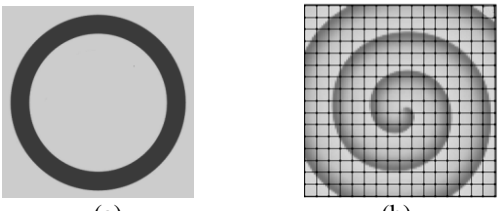
The cellular automaton model uses a simple set of rules to represent the complex physiological processes that result in electrical impulse generation, conduction and propagation. The simplicity of the assumptions allows one to simulate wave propagation within a realistic whole heart model [3] To develop the simplest form of cellular automata model for cardiac conduction, we consider the nature of propagation of electrical activity by cardiac action potentials to represent a form of information transmission on a discrete lattice of points through space, representing the volume of the myocardium [4]

The heart tissue is an excitable media. Some researchers have approached the spread of the activation process mathematically in the form of a wave propagation problem[3] . One of the most important properties of wave propagation in excitable Medias is their propagation patterns. Ring pattern and spiral wave pattern [5] [5] can be mentioned as some examples of propagation patterns (see fig. 1). The model presented for action potential propagation in excitable media must be able to show these patterns. Ideally waves generated by computer models should be as circular as possible avoiding flat edges.



(a) (b)

Fig. 1 a- Von Neumann neighborhood b- Moore neighborhood



(a) (b)

Fig. 2 Wave propagation patterns including a- target pattern b-spiral pattern

In this paper, we presented a new cellular automata model for simulating the propagation of ventricular action potential.

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We discuss the effect of changing the parameters of the model on result patterns and analyze the results. We also find the minimum neighborhood between Moore and Von Neumann neighborhoods for optimization of our model.

## II. Cellular Automata models of propagation in Excitable media

### A. The Moe model

Moe et al. [6] had presented a primitive model for atrial fibrillation by using CA concepts. He considered five states for his model; consist of one state for resting, one state for being fully excited and three intermediate states for describing different refractory levels. He assumed six neighbors for each cell with regard to hexagonal shape cells. This model had been considered strongly as the first action potential propagation based on CA. The only problem of this model is its lack of isotropy means the model does not provide precise representation of the shape of cardiac spiral wave. Therefore, future models were presented more convenient model for excitable media relying on the principles used in this model.

A spiral wave generated by Moe model in cardiac tissue after 127 sec. is shown in fig. 4 below.

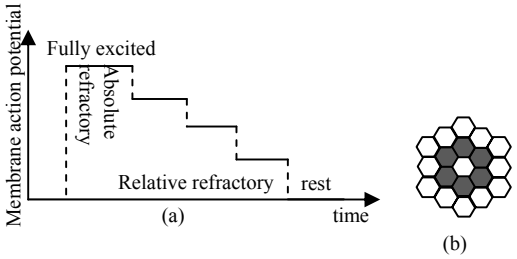



Fig. 3 a- schematic representation of the live states of activity. b- 6 neighbours of a central cell in Moe method.

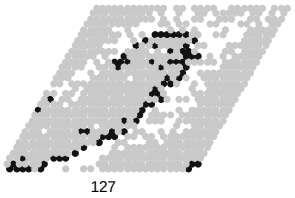



Fig. 4 Spiral wave produced by Moe model in arbitrary time (t=127) is displayed in which black colors shows fully excited cells. It also shows resting and refractory states by White and gray colored cells respectively

### B. The Gerdhardt model

Gerhardt et al. [7] introduced two variable u and v for the excitation and the recovery value of a cell to reproduce wave curvature with CA concept. The variable u can have a value of 0 or 1, while the variable v can have a value between 0 and $v_{max}$ which is determined before. This model presented a near isotropic pattern by using square neighborhood with a radius of 3(containing 48 neighbors for a central cell). Although the model used large number of neighbors for a central cell, flat edges in result patterns were observed. The other problem with this model is its running time. By using this amount of neighbors, the advantage of applying CA was ignored and the speed of simulation in large scale reduced significantly.

### C. The Markus model

Another model was proposed by Markus and Hess [8] by creating some changes in Gerhardt idea. He used a variable S instead of two variables u and v. this new variable can have the value between 0 and N+1. S=0 and S=N+1 were the representative of resting state and fully exciting state, respectively. The recovery state of a cell was shown by any value of S between 1 and N. a special kind of neighborhood was used in this model. Each cell had a point placed at a random position inside of it. A cell's neighbors are those which have their random point within a circular radius of the local cell's own random point (figure 5 (a)). By this kind of method, Markus achieved Isotropy. The achieved spiral pattern was shown in figure 5 (b). using this kind of neighborhood and calculating circular distance were this model's problem. Because of this circular neighborhood, a square root operation was needed for each pair of 2 points and therefore the simulation was taken long time.

### D. The Weimar models

The other models were presented by Weimar [10] [9] containing weighted mask for expressing the premiership of nearer and farther neighbors. These weights were proceeded to 19 or 20 for close neighbors. A square neighborhood with the radius of 7 was used in this model. Applying this large amount of neighbors is one of the important disadvantages of this model.

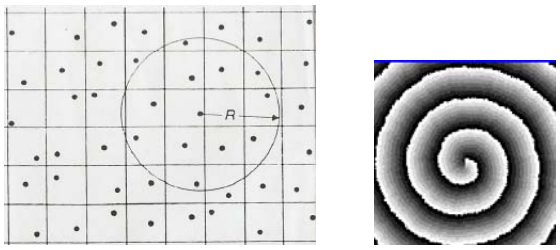



Fig. 5 a- an example of Circular neighborhood of the Markus mode[8] b- A spiral wave generated by the Markus model

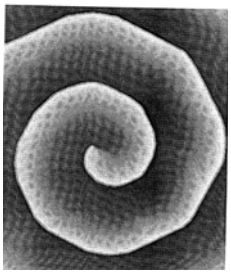


Fig. 6 Spiral wave on a 686*960 cell domain [10]

## III. Method

In this paper, we simulate action potential propagation by using fewer neighborhoods with the idea of Markus model. In this case, we consider both Moore and Von Neumann neighborhoods. (See fig. 1). In addition, we introduced $S^t_{mn}$ variable like the one in Markus model. M, n and t variables denotes the row number, column number, and the time step, respectively, when the situation will be studied.

Here $S^t_{mn}$ is defined by the sum of values of the states $u^t_{mn}$ at the time t over the neighboring cells. In fact, we use this method to eliminate flat edges in result patterns. $u^t_{mn}$ and $v^t_{mn}$ variables are introduced like Gerhard's ones. But in our model, each of the state variables can take values from 0 up to N-1. N is a parameter of the model which shows the number of discrete states between resting and fully excited in both excitability ($u^t_{mn}$) and recovery ($v^t_{mn}$) variables.

The cell first increases its *u* value by $u_{up}$ at each time step until *u*=*N*-1. Then; *v* rises by $v_{up}$ at each time step until *v*=*N*-1. Next; *u* decreases by $u_{Down}$ at each time step until *u*=0. Finally; *v* begins decreasing by $v_{Down}$ at each time step until *v*=0. At this point; *u*=0 and *v*=0, and the cell is back at its relaxed state.

In other words; the transition rule is as follows:

(1) If $S^t_{mn}$ is greater than the threshold of excitation (Δ) and $v^t_{mn}$ =$u^t_{mn}$=0, the cell will be excited in next time step. In this case, $u^{t+1}_{mn}$= $u_{up}$ and $v^{t+1}_{mn}$=0.

(2) If $S^t_{mn}$ < Δ and $v^t_{mn}$ =$u^t_{mn}$=0, the cell stays at its previous state. This means $v^{t+1}_{mn}$ =$u^{t+1}_{mn}$= $v^t_{mn}$ =$u^t_{mn}$=0. We should remind that Δ is a positive constant and must be in the range of 0<Δ<2N in Moore neighborhood and 0<Δ<N in Von Neumann neighborhood.

(3) Once $v^t_{mn}$+$u^t_{mn}$≠0 and a cell has enough excited neighbors to meet its excitability variable, the cell moves through the transitions given in figure 7.

After discussing the model with constant parameter Δ, two different threshold $\Delta_1$ and $\Delta_2$ with probability of P will be used in fallowing sections. We can achieve the isotropy by adding $\Delta_1$ and $\Delta_2$ randomly over the cells as shown in figure 8.

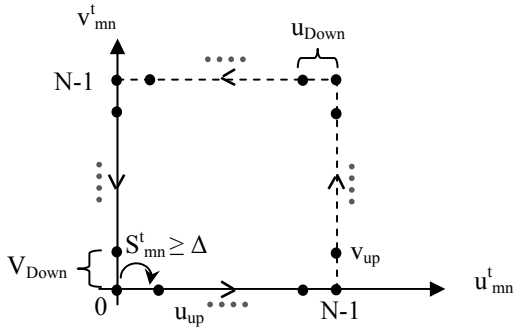



Fig. 7 The diagram which represent the state transitions of a cell

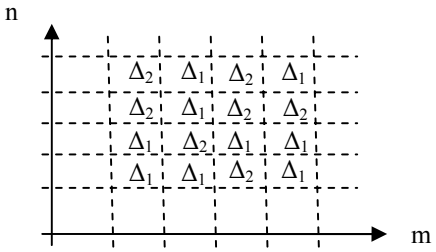



Fig. 8 two different thresholds ($\Delta_1$ and $\Delta_2$) are distributed randomly over the lattice

## IV. Results

The Cellular Automata model described above was implemented in both dev-C++ and Matlab software package. The source code is available to interested parties as per request to the author.

It is obvious that this model is faster than Markus model as it needs no complex operations such as square root calculations. The Markus model used circular neighborhoods but the calculation of distances using square root calculations proved extremely slow. However, in our model the transition rule depend on the summation of the excitability attributes of excited neighboring cells.

The effects of model parameters are tested in network with 50×50 and 150×150 cells. The results are as follows:

### *A.* The effect of Δ and N

The effect of Δ on producing or eliminating flat edges in result patterns is studied in this section. As shown in fig. 9, action potential propagation is simulated with $u_{up}$=3, $u_{Down}$=2, $v_{up}$=$v_{Down}$=1, the N value of 4, Δ=2 and Δ=3. This figure shows that the threshold value of 3 gets octagonal pattern. And a Quadrilateral pattern is obtained for Δ=2 and a dodecagonal pattern for Δ=6. By greater Δ, the result pattern has less flat edges and it is more similar to spiral pattern.

Fig. 10 shows the effect of N on result patterns (in only ring pattern). It is obvious that the result do not impress by various values of N. By greater N, the thickness of pattern is increased.

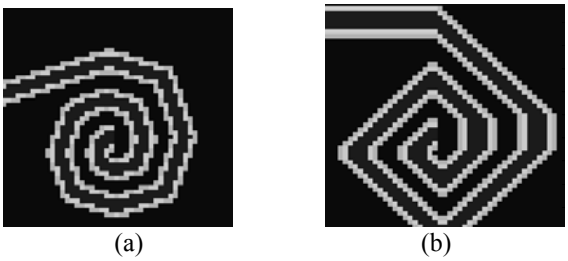
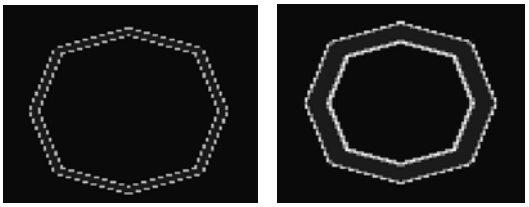



Fig. 9 Spiral pattern obtained by above method with N=4 and a- Δ=3 b- Δ=2. Part a in this figure is more similar to fig. 2 which is shown ideal spiral pattern.



(a) (b)

Fig. 10 effect of N on presented model at a network of 2500 cells with uup=3, uDown=2, vup=vDown=1, Δ=3 and a- N=4 b- N=10

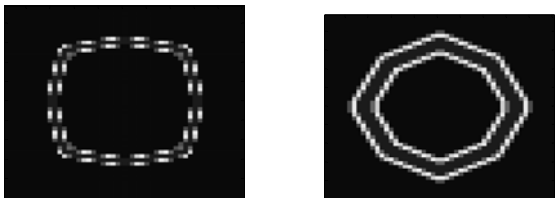
According to fig. 9 and fig. 10, we can control the shape and propagation speed of the generated patterns by choosing an appropriate value of the threshold.

*B. The effect of different neighborhoods*

In fig. 11 and fig. 12, the comparison of two different neighborhoods used in this paper is mentioned. It can be seen that using Moore neighborhood has appropriate result in eliminating flat edges. In fact, Generating isotropy by reducing the neighbors from Moore up to von Neumann proved less successful. So we will continue to use a Moore neighborhood for the remainder of our work.

(a) (b)

Fig. 11 spiral wave generated by using a- Moore neighborhood b- Von Neumann neighborhood



(a) (b)

Fig. 12 ring pattern obtained by using a- Von Neumann neighborhood b- Moore neighborhood

*C. The effect of using distinct values of $\Delta_1$ and $\Delta_2$*

In fig. 13 (a), wave propagation with Δ=4 is shown in a network of 22500 cells. However in part b of this figure, two different threshold values $\Delta_1$ and $\Delta_2$ are used. Using this method can generate isotropic patterns as shown in fig. 13.

(a) (b)

Fig. 13 A network of 22500 cells with N=6 and a- Δ=4 b- $\Delta_1$=6 and $\Delta_2$=4

*A. Action Potential Propagation in a 2-D cardiac tissue*

In this section, we show propagation of AP on a 2-D square lattice with the above simple rule, using Moore neighborhood. The membrane potential is represented depolarized and hyperpolarized tissue by white and black colors, respectively.

Abnormal action potential in 2-D cardiac tissue based on our method is shown in fig. 15. As it can be seen, the spiral wave is more isotropic and is similar to ideal one shown in fig. 2-b.

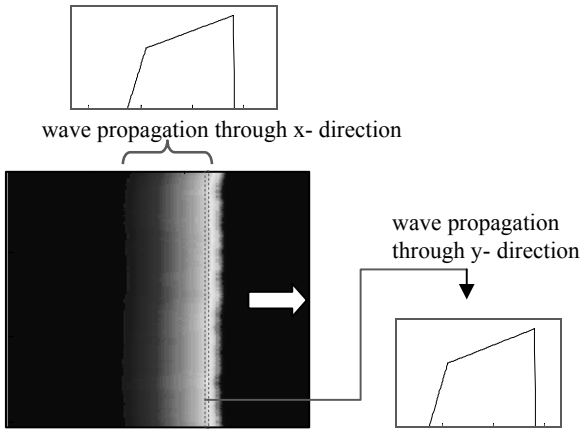



Fig. 14 Linear wavefront propagation in 2-D cardiac tissue. The membrane potential is color-coded according to the bar in the figure, with red representing depolarized tissue and blue hyperpolarized tissue.

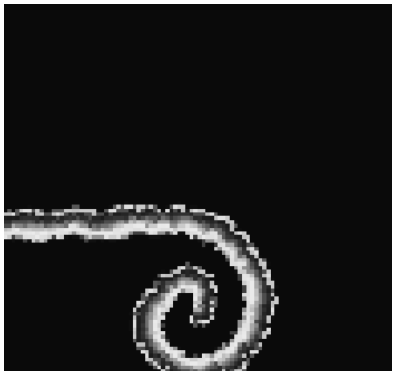


Fig. 15 Spiral wave generated by presented model.

## V.CONCLUSION

CA models aiming for wave propagation without curvature (square wave propagation) can easily achieve adequate

performance when curvature is attempted the calculation becomes too complex to maintain such performance. In this paper, a new cellular automata model for wave propagation is presented with fewer neighbors compared to previous studies.

It was seen that the calculation is simple enough to be performed across a large grid of cells in short period of time. The effect of model parameters (Δ and N) on the isotropy and speed of run time was survived in this research. At last, the minimum neighborhood was achieved for the presented model.

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