World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:6, 2007

An Adaptive Approach to Synchronization of Two Chua's Circuits

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***Abstract*—**This paper introduces an adaptive control scheme to synchronize two identical Chua's systems. Introductory part of the paper is presented in the first part of the paper and then in the second part, a new theorem is proposed based on which an adaptive control scheme is developed to synchronize two identical modified Chua's circuit. Finally, numerical simulations are included to verify the effectiveness of the proposed control method.

***Keywords*—**Chaos synchronization, Adaptive control, Chua's circuits.

I. INTRODUCTION

CHOS control and synchronization have attracted a great deal of attention of researchers from many different fields and is very important in many physical systems such as secure communication, space engineering, electronic systems, and semiconductor laser [1]. The control problem attempts to stabilize a chaotic attractor to either a periodic orbit or an equilibrium point [2]. Recently several control strategies have been proposed to stabilize chaotic systems. There are two main approaches to control chaos: non-feedback control [3, 4] and feedback control [5-7].

The idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll [8, 9]. Different approaches including conventional linear control techniques as well as advanced nonlinear control schemes have been already applied to the mentioned problem [1, 10]. In the most of the existing works, it is essential to know the system parameters. In practical situations, however, the parameters are usually unknown. Therefore, the derivation of an adaptive controller for the control and synchronization of chaotic systems is important.

Chua's circuit has been studied extensively as a prototypical electronic system by means of theoretical analysis and laboratory experiments. In recent years synchronization of two Chua's circuits has been studied [2, 11 and 12]. In this paper we present a new approach to adaptively synchronize two Chua's systems.

II. ADAPTIVE SCHEME

The Chua's system is described by the following state space representation;

$$\begin{cases} \dot{x} = p[y - x - f(x)] \\ \dot{y} = x - y + z \\ \dot{z} = -qy \end{cases} \tag{1}$$

where

$$\begin{aligned} & f(x) = bx + 0.5(a-b)(|x+E| - |x-E|) \\ & a < b < 0,\ 0 < E < \infty \end{aligned} \tag{2}$$

and a, b, p and q are constant parameters. When $a = -1.27, b = -0.68, p = 10, q = 14.87$ and $E = 1$ this system generates chaotic attractor [3].

Lets consider system (3) as the master and system (4) as the slave system.

Master system;

$$\begin{cases} \dot{x}_1 = p[y_1 - x_1 - f(x_1)] \\ \dot{y}_1 = x_1 - y_1 + z_1 \\ \dot{z}_1 = -qy_1 \end{cases} \tag{3}$$

Slave system;

$$\begin{cases} \dot{x}_2 = p[y_2 - x_2 - f(x_2)] + u_1 \\ \dot{y}_2 = x_2 - y_2 + z_2 + u_2 \\ \dot{z}_2 = -qy_2 + u_3 \end{cases} \tag{4}$$

where u_1, u_2 and u_3 are controllers to be determined. Let the state error signals be;

$$\begin{cases} e_x = x_1 - x_2 \\ e_y = y_1 - y_2 \\ e_z = z_1 - z_2 \end{cases} \tag{5}$$

So the error dynamics of the systems (3) and (4) would be as;

$$\begin{cases} \dot{e}_x = pe_y - pe_x - p[f(x_1) - f(x_2)] - u_1 \\ \dot{e}_y = e_x - e_y + e_z - u_2 \\ \dot{e}_z = -qe_y - u_3 \end{cases} \tag{6}$$

***Theorem*:** The slave system (4) can be synchronized with the master system (3) using the adaptive rule $u_2 = u_3 = 0, u_1 = ke_x$ where $\dot{k} = Ae_x^2, A > 0$.

Proof: The error dynamics are;

$$\begin{cases} \dot{e}_x = pe_y - pe_x - p[f(x_1) - f(x_2)] - ke_x \\ \dot{e}_y = e_x - e_y + e_z, \quad \dot{e}_z = -qe_y \end{cases} \tag{7}$$

Consider the Lyapanov candidate function of the form;

$$V(e) = qe_x^2 + pqe_y^2 + pe_z^2 - ne_y e_z + D(k - \hat{k})^2 \tag{8}$$

where $D > 0$ and $\hat{k}$ are constants. Define adaptive parameter error as;

$$e_k = k - \hat{k} \ . \tag{9}$$

Then one can write (8) in compact form as;

$$V(e) = e^T G e \tag{10}$$

$$e = [e_x \ e_y \ e_z \ e_k]^T \tag{11}$$

$$G = \begin{bmatrix} q & 0 & 0 & 0 \\ 0 & pq & -n/2 & 0 \\ 0 & -n/2 & p & 0 \\ 0 & 0 & 0 & D \end{bmatrix} \tag{12}$$

G is positive definite when the followings are satisfied.

$$q > 0, \ pq > 0, \ p^2 q - n^2/4 > 0 \tag{13}$$

It is easy to show that the last condition is satisfied when $0 < n < \sqrt{4p^2 q}$. When G is positive definite then $e^T G e > 0$ and therefore $V > 0$.
Derivate of V along the system (6) dynamics is as follows.

$$\begin{aligned} dV/dt &= 2qe_x\dot{e}_x + 2pqe_y\dot{e}_y + 2pe_z\dot{e}_z - ne_y\dot{e}_z - ne_z\dot{e}_y + D(k-\hat{k})\dot{k} \\ &= e_x\{(-2pq - 2qk + AD(k-\hat{k}))e_x - 2pq[f(x_d) - f(x_r)]\} \\ &\quad + 4pqe_xe_y - 2pqe_y^2 - ne_xe_z + ne_ye_z - ne_z^2 + nqe_y^2 \end{aligned} \tag{14}$$

It is easy to find that;

$$f(x_d) - f(x_r) = k_e(x_d, x_r)e_x, \ a \le k_e \le b \le 0$$

Let $AD = 2q$, $L = q(pa + \hat{k})$

So we have;

$$\begin{aligned} dV/dt &\le -2q(pa + \hat{k} + p)e_x^2 + 4pqe_xe_y + (nq - 2pq)e_y^2 \\ &\quad - ne_xe_z + ne_ye_z - ne_z^2 = [e_x \ e_y \ e_z]Q[e_x \ e_y \ e_z]^T \end{aligned} \tag{15}$$

$$Q = \begin{bmatrix} -2L - 2pq & 2pq & -n/2 \\ 2pq & -2pq + nq & n/2 \\ -n/2 & n/2 & -n \end{bmatrix} \tag{16}$$

Q is negative definite if and only if the following conditions are hold.

$$\begin{gathered} -2L - 2pq < 0 \\ \begin{vmatrix} -2L - 2pq & 2pq \\ 2pq & -2pq + nq \end{vmatrix} > 0 \\ \begin{vmatrix} -2L - 2pq & 2pq & -n/2 \\ 2pq & -2pq + nq & n/2 \\ -n/2 & n/2 & -n \end{vmatrix} < 0 \end{gathered} \tag{17}$$

The above three conditions can be replaced by the following two conditions [13].

$$\begin{bmatrix} -2L - 2pq & 2pq \\ 2pq & -2pq + nq \end{bmatrix} < 0 \tag{18}$$

and

$$\begin{aligned} &\begin{bmatrix} -2L - 2pq & 2pq \\ 2pq & -2pq + nq \end{bmatrix} - \frac{-1}{n}\begin{bmatrix} -n/2 \\ n/2 \end{bmatrix}\begin{bmatrix} -n/2 & n/2 \end{bmatrix} \\ &= \begin{bmatrix} -2L - 2pq + n/4 & 2pq - n/4 \\ 2pq - n/4 & -2pq + nq + n/4 \end{bmatrix} < 0 \end{aligned} \tag{19}$$

Inequality (18) is satisfied when;

$$0 < n < 2p \tag{20}$$

$$(-2L - 2pq)(-2pq + \eta q) > 4p^2 q^2 \tag{21}$$

So we have;

$$0 < n < \frac{2Lp}{L + pq} \tag{22}$$

Furthermore inequality (19) is satisfied when;

$$0 < n < 8L + 8pq, \quad 0 < n < \frac{2pq}{q + 1/4} \tag{23}$$

$$(-2L - 2pq + n/4)(-2pq + n(q + 1/4)) - (2pq - n/4)^2 > 0 \tag{24}$$

So we conclude that:

$$\begin{aligned} &0.25(q + 1/4)n^2 + [(-2L - 2pq)(q + 1/4) - pq/2]n \\ &\quad + 4p^2q^2 + 4pqL - n^2/16 + pqn - 4p^2q^2 \\ &\quad = \frac{q}{4}n^2 + [(-2L - 2pq)q - L/2]n + 4pqL \\ &\quad = \alpha n^2 + \beta n + \gamma > 0 \end{aligned} \tag{25}$$

is hold in which the following definitions were employed.

$$\begin{gathered} \alpha = q/4 > 0 \\ \beta = (-2L - 2pq)q - L/2 < 0 \\ \gamma = 4pqL > 0 \end{gathered} \tag{26}$$

When $\beta^2 \leq 4\alpha\gamma$ then any positive number n is a solution of (25). Hence, n can be selected as;

$$0 < n < \min\left\{\sqrt{4p^2q}, 2p, \frac{2Lp}{L+pq}, 8L+8pq, \frac{2pq}{q+1/4}\right\} \qquad (27)$$

When $\beta^2 > 4\alpha\gamma$ we let

$$n_1 = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \qquad (28)$$

Then any n can satisfy above inequality considering the following condition.

$$0<n<\min\left\{\sqrt{4p^2q}, 2p, \frac{2Lp}{L+pq}, 8L+8pq, \frac{2pq}{q+1/4}, n_1\right\} \qquad (29)$$

By this selection Q would be negative definite. In conclusion, we can always choose $0 < n \leq 1$ such that G be positive definite and Q be negative definite. Hence the system (3) and (4) can be adaptively synchronized.

III. Simulation Results

To verify the effectiveness of the proposed synchronizing controller we illustrate a numerical simulation. We use two identical Chua's circuits with parameters:

$$a = -1.27, b = -1.68, \ p = 10.0, \ q = 14.87, \ E = 1$$

The initial values for the master system are $x_{10} = 0.5$, $y_{10} = 0.5, z_{10} = -0.5$ and for the slave system are $x_{20} = 0.1$, $y_{20} = 0.1, z_{20} = -0.1$ respectively. The control parameter is initiated as $k_0 = 0.5$. The trajectory of the master system's states is shown in Fig. 1.

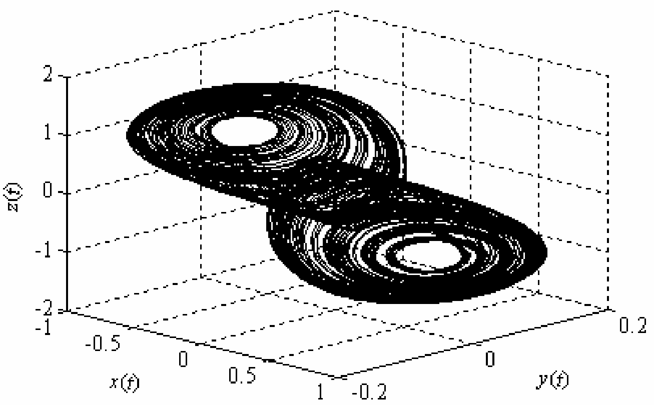

Fig. 1 The chaotic attractor of the Chua's circuit

The time responses for the state adaptive synchronization errors are shown in the Figs. 2, 3, 4. It is easily seen that the errors are diminished after few seconds.

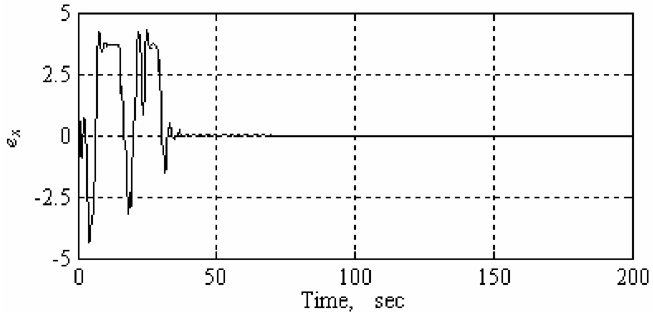

Fig. 2 The synchronization error for state x

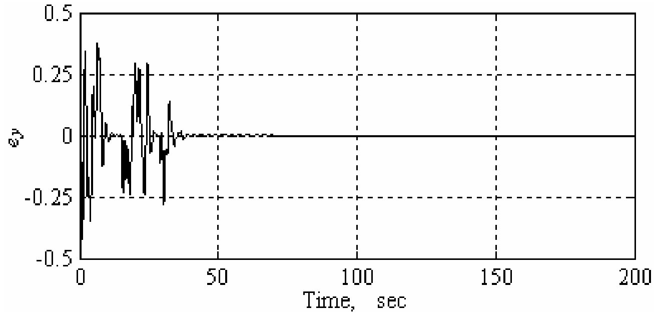

Fig. 3 The synchronization error for state y

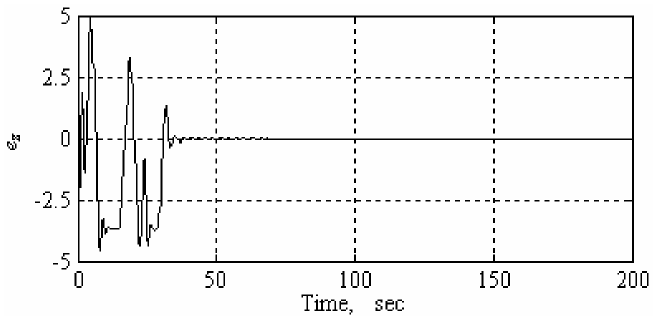

Fig. 4 The synchronization error for state z

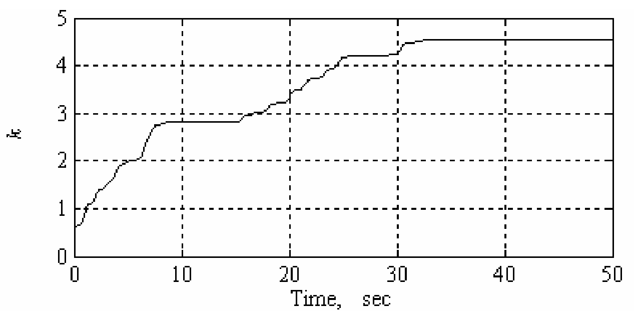

Fig. 5 Parameter k with initial condition $k_0 = 0.5$

The parameter adaptation is shown in Fig. 5. Figs. 6 and 7 indicate effect of the parameter initial condition on the synchronization time. The final value of the parameter is also affected.

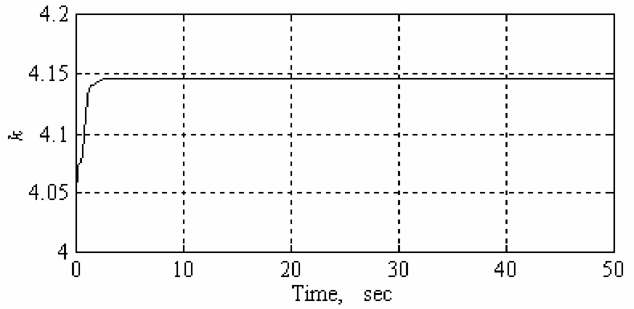

Fig. 6 Parameter k with initial condition $k_0 = 4$

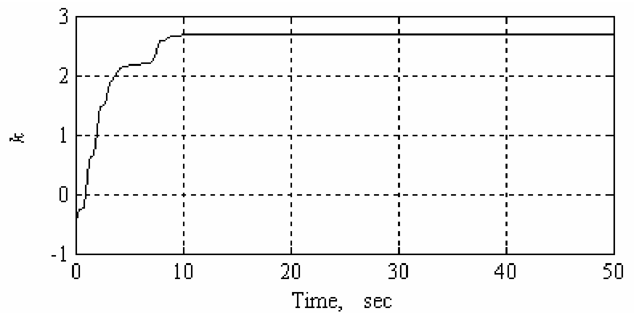

Fig. 7 Parameter k with initial condition $k_0 = -0.5$

IV. Conclusion

In this paper we have introduced a new adaptive scheme to synchronize two Chua's circuit. The numerical simulations were presented to show the effectiveness of the proposed method. We have also shown that the synchronizing delay time can be decreased by proper choice of the initial condition of the control parameter.

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