

# Mathematical Modeling of the Influence of Hydrothermal Processes in the Water Reservoir

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**Abstract**—In this paper presents the mathematical model of hydrothermal processes in thermal power plant with different wind direction scenarios in the water reservoir, which is solved by the Navier - Stokes and temperature equations for an incompressible fluid in a stratified medium. Numerical algorithm based on the method of splitting by physical parameters. Three dimensional Poisson equation is solved with Fourier method by combination of tridiagonal matrix method (Thomas algorithm).

**Keywords**—thermal power plant, hydrothermal process, large eddy simulation, water reservoir

## I. INTRODUCTION

NOW hydrophysical problems associated with the disposal in ponds and watercourse of heated water in the operation of thermal and nuclear power plants (TPPs and NPPs) becomes important.

To address emerging resetting the heated water hydrologic engineering and environmental problems need to be able to predict and control the temperature of the water and impurity content in the reservoirs and rivers. To study the processes occurring in the reservoir requires a comprehensive study of the problem: field measurements and mathematical modeling of hydrothermal processes in the pond. To construct the mathematical models should be considered the main characteristic feature of flow in the water - turbulent fluid motion. It is known that the turbulence is observed in practically all flows, irrespective of whether they occur in natural conditions or technological installations. An example of the latter may be questions related to the supply and discharge of water for thermal power plants.

Both thermal and nuclear power plants require reservoirs. Production of electricity worldwide is growing, and is doubling every 7-10 years. Over 80% of world electricity is generated by thermal and nuclear power plants. To operate these plants require large amounts of water for cooling units at an average of 40-50 cubic meters per second to 1000 MW of installed capacity. Hence it is evident that for thermal power plants of 2000-4000 MW is required 80-200 cubic meters of water per second. Therefore, when deciding where to build thermal power plants and nuclear power becomes essential questions of their water supply. Naturally, the large thermal power stations should be located on the banks of large rivers, ponds and lakes or artificial reservoirs.

The creation of artificial reservoirs requires a lot of money, so the thermal power plants tend to have on existing reservoirs or lakes. The impact of thermal and nuclear power plants in the hydrological and biological regime of the reservoir varied. Most importantly in the thermal pollution is reaching the superheat water to 30-35 degrees. This increases water temperatures adversely affect the hydrobiological state of self-purification processes as water reservoirs.

Industrial facilities often located on the shores of reservoirs and lakes thrown together with warm water and waste in the form of impurities. For cooling thermal power plants and industrial facilities are used 10% water, and the issues of optimal and efficient use of the reservoir to cool the plants become very relevant. In solving these problems need to be able to predict and control the temperature of water in the reservoir. The distribution of temperature influenced not only by processes of heat and mass transfer, but also by the stratification. Stratification appears in connection with the difference of temperature of water discharged to the ambient temperature in the reservoir, or the presence of impurities in the discharged water. For example, warm water is lighter, so it is in the form of jets or standing stretches near the free surface. Stable density stratification of the water helps to reduce the turbulent exchange between the vertical layers of liquids, especially in the density jump. In general, hydrothermal regime of the reservoir is formed under the influence of uncontrollable natural factors (solar and atmospheric radiation, wind, convective heat transfer, evaporation) and the factors that are amenable to regulation (amount and temperature of the discharged water, the presence of impurities, a selective sampling, etc.).

In recent years, put a hard limit protect the environment. According to the designer usually has to follow the rules, which limit the maximum size of the "zone transfer" of hot water fault, so that they do not exceed half the width of the river and took no more than half of the total cross-sectional area and flow. Non-compliance with these rules may lead to short time or long time stopping power and, therefore, the accuracy requirements to the structural analysis are very strict. In fact arising under this hydrodynamic problem can be described as a fully three-dimensional, with irregular borders, with the presence of mass and buoyancy forces from the main flow velocities, which can vary by an order, sometimes so fast that the important role played by the effects of nonstationarity.

In addition, under certain combinations of conditions, when the heated water is practically fault drawn into the upstream region of the cooling water intake, there are extensive areas of recycling. The result may be a significant loss of overall operating efficiency of the system.

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From the above it follows that the construction of a theoretical model that adequately the real processes occurring in the water - cooler is quite a challenge.

Thermal power plants are divided into the condensing (CPS) which are intended to provide only an electric energy, and combined heat and power (TPC) which are producing heat in the form of hot water and steam in addition to electrical power. Large CPSs with subordination are called State district power plants (SDPP).

Ekibastuz SDPP -1 is taken as an example of such effects of the TPP to the aquatic environment, located in Pavlodar region in 17 km to the North-East of the city Ekibastuz, Kazakhstan.

Technical SDPP -1 was carried out on the back of the circuit with circulating cooling water. The surface of the reservoir is located at 158.5 m, the area is 19.6 square meters km, the maximum size of  $4 \times 6$  km, average depth 4.6 m, maximum depth of 8.5 m at the water intake, the volume of the reservoir is 80 million cubic meters meter, in the reservoir is used selective intake and spillway of the combined type. Waste water enters the pre-channel mixer, then through the filtration dam is uniformly supplied to the cooling pond. Water intake is at a distance of 40 meters from the dam at a depth of 5 meter. Project water flow 120 cubic meters per second, and the actual consumption varies depending on the mode of TPP within the 80-120 cubic meters per second.

## II. MATHEMATICAL MODEL

In reservoirs - coolers spatial temperature variation is small (generally less than  $25^{\circ}C$ ). The corresponding change in density is much smaller than the magnitude of the density of water. Therefore, stratified flow in a reservoir - the cooler can be described by the equations in the Boussinesq approximation. In view of the above, the starting point for describing the laminar flow are the Navier - Stokes equations and the conservation of energy:

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial}{\partial x_j} \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) + \beta g_i (T - T_0) - \frac{\partial \tau_{ij}}{\partial x_j} \quad (1)$$

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad (i=1,2,3). \quad (2)$$

$$\frac{\partial T}{\partial t} + \frac{\partial \bar{u}_j T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \chi \frac{\partial T}{\partial x_j} \right) \quad (3)$$

$$\text{where } \tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \quad (4)$$

$g_i$  - is the gravity acceleration,  $\beta$  - the coefficient of volume expansion,  $u_i$  - velocity components,  $\chi$  - thermal diffusivity coefficient,  $T_0$  - the equilibrium temperature,  $T$  - deviation of temperature from the balance.

In writing equations was assumed that the medium is incompressible, constant physical properties except for the temperature, the volumetric flow of heat due to absorption of solar radiation in the water column is neglected.

As for constructing model of turbulence we used dynamic model of Smagorinsky, the following is the underlying principle of the dynamic model for extracting information concerning a given eddy-viscosity model via a double filtering in physical space. It is worth to admit that the most of the historical developments have been done with Smagorinsky's model.

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2\nu_{sgs} \bar{s}_{ij}$$

$$\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases} \quad \text{Kroneker symbol}$$

$$\text{where } \nu_{sgs} = (C_s \Delta)^2 \sqrt{2s_{ij} s_{ij}},$$

$$\bar{s}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right), \quad \Delta = (\Delta x \Delta y \Delta z)^{1/3}$$

$$C_s = \frac{1}{\pi} \left( \frac{3C_k}{2} \right)^{-3/4}, \quad C_s = 0.18 \quad \text{for a Kolmogorov constant of 1,4.}$$

But the dynamic procedure applies in fact to the types of eddy viscosities such as those used in the structure-function model.

We start with regular LES corresponding to a "bar-filter" of width  $\Delta x$ , an operator associating an function  $\bar{f}(x, t)$ . We then define a second "test filter" tilde of large width  $2\Delta x$  associating  $\tilde{f}(x, t)$ . Let us first apply this filter product to the Navier-Stokes equation. The subgrid-scale tensor of the field  $\tilde{u}_i$  is obtained from equation (4) with the replacement of the filter bar by the double filter and tilde filter:

$$\tau_{ij} = \tilde{\overline{u_i u_j}} - \tilde{u}_i \tilde{u}_j \quad (5)$$

$$l_{ij} = \tilde{\overline{u_i u_j}} - \tilde{u}_i \tilde{u}_j \quad (6)$$

We now apply the tilde filter to equation (4), which leads to

$$\tilde{\tau}_{ij} = \tilde{\overline{u_i u_j}} - \tilde{u}_i \tilde{u}_j \quad (7)$$

Adding equations (6) and (7) and using equation (5), we obtain

$$l_{ij} = \tau_{ij} - \tilde{\tau}_{ij}$$

We use Smagorinsky model expression for the subgrid stresses related to the bar filter and tilde-filter it to get

$$\tilde{\tau}_{ij} - \frac{1}{3} \delta_{ij} \tilde{\tau}_{kk} = -2C \tilde{A}_{ij} \quad \text{where } A_{ij} = (\Delta x)^2 \left| \overline{S_{ij}} \right| \quad (8)$$

We now have to determine  $\tau_{ij}$ , the stress resulting from the filter product. This is again obtained using the Smagorinsky model, which yields

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2CB_{ij} \text{ where } B_{ij} = (2\Delta x)^2 \left| \bar{S} \right| \bar{S}_{ij} \quad (9)$$

Subtracting (8) from (9) yields with the aid of Germano's identity

$$l_{ij} - \frac{1}{3} \delta_{ij} l_{kk} = 2CB_{ij} - 2C \bar{A}_{ij}$$

$$l_{ij} - \frac{1}{3} \delta_{ij} l_{kk} = 2CM_{ij}$$

$$\text{where } M_{ij} = B_{ij} - \bar{A}_{ij} \quad (10)$$

All the terms of equation (10) may now be determined with the aid of  $\bar{u}$ . Unfortunately, there are five independent equations for only one variable C, and thus the problem is overdetermined. A first solution proposed by Germano is to multiply (10) tensorially by  $\bar{S}_{ij}$  to get

$$C = \frac{1}{2} \frac{l_{ij} \bar{S}_{ij}}{M_{ij} \bar{S}_{ij}}$$

This provides finally dynamical evaluation of C, which can be used in the LES of the bar field  $\bar{u}$  [6,8].

Initial and boundary conditions are defined for the non-stationary 3D equations of motion, continuity and temperature, satisfying the equations.

### III. NUMERICAL ALGORITHM

Numerical solution of (1) - (3) is carried out on the posted grid using the scheme against a stream of the second type

$$\frac{\partial u \zeta}{\partial x} = \frac{u_R \zeta_R - u_L \zeta_L}{\Delta x} \text{ where } \zeta \text{ can be } u, v, w$$

$$u_L = \frac{u_i + u_{i-1}}{2}, \quad u_R = \frac{u_{i+1} + u_i}{2}$$

$$\zeta_L = \begin{cases} \zeta_{i-1}, u_L > 0 \\ \zeta_i, u_L < 0 \end{cases}, \quad \zeta_R = \begin{cases} \zeta_i, u_R > 0 \\ \zeta_{i+1}, u_R < 0 \end{cases}$$

and compact approximation for convective terms [1,2,3,4,7].

Scheme of splitting on physical parameters is used to solve the problem in view of the above with the proposed model of turbulence [5]. It is anticipated that at the first stage the transfer of momentum occurs only through convection and diffusion. Intermediate field of speed is handled by using method of fractional steps through the tridiagonal matrix method (Thomas algorithm). In the second phase is for pressure which is found by the help of intermediate field of speed. Poisson equation for pressure is solved by Fourier method in combination with the tridiagonal matrix method (Thomas algorithm) that is applied to determine the Fourier coefficients [10,11].

At the third stage, it is supposed that the transfer is carried out only by the pressure gradient. The algorithm was parallelized on the high-performance system. [9,10]

$$I) \frac{\bar{u}^* - \bar{u}^n}{\tau} = -(\nabla \bar{u}^n \bar{u}^* - \nu \Delta \bar{u}^*)$$

$$II) \Delta p = \frac{\nabla \bar{u}^*}{\tau}$$

$$III) \frac{\bar{u}^{n+1} - \bar{u}^*}{\tau} = -\nabla p.$$

### IV. RESULTS OF COMPUTATIONAL MODELING

For modeling of hydrothermal processes in the reservoir with stratified force was used LES method. Using this method allows to improve the accuracy and informative description of turbulent flows.

For a detailed study of hydrothermal processes are solved two problems. The first problem deals with modeling of hydrothermal processes in the west wind, and the second problem is devoted to modeling hydrothermal processes in the north-west wind. Calculations were carried out in a rectangular area with dimensions 4 km in both horizontal directions. Initial and boundary conditions were given.

Fig. 1 shows the contour and the isolines of the temperature distribution at different time period after the start of the SDPP -1, on the water surface at the western wind, the top view. Fig. 2 shows the spatial contour and isolines distribution of temperature contours at different time period after the start of the SDPP -1, on the water surface at the western wind, with different side views.

Fig. 3 and 4 show the contour and isolines on the surface of the water in the north-west wind and the spatial contour and isolines of temperature distribution at different time period after the start of the SDPP -1.

In all the figures show that the temperature distribution with distance from the flow close to isothermal. The results show that the temperature distribution is distributed over a larger area, depending on wind direction. Temperature distribution has elongated shape.

The resulting temperature distribution widens in the horizontal directions, so if do not take any action the temperature will raise in the reservoir by time period.

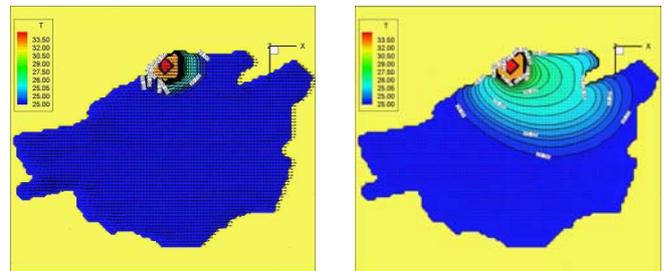


Fig. 1 Contour and isolines of temperature distribution at 12 and 24 h after the start of the SDPP-1, on the water surface at the western wind, the top view

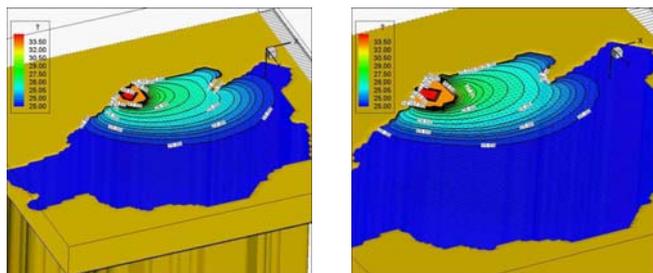


Fig. 2 Contour and isolines of temperature distribution at 12 and 24 h after the start of the SDPP-1, on the water surface at the western wind, with different side views

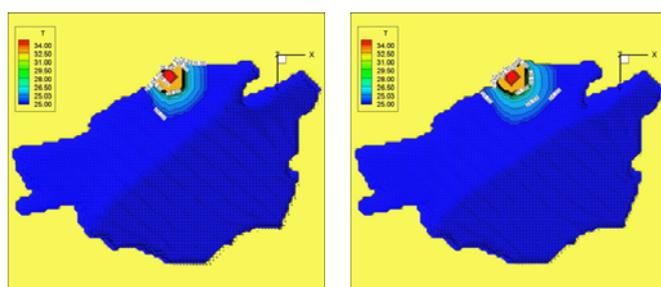


Fig. 3 Contour and isolines of temperature distribution at 8 and 12 h after the start of the SDPP-1, on the surface of water at the north-west wind, the top view

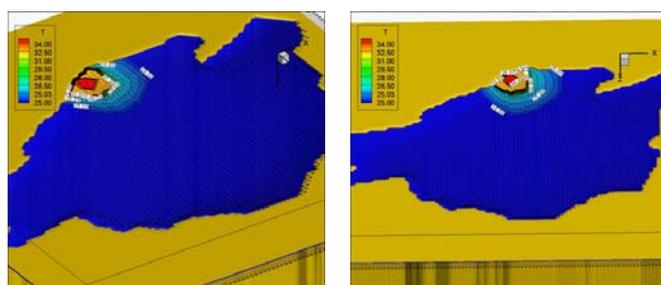


Fig. 4 Contour and isolines of temperature distribution at 8 and 12 h after the start of the SDPP-1, on the surface of water at the north-west wind, with different side views

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