

# Denoising and Compression in Wavelet Domain via Projection onto Approximation Coefficients

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**Abstract**—We describe a new filtering approach in the wavelet domain for image denoising and compression, based on the projections of details subbands coefficients (resultants of the splitting procedure, typical in wavelet domain) onto the approximation subband coefficients (much less noisy). The new algorithm is called Projection Onto Approximation Coefficients (POAC). As a result of this approach, only the approximation subband coefficients and three scalars are stored and/or transmitted to the channel. Besides, with the elimination of the details subbands coefficients, we obtain a bigger compression rate. Experimental results demonstrate that our approach compares favorably to more typical methods of denoising and compression in wavelet domain.

**Keywords**—Compression, denoising, projections, wavelets.

## I. INTRODUCTION

AN image is affected by noise in its acquisition and processing. The denoising techniques are used to remove the additive noise while retaining as much as possible the important image features. In the recent years there has been an important amount of research on wavelet thresholding and threshold selection for images denoising [1]-[51], because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficient due to important signal features [1]-[3]. These small coefficients can be thresholded without affecting the significant features of the image.

In fact, the thresholding technique is the last approach based on wavelet theory to provide an enhanced approach for eliminating such noise source [4], [5] and ensure better image quality [6], [7]. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its basic form, each coefficient is thresholded by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the essential

signal characteristics and with less noise. Since the work of Donoho & Johnstone [3], there has been much research on finding thresholds, however few are specifically designed for images [14]-[51].

Unfortunately, this technique has the following disadvantages:

- 1) it depends on the correct election of the type of thresholding, e.g., OracleShrink, VisuShrink (soft-thresholding, hard-thresholding, and semi-soft-thresholding), Sure Shrink, Bayesian soft thresholding, Bayesian MMSE estimation, Thresholding Neural Network (TNN), due to Zhang, NormalShrink, , etc. [1]-[5], [8]-[38]
- 2) it depends on the correct estimation of the threshold which is arguably the most important design parameter,
- 3) it doesn't have a fine adjustment of the threshold after their calculation,
- 4) it should be applied at each level of decomposition, needing several levels, and
- 5) the specific distributions of the signal and noise may not be well matched at different scales.

Therefore, a new method without these constraints will represent an upgrade. On the other hand, similar considerations should be kept in mind regarding the problem of image compression based on wavelet thresholding.

The Bidimensional Discrete Wavelet Transform and the method to reduce noise and to compress by wavelet thresholding is outlined in Section II. The new approach as denoiser and compression tools in wavelet domain is outlined in Section III. In Section IV, we discuss briefly the more appropriate metrics for denoising and compression. In Section V, the experimental results using the proposed algorithm are presented. Finally, Section VI provides a conclusion of the paper.

## II. BIDIMENSIONAL DISCRETE WAVELET TRANSFORM

The Bidimensional Discrete Wavelet Transform (DWT-2D) [6]-[7], [12]-[51] corresponds to multiresolution approximation expressions. In practice, multiresolution analysis is carried out using 4 channel filter banks (for each level of decomposition) composed of a low-pass and a high-pass filter and each filter bank is then sampled at a half rate (1/2 down sampling) of the previous frequency. By repeating this procedure, it is possible to obtain wavelet transform of any order. The down sampling procedure keeps the scaling parameter constant

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(equal to  $\frac{1}{2}$ ) throughout successive wavelet transforms so that is benefits for simple computer implementation. In the case of an image, the filtering is implemented in a separable way by filtering the lines and columns.

Note that [6], [7] the DWT of an image consists of four frequency channels for each level of decomposition. For example, for  $i$ -level of decomposition we have:

- LL<sub>n,i</sub>: Noisy Coefficients of Approximation.
- LH<sub>n,i</sub>: Noisy Coefficients of Vertical Detail,
- HL<sub>n,i</sub>: Noisy Coefficients of Horizontal Detail, and
- HH<sub>n,i</sub>: Noisy Coefficients of Diagonal Detail.

The LL part at each scale is decomposed recursively, as illustrated in Fig. 1 [6], [7].

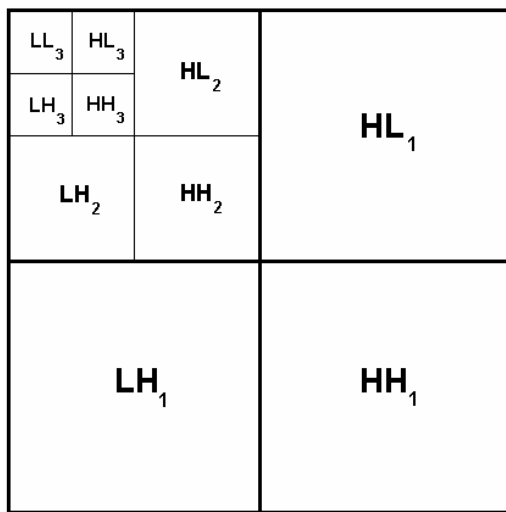


Fig. 1 Data preparation of the image. Recursive decomposition of LL parts

To achieve space-scale adaptive noise reduction, we need to prepare the 1-D coefficient data stream which contains the space-scale information of 2-D images. This is somewhat similar to the “zigzag” arrangement of the DCT (Discrete Cosine Transform) coefficients in image coding applications [42]. In this data preparation step, the DWT-2D coefficients are rearranged as a 1-D coefficient series in spatial order so that the adjacent samples represent the same local areas in the original image [44].

Figure 2 shows the interior of the DWT-2D with the four subbands of the transformed image [51], which will be used in Fig.3. Each output of Fig. 2 represents a subband of splitting process of the 2-D coefficient matrix corresponding to Fig. 1.

#### A. Wavelet Noise Thresholding

The wavelet coefficients calculated by a wavelet transform represent change in the image at a particular resolution. By looking at the image in various resolutions it should be possible to filter out noise, at least in theory. However, the defini-

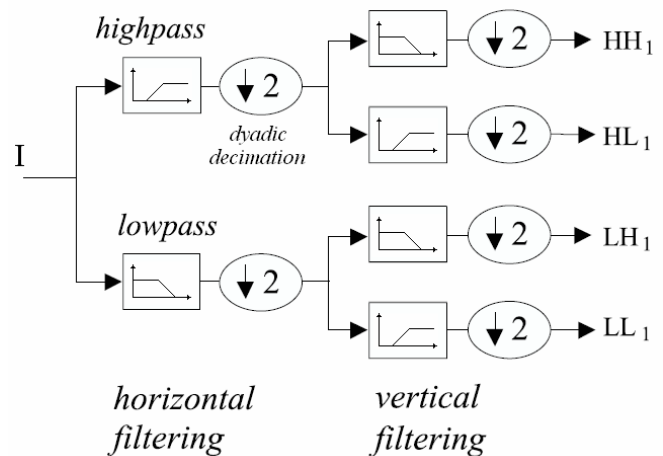


Fig. 2 Two dimensional DWT. A decomposition step. Usual splitting of the subbands.

tion of noise is a difficult one. In fact, "one person's noise is another's signal". In part this depends on the resolution one is looking at. One algorithm to remove Gaussian white noise is summarized by D.L. Donoho and I.M. Johnstone [2], [3], and synthesized in Fig. 3.

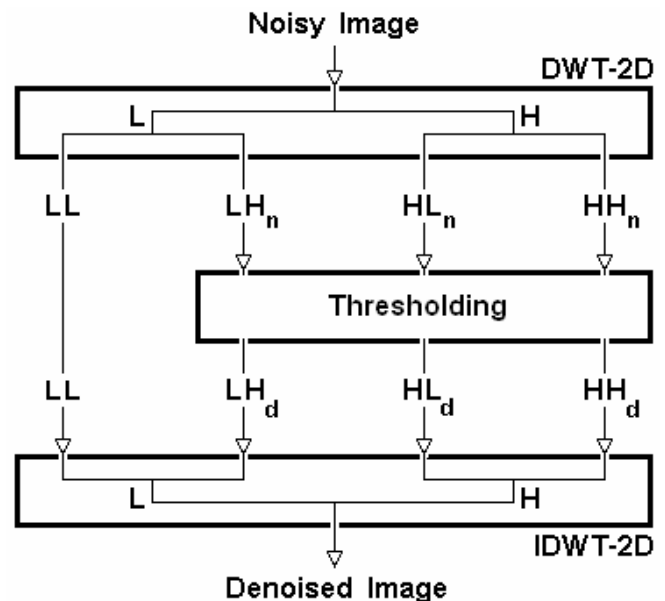


Fig. 3 Thresholding Techniques

The algorithm is:

- 1) Calculate a wavelet transform and order the coefficients by increasing frequency. This will result in an array containing the image average plus a set of coefficients of length 1, 2, 4, 8, etc. The noise threshold will be calculated on the highest frequency coefficient spectrum (this is the largest spectrum).
- 2) Calculate the *median absolute deviation* (mad) on the largest coefficient spectrum. The median is calculated from the absolute value of the coefficients. The equation for the median absolute deviation is shown below:

$$\delta_{mad} = \frac{\text{median}(|C_{n,i}|)}{0.6745} \quad (1)$$

where  $C_{n,i}$  may be  $LH_{n,i}$ ,  $HL_{n,i}$ , or  $HH_{n,i}$  for  $i$ -level of decomposition. The factor 0.6745 in the denominator rescales the numerator so that  $\delta_{mad}$  is also a suitable estimator for the standard deviation for Gaussian white noise [5], [42], [44].

- 3) For calculating the noise threshold  $\lambda$  we have used a modified version of the equation that has been discussed in papers by D.L. Donoho and I.M. Johnstone. The equation is:

$$\lambda = \delta_{mad} \sqrt{2 \log[N]} \quad (2)$$

where  $N$  is the number of pixels in the subimage, i.e., HL, LH or HH.

- 4) Apply a thresholding algorithm to the coefficients. There are two popular versions:

4.1. Hard thresholding. Hard thresholding sets any coefficient less than or equal to the threshold to zero, see Fig. 4(a).

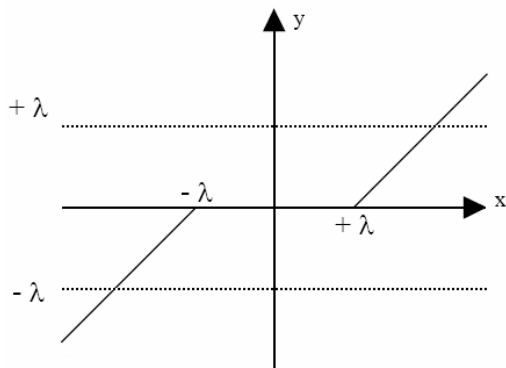


Fig. 4 (a) Soft-Thresholding

where  $x$  may be  $LH_{n,i}$ ,  $HL_{n,i}$ , or  $HH_{n,i}$ ,  $y$  may be  $HH_{d,i}$ :  
 Denoised Coefficients of Diagonal Detail,  
 $HL_{d,i}$ : Denoised Coefficients of Horizontal Detail,  
 $LH_{d,i}$ : Denoised Coefficients of Vertical Detail,  
 for  $i$ -level of decomposition.

The respective code is:

```
for row = 1:N1/2
for column = 1:N1/2
if |Cn,i[row][column]| <= λ,
Cn,i[row][column] = 0.0;
end
end
end
```

4.2. Soft thresholding. Soft thresholding sets any coefficient less than or equal to the threshold to zero, see Fig. 4(b). The threshold is subtracted from any coefficient

that is greater than the threshold. This moves the image coefficients toward zero.

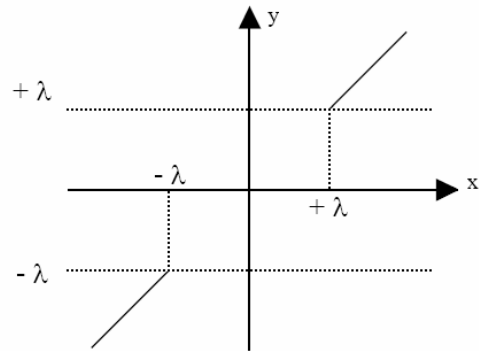


Fig. 4 (b) Hard-Thresholding

The respective code is:

```
for row = 1:N1/2
for column = 1:N1/2
if |Cn,i[row][column]| <= λ,
Cn,i[row][column] = 0.0;
else
Cn,i[row][column] = Cn,i[row][column] - λ;
end
end
end
```

### III. PROJECTION ONTO APPROXIMATION COEFFICIENTS

As a natural consequence of Projection Onto Span Algorithm (POSA), which was introduced by Mastroianni [51], the POAC is based on the Orthogonality Principle too [52]. [53].

#### A. Denoising via POAC inside wavelet domain

In this section, the denoising of an image corrupted by white Gaussian noise will be considered, i.e.,

$$I_n = I + n \quad (3)$$

where  $n$  is independent Gaussian noise. We observe  $I_n$  (a noisy image) and wish to estimate the desired image  $I$  as accurately as possible according to some criteria.

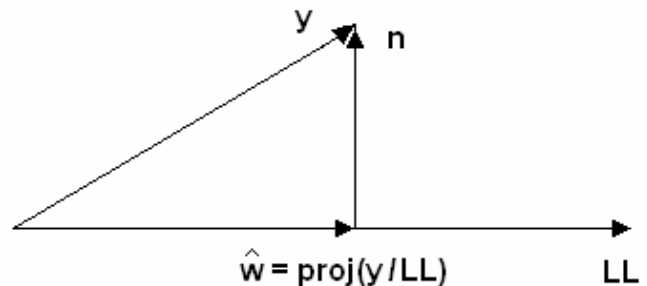


Fig. 5 Orthogonality Principle inside wavelet domain

Inside wavelet domain, if we use an orthogonal wavelet trans-

form, the problem can be formulated as  

$$y = \hat{w} + n \quad (4)$$

where  $y$  noisy wavelet coefficient (LH, HL and HH),  $\hat{w}$  true coefficient, and  $n$  noise, which is independent Gaussian. This is a classical problem in estimation theory [52]. Our aim is to estimate from the noisy observation. A estimator based on the orthogonality principle will be used for this purpose [52], [53].

Such estimators have been widely advocated for image restoration and reconstruction problems [51], [54]. In this particular case, and based on Fig.5, we have

$$\hat{w} = proj(y / LL) \quad (5)$$

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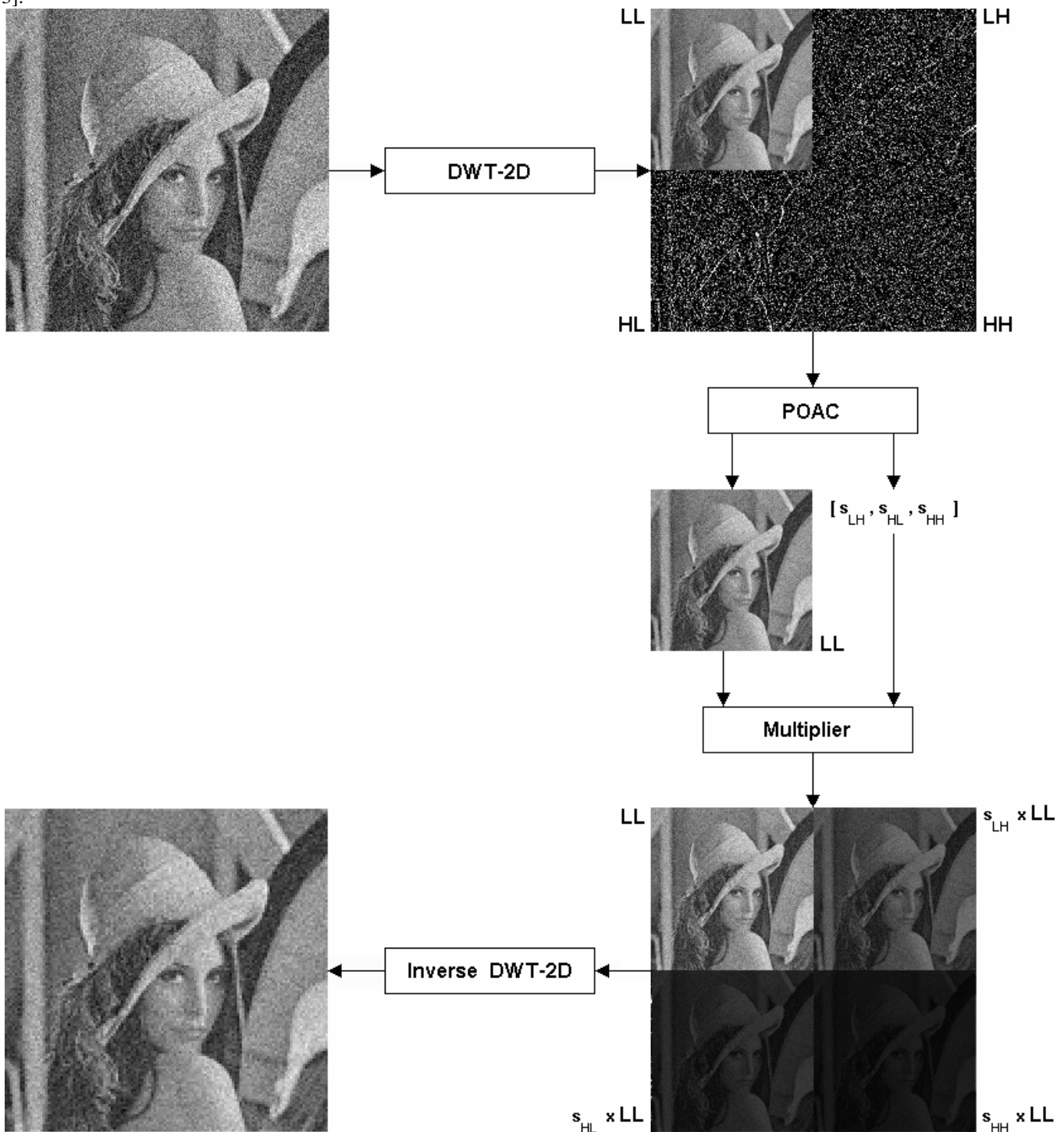


Fig. 6 POAC algorithm as denoiser

That is to say,  $\hat{w}$  is the projection of  $y$  (LH, HL and HH) onto LL. Therefore,

$$\hat{w} = \frac{\text{trace}(LL y^T)}{\text{trace}(LL LL^T)} LL \quad (6)$$

arising three possibilities, i.e.,

$$\hat{w} = s_{LH} LL \quad (7.a)$$

$$\hat{w} = s_{HL} LL \quad (7.b)$$

$$\hat{w} = s_{HH} LL \quad (7.c)$$

where

$$s_{LH} = \frac{\text{trace}(LL LH^T)}{\text{trace}(LL LL^T)} \quad (8.a)$$

$$s_{HL} = \frac{\text{trace}(LL HL^T)}{\text{trace}(LL LL^T)} \quad (8.b)$$

$$s_{HH} = \frac{\text{trace}(LL HH^T)}{\text{trace}(LL LL^T)} \quad (8.c)$$

That is to say, they are three scalars that arising as a consequence of POAC application inside wavelet domain (see Fig.6). This allows generate three new denoised detail coefficient matrices, uncorrelated regarding the noise and correlated with the approximation coefficient matrix LL, the less noisy one of all.

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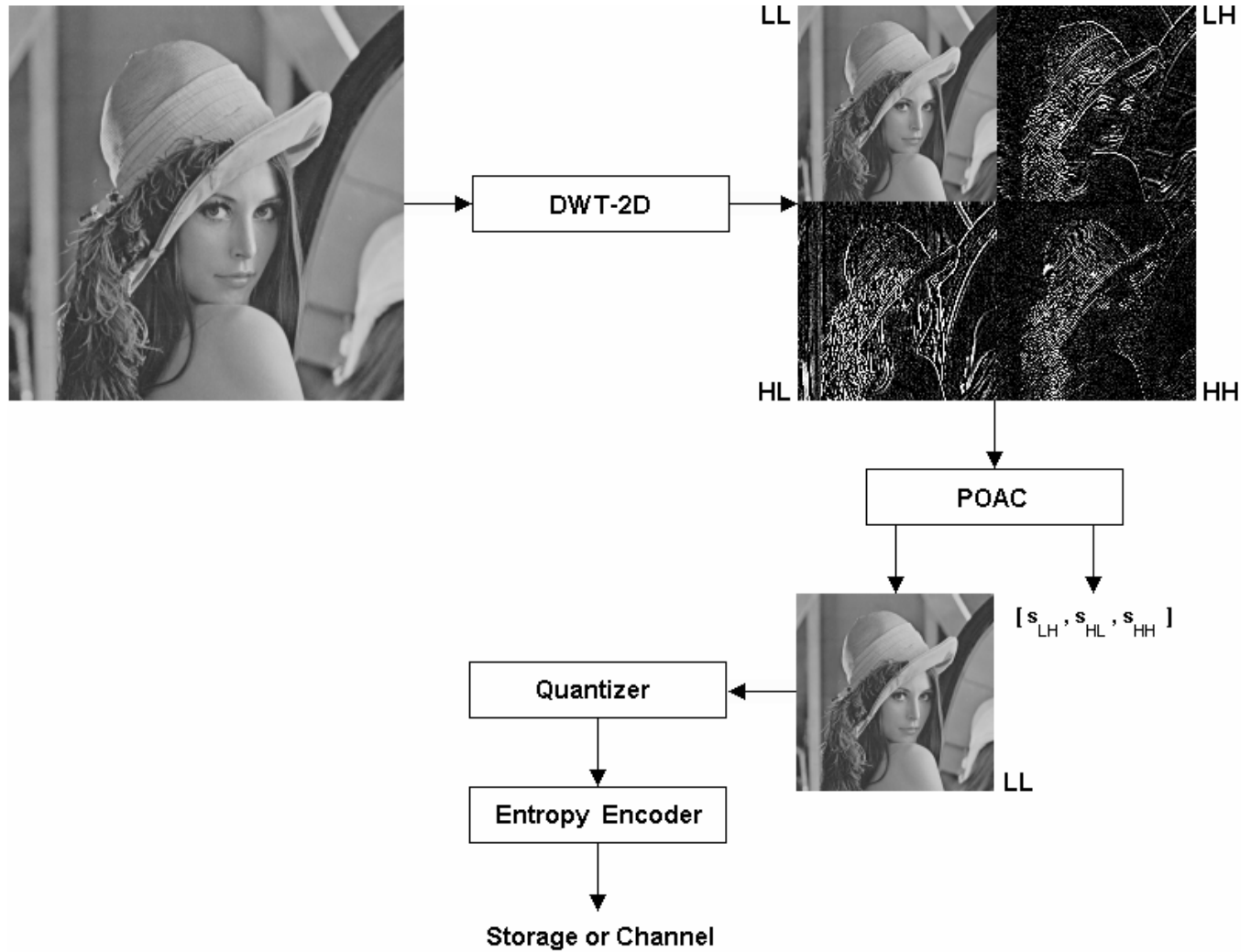


Fig. 7(a) POAC algorithm as compressor. ENCODER

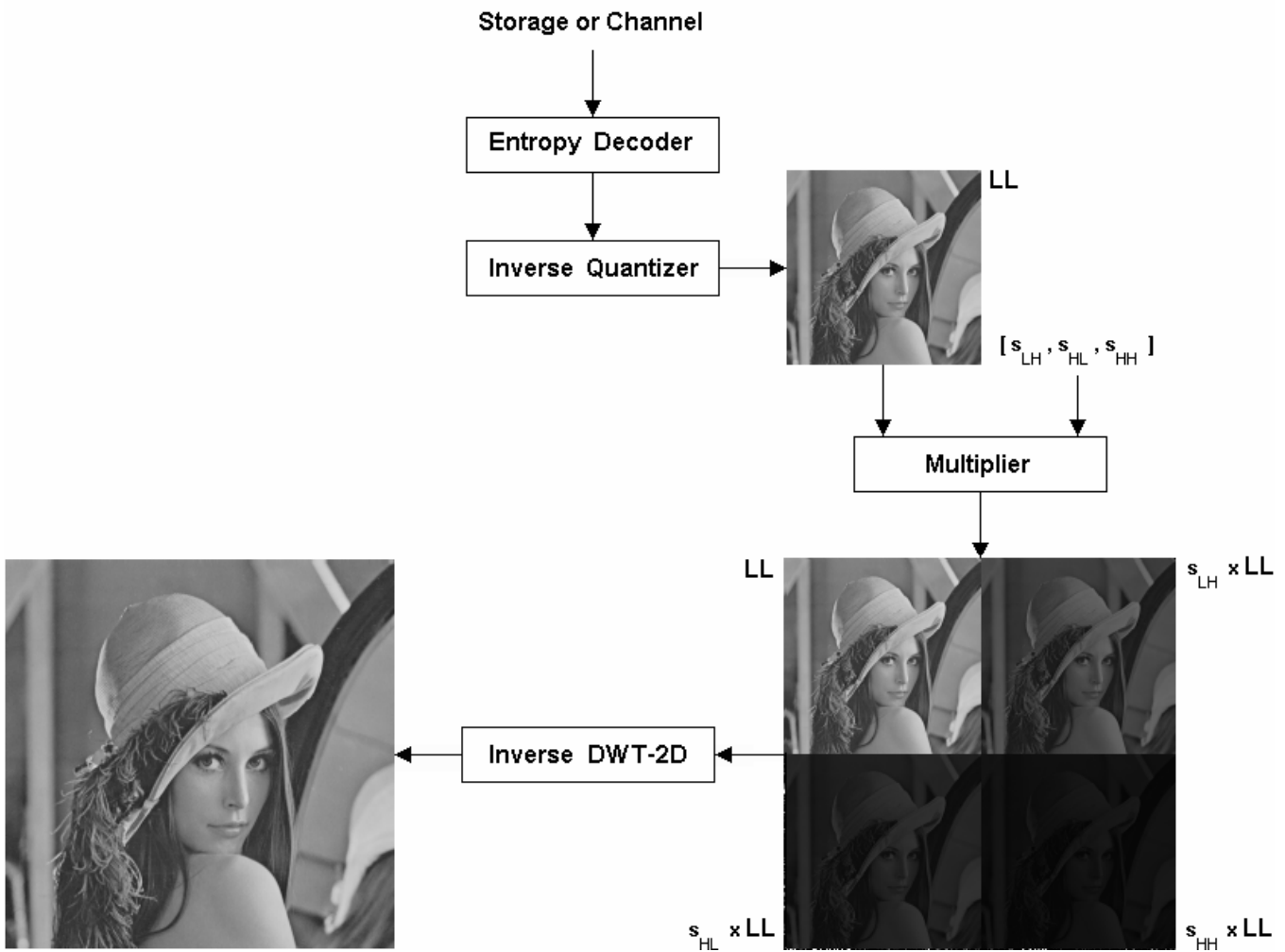


Fig. 7(b) POAC algorithm as compressor. DECODER

**B. Compression thanks to POAC inside wavelet domain**

As we could see in the previous section, the input of POAC inside wavelet domain, are the four subbands, i.e., LL, LH, HL and HH, while its output is the approximation subband LL plus three scalars  $s_{LH}$ ,  $s_{HL}$  and  $s_{HH}$ . This intrinsically, represents a compression approach with a compression rate of 4:1, approximately.

The Figures 7(a) and 7(b) represents the encoder and decoder architecture for compression thanks POAC inside wavelet domain.

**IV. METRICS**

**A. Data Compression Ratio (CR)**

Data compression ratio, also known as compression power, is a computer-science term used to quantify the reduction in data-representation size produced by a data compression algorithm. The data compression ratio is analogous to the

physical compression ratio used to measure physical compression of substances, and is defined in the same way, as the ratio between the *uncompressed size* and the *compressed size* [54]:

$$CR = \frac{\text{Uncompressed Size}}{\text{Compressed Size}} \tag{9}$$

Thus a representation that compresses a 10MB file to 2MB has a compression ratio of  $10/2 = 5$ , often notated as an explicit ratio, 5:1 (read "five to one"), or as an implicit ratio, 5X. Note that this formulation applies equally for compression, where the uncompressed size is that of the original; and for decompression, where the uncompressed size is that of the reproduction.

**B. Percent Space Savings (PSS)**

Sometimes the space savings is given instead, which is defined as the reduction in size relative to the uncompressed



Noisy



Soft-thresholding (ST)



Hard-thresholding (HT)



POAC

Fig. 8 Noisy and filtered images

size:

$$PSS = (1 - \frac{1}{CR}) * 100\% \quad (10)$$

Thus a representation that compresses 10MB file to 2MB would yield a space savings of  $1 - 2/10 = 0.8$ , often notated as a percentage, 80%.

### C. Peak Signal-To-Noise Ratio (PSNR)

The phrase peak signal-to-noise ratio, often abbreviated PSNR, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic deci-bel scale.

The PSNR is most commonly used as a measure of quality of

reconstruction in image compression etc [54]. It is most easily defined via the mean squared error (MSE) which for two  $NR \times NC$  (rows-by-columns) monochrome images  $I$  and  $I_d$ , where the second one of the images is considered a denoised approximation of the other is defined as:

$$MSE = \frac{1}{NR \times NC} \sum_{nr=0}^{NR-1} \sum_{nc=0}^{NC-1} \|I(nr, nc) - I_d(nr, nc)\|^2 \quad (11)$$

The PSNR is defined as [ ]:

$$PSNR = 10 \log_{10} \left( \frac{MAX_I^2}{MSE} \right) = 20 \log_{10} \left( \frac{MAX_I}{\sqrt{MSE}} \right) \quad (12)$$



Fig. 9 Original and compressed images

Here,  $MAX_i$  is the maximum pixel value of the image. When the pixels are represented using 8 bits per sample, this is 255. More generally, when samples are represented using linear pulse code modulation (PCM) with B bits per sample, maximum possible value of  $MAX_i$  is  $2^B - 1$ .

For color images with three red-green-blue (RGB) values per pixel, the definition of PSNR is the same except the MSE is the sum over all squared value differences divided by image size and by three [54].

Typical values for the PSNR in lossy image and video compression are between 30 and 50 dB, where higher is better.

#### V. EXPERIMENTAL RESULTS

The simulations demonstrate that the POAC technique improves the noise reduction and compression performances in wavelet domain to the maximum.

Here, we present a set of experimental results using one typical image. Such images were converted to bitmap file format for their treatment [54]. Figure 8 shows the noisy (Gaussian white noise, with mean value = 0, and standard deviation = 0.01) and filtered images, with 256-by-256 (pixel) by 256 (gray levels) bitmap matrix. Table I summarizes the assessment parameters vs. filtering techniques for Fig.8, where ST means Soft-Thresholding and HT means Hard-Thresholding. On the other hand, Fig.9 shows the original and compressed /decompressed images via ST, HT and POAC techniques. Table II summarizes the assessment parameters vs. compressed techniques for Fig.9. The quality is similar with very different CR and PSS.

TABLE I  
 ORIGINAL VS DENOISED IMAGES

METRIC	ST	HT	POAC
MSE	229.2780	229.2803	228.6764
PSNR	24.5272	24.5271	24.5386



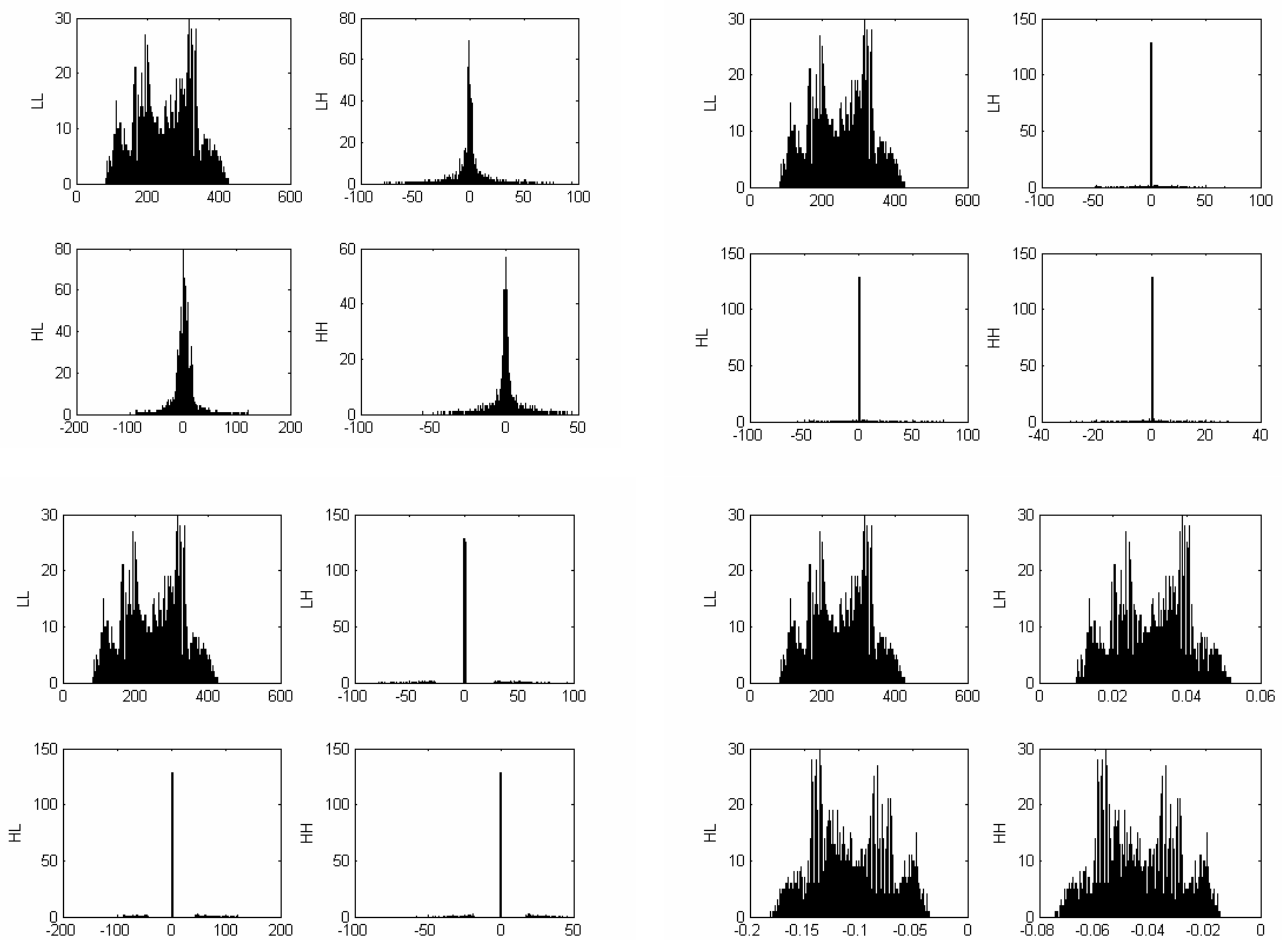


Fig. 10 Histograms of wavelet coefficients: (a) before thresholding, (b) after soft-thresholding, (c) after hard-thresholding, and (d) after POAC

TABLE II  
 ORIGINAL VS DECOMPRESSED IMAGES

METRIC	ST	HT	POAC
CR	3.065	3.044	5.055
PSS [%]	67.37	67.15	80.21
MSE	48.3938	34.9821	63.6266
PSNR	31.2829	32.6923	30.0944

On the other hand, Fig.10 shows the histograms of approximation (LL) and detail (LH, HL and HH) wavelet coefficients before and after the thresholding techniques. Observe, the damage caused for the pruning of ST and HT techniques, and the histogram affinity between LL (less noisy) and wavelet coefficients (LH, HL and HH) after POAC technique; considering that the histogram depends on the noisy presence in the wavelet coefficients.

Wavelet basis employed in the experiments were Daubechies 1, 2 and 4, with only one level of decomposition.

Finally, all techniques (denoising and compression) were implemented in MATLAB® (Mathworks, Natick, MA) on a PC with an Athlon (2.4 GHz) processor.

## VI. CONCLUSION

In this paper we have developed a *Projection Onto Approximation Coefficients* technique for image filtering and compression inside wavelet domain. The simulations show that the POAC have better performance than the most commonly used thresholding technique for compression and denoising (for the studied benchmark parameters) which include Soft-Thresholding and Hard-Thresholding.

Besides, the novel demonstrated to be efficient to remove multiplied noise, and all uncle of noise in the undecimated wavelet domain. Finally, cleaner images suggest potential improvements for classification and recognition.

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