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Contact Stress Analysis of Spur Gear Teeth Pair

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Abstract**—**Contact stress analysis between two spur gear teeth was considered in different contact positions, representing a pair of mating gears during rotation. A programme has been developed to plot a pair of teeth in contact. This programme was run for each 3° of pinion rotation from the first location of contact to the last location of contact to produce 10 cases. Each case was represented a sequence position of contact between these two teeth. The programme gives graphic results for the profiles of these teeth in each position and location of contact during rotation. Finite element models were made for these cases and stress analysis was done. The results were presented and finite element analysis results were compared with theoretical calculations, wherever available.

Keywords**—**Contact stress, Spur gear, Contact ratio, Finite elements.

I. Introduction

PITTING is a surface fatigue failure resulting from repetitions of high contact stress. The surface fatigue mechanism is not definitively understood. The contact-affected zone, in the absence of surface shearing tractions, entertains compressive principal stresses. Rotary fatigue has its cracks grown at or near the surfaces in the presence of tensile stresses, which are associated with crack propagation, ends to catastrophic failure. Because engineers had to design durable machinery before the surface fatigue phenomenon was understood in detail, they had taken the posture of conducting tests, observing pits on the surface, and declaring failure at an arbitrary projected area of hole, and they related this to the Hertzian contact pressure. When loads are applied to the bodies, their surfaces deform elastically near the point of contact; so that a small area of contact is formed. It is assumed that, as this small area of contact forms, points that come into contact are points on the two surfaces that originally were equal distances from the tangent plane [3]. Pitting commonly appears on operating surfaces of gear teeth, a fundamental cause is excessive loading that raised contact stresses beyond critical levels. Contact stress has been expressed clearly in this work by 10 finite element models; each model represents a case of contact between a pair of teeth at a different position during the rotating operation. The point of contact was moving from the tip to the root of tooth according to the angular motion. The domain in this study was the angular location of the point of contact on the arc of action from the beginning to the end of contact between this pair of teeth. Target and contact elements were used in both sides of the contact pattern between the surfaces of this pair of teeth.

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II. Spur Gear Contact

The transfer of power between gears takes place at the contact between the acting teeth. The stresses at the contact point are computed by means of the theory of Hertz. The theory provides mathematical expressions of stresses and deformations of curved bodies in contact. Fig. 1 shows a model applied to the gear-two parallel cylinders in contact.

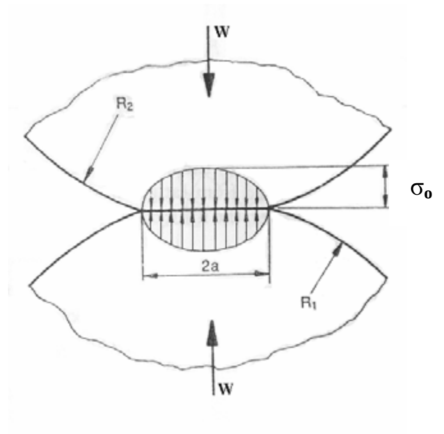


Fig. 1 Parallel cylinders in contact

According to the theory, the deformed distance (a) equals,

$$a = 2\sqrt{\frac{W(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{F\pi(1/R_1 + 1/R_2)}} \tag{1}$$

The Hertz theory assumes an elliptic stress distribution, as seen in the Fig. 1; the maximum stress is in the middle and equals:

$$\sigma_o = \sqrt{\frac{W(1/R_1 + 1/R_2)}{F\pi[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]}} \tag{2}$$

Where W is the load, E_1 and E_2 are the Modulus of Elasticity of pinion and gear respectively, υ_1 and υ_2 are the Poisson's ratios of pinion and gear respectively and F is the face width of pinion. Same equation can be apply for teeth, assuming for R_1 and R_2 the respective radii of the involute curve at the contact point, as shown in Fig. 2. Let us assume that the contact takes place at point 1, and then the respective radii are equal to:

$R_1 = r_{p1} \sin \phi$

$R_2 = r_{p2} \sin \phi$

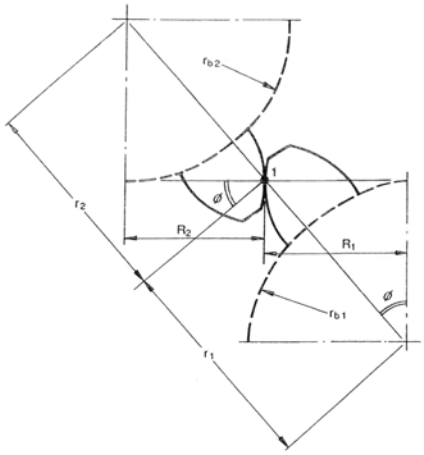


Fig. 2 Two involute teeth in contact

Hence, the Hertz equation for contact stresses in the teeth becomes,

$$\sigma_o = \sqrt{\frac{W(1 + r_{p1}/r_{p2})}{r_{p1} F \pi [(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2] \sin \phi}} \quad (3)$$

Where r_{p1} and r_{p2} are the pitch radii of the pinion and gear and ϕ is the pressure angle. The stress correlations derived heretofore and Eq. (3) are based on a number of simplifying assumptions, such as pure bending of short beam and elliptic distribution of stresses at tooth contact. A question therefore arises concerning their accuracy [12]. The elastic compression of two-dimensional bodies in contact can not be calculated solely from the contact stresses given by the Hertz theory. Some account must be taken for the shape and size of the bodies themselves and the way in which they are supported. In most practical circumstances such calculations are difficult to perform, which have resulted in a variety of approximate formulae for calculating the elastic compression of bodies in line contact such as gear teeth and roller bearings in line contact [10]. The pitting problems, design needs and safety requirements make far in depth and complicated study of this contact. The current project aims to arriving at these very solutions.

III. CONTACT RATIO (C_R)

Fig. 3 shows a gear mesh with the driving pinion tooth on the left just coming into contact at point (A) and the two teeth on the right in contact at point S. Notice that contact starts at point (A) the highest point of contact on the gear tooth where the outside diameter of the gear crosses the line of action and ends where the outside diameter of the pinion crosses the line of action at point (B) the lowest point of contact on the gear tooth. AB is the length of contact line [7].

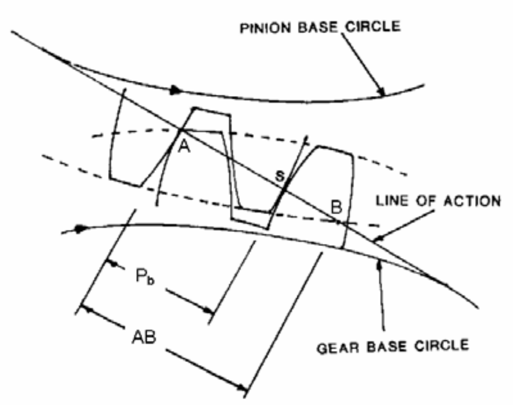


Fig. 3 Gear tooth action

The contact ratio (C_R) is defined as the average number of teeth in contact at one time; it is calculated from:

$$C_R = \frac{\overline{AB}}{P_b} \quad (4)$$

For some period of time one tooth mesh carries the load and for another period of time two tooth meshes share the load. At contact ratio C_R of (1.0) it would mean that one tooth is in contact (100%) of the time. Contact ratio of (1.6) means two pairs of teeth are in contact (60%) of the time and one pair carries the load (40%) of the time. Contact ratios for conventional gearing are generally in the range (1.4–1.6). The zone of action of meshing gear teeth shows in Fig. 4, that tooth contact begins and ends at the intersections of the two addendum circles with the pressure line (line of action). In Fig. 4 initial contact occurs at A and B, respectively. As shown, the distance aP is called the arc of approach qa, and the distance Pb, the arc of recess q_r. The sum of these is the arc of action q_t.

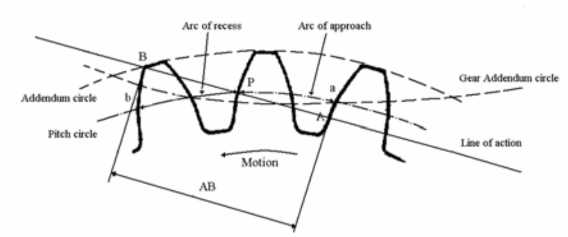


Fig. 4 Approach and recess angles

Now, consider a situation in which the arc of action is exactly equal to the circular pitch P_c, that is, $q_t = P_c$. This means that one tooth and its space will occupy the entire arc ab. In other words, when a tooth is just beginning contact at A, the previous tooth is simultaneously ending its contact at B. Therefore during the tooth action from A to B, there will be exactly one pair of teeth in contact. Next, consider a situation in which the arc of action is greater, say, $q_t = 1.2\ P_c$. This means that when one pair of teeth is just entering contact at A, another pair, already in contact, will not yet have reached B. Thus, for a short period of time, there will be two teeth in contact, one in the vicinity of (a) and another near (b). As the meshing proceeds, the pair near (b) must cease contact, leaving only a single pair of contacting teeth, until the procedure repeats itself. Because of the nature of this tooth action, since either one or two pairs of teeth are in contact, it is convenient to define the term contact ratio C_R as:

$$C_R = \frac{q_t}{P_c} \tag{5}$$

Where, q_t is the arc of action. In other words, contact ratio is a number which indicates the average number of pairs in contact. Note that this ratio is also equal to the length of the path of contact divided by the base pitch. Gears should not generally be designed having contact ratios less than about 1.2, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level. An easier way to obtain the contact ratio is to measure the line of action AB instead of the arc distance ab. Since AB in Fig. 4 is a tangent to the base circle when extended, the base pitch P_b must be used to calculate C_R instead of the circular pitch P_c [11], as in Eq. (4). But for corrected gears (non standard gears) the corrected values of outer radii of pinion and gear (r_{a1} and r_{a2} respectively) and the center distance C are to be inserted. Also, ϕ will be replaced by the operating (working) pressure angle ϕ_{op} [8], hence:

$$\mathrm{inv}\phi_{op} = \frac{2(C_{f1} + C_{f2})\tan\phi}{Z_1 + Z_2} + \mathrm{inv}\phi \tag{6}$$

Where C_{f1} and C_{f2} are the profile shift factor of pinion and gear, and Z_1 and Z_2 are pinion and gear number of teeth respectively. Referring to Fig. 3, the positions of the end points of the path of contact are then as follows [4]:

$$S_A = -r_{b1}\tan\phi_{op} + \sqrt{(r_{a1})^2 - (r_{b1})^2} \tag{7}$$

$$S_B = r_{b2}\tan\phi_{op} - \sqrt{(r_{a2})^2 - (r_{b2})^2} \tag{8}$$

Where r_{b1} and r_{b2} are the base radii of pinion and gear and r_{a1} and r_{a2} are the outer radii of pinion and gear respectively. Another useful equation to obtain the value of working pressure angle ϕ_{op} is:

$$\cos(\phi_{op}) = \frac{r_{b1} + r_{b2}}{C} \tag{9}$$

So, $AB = S_A - S_B$

$$\overline{AB} = \sqrt{(r_{a1})^2 - (r_{b1})^2} + \sqrt{(r_{a2})^2 - (r_{b2})^2} - (r_{b1} + r_{b2})\tan\phi_{op} \tag{10}$$

Or

$$\overline{AB} = \sqrt{(r_{a1})^2 - (r_{b1})^2} + \sqrt{(r_{a2})^2 - (r_{b2})^2} - (r_{p1} + r_{p2})\sin\phi_{op} \tag{11}$$

By substituting in Eq. (4) yields:

$$C_R = \frac{1}{P_b}\left[\sqrt{(r_{a1})^2 - (r_{b1})^2} + \sqrt{(r_{a2})^2 - (r_{b2})^2} - (r_{b1} + r_{b2})\tan\phi_{op}\right] \tag{12}$$

From above equations the value of contact ratio must be greater than (1.0) provided the length of path of contact is greater than base pitch (P_b). The operation of the gears is impossible unless the value of (C_R) is at least (1.0) and in general the higher value of C_R makes the gear pair runs smoothly. However, for spur gears a contact ratio of at least (1.4) is generally recommended. Referring to Fig. 4 the angle corresponding to the arc of action or to the length of path of contact can be found by calculating the angle of approach and the angle of recess. Where the angle of approach is given by [4]:

$$\phi_{App} = \frac{-S_A}{r_{b1}} \tag{13}$$

the angle of recess is given by:

$$\phi_{Rec} = \frac{S_B}{r_{b1}} \tag{14}$$

So, the rolls angle ϕ_c (the angle corresponding to the arc of action or to the length of path of contact) from the first point of contact A to the last point of contact B of one pair of teeth at contact operation is:

$$\phi_c = \phi_{App} + \phi_{Rec} \tag{15}$$

IV. CONTACT PERIODS

It is clear that during the rotation operation of the mating gears each tooth will share load in double tooth contact stage and will carry the entire load at single tooth contact, and the contact ratio is the most important parameter which plays in this situation. In section 3 it is explained how to obtain the contact path length, the contact ratio and the angle which is corresponding to this contact length which is the most important feature here in this section compared to the other following sections, because the contact stages will be investigated and analyzed by dividing this angle into any desirable angular periods or angular intervals.

V. MODEL GEOMETRY

To apply consider to formulation case of 1mm module, 20 teeth pinion and 60 teeth gear, pinion operating in 1440rpm, where the pressure angle is 20° and profile shift factor is zero. The material conditions of the pinion and gear are; 2.15MPa and 2.08MPa respectively as Modulus of elasticity and 0.29 and 0.3 respectively as Poisson ratio. By using QBASIC Language a computer programme has been developed to plot one pair of teeth in contact at different situations of contact depending on the formulation in section 3. From Equations (11) and (12) the length of contact and contact ratio are computed and they are 4.929mm and 1.6 respectively. While from Eq. (15) the corresponding angle of contact is 0.524radian or ≈ 30°, considering that the contact will start at angle 0° and end at angle 30°. The selected angular interval value is 3°, so the progress of contact will be studied for each 3°, which means there are 10 cases of contact under consideration.

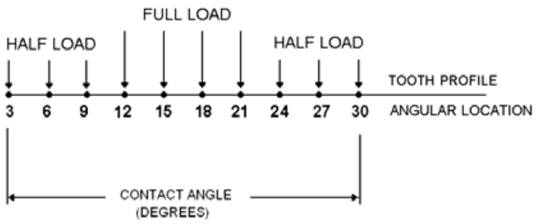


Fig. 5 Angular intervals

Fig. 5 shows the angular intervals during the contact operation of the domain and it also shows the load distribution within these contact locations along with the angle of contact [1]. The outputs of the developed programme represent the selected 10 cases of contact. Each case gives the contact situation after rolling 3° from the previous case. The first case shows the contact after the first 3°, the second case shows the contact after 6° and third case shows the contact after 9°, and so on. Figures 6 and 7 show programme outputs for cases (3 and 9) respectively, for contact under conditions of 1mm module, 20 teeth, zero profile shift factor and (3) gear ratio:

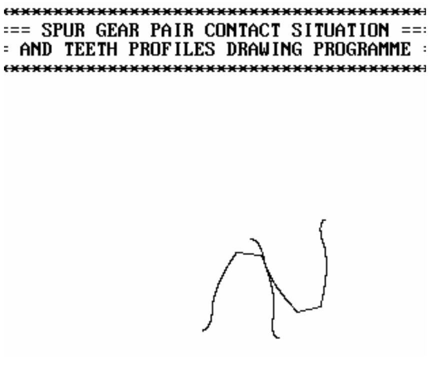


Fig. 6 Tooth profile of contact case 3

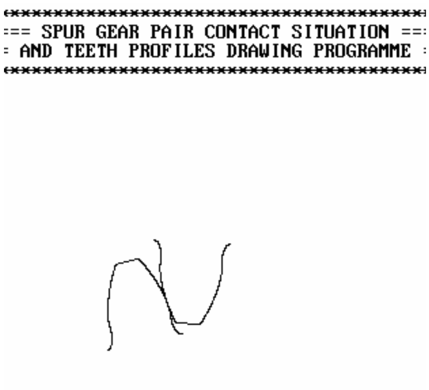
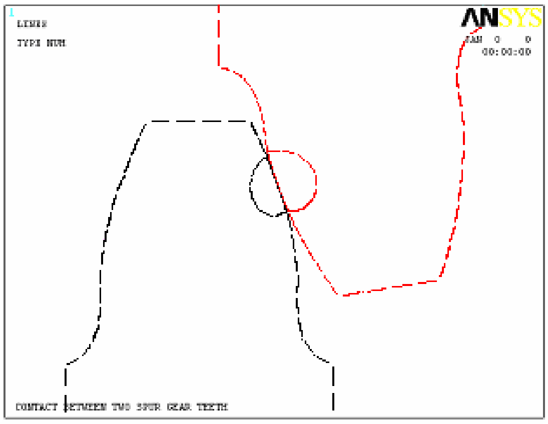


Fig. 7 Tooth profile of contact case 9

VI. ANSYS CONTACT TECHNIQUE

ANSYS supports three contact models: node-to-node, node-to-surface, and surface-to-surface [2]. Each type of model uses a different set of ANSYS contact elements and is appropriate for specific types of problems. To model a contact problem, firstly it must identify the parts to be analyzed for their possible interaction. If one of the interactions is at a point, the corresponding component of the model is a node. If one of the interactions is at a surface, the corresponding component of a model is an element: beam, shell, or solid element. The finite element model recognizes possible contact pairs by the presence of specific contact elements. These contact elements are then interpreted with the model exactly where they are being analyzed for interaction. A problem of contact stress has been chosen to contain two teeth in different contact positions, representing a mating pair of gears during the rotating operation. An eight noded iso-parametric plane stress quadratic quadrilateral element was used to build the finite element models of these two teeth. The type of contact was point to line or node to surface. The target surface was chosen in the gear tooth and meshed by 2D target element while the contact surface was chosen in the pinion tooth with 2D contact element. In ANSYS software, these two elements are (Targe169) and (Conta175) respectively as it shown in Fig. 8 and its enlargement in Fig. 9.

Fig. 8 Target and contact regions



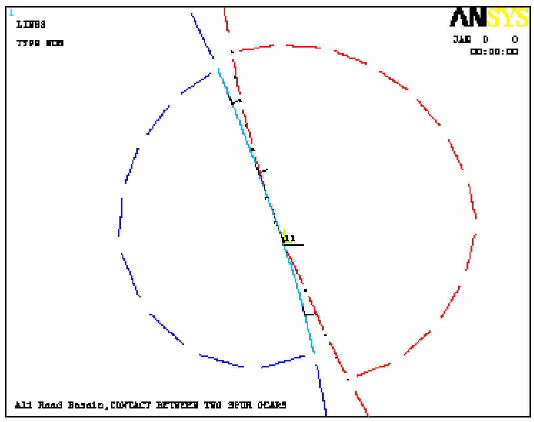


Fig. 9 Enlarged contact regions

Fig. 10 shows the finite element mesh of the contact pair of teeth along with the target and contact regions, while Fig. 11 expresses the boundary conditions of the model.

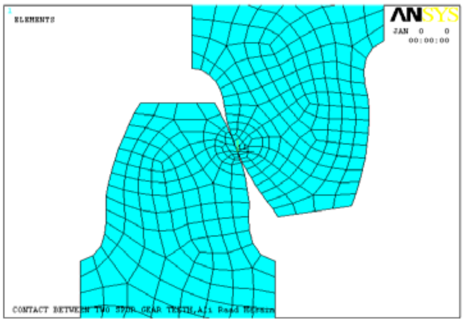


Fig. 10 Contact model grids.

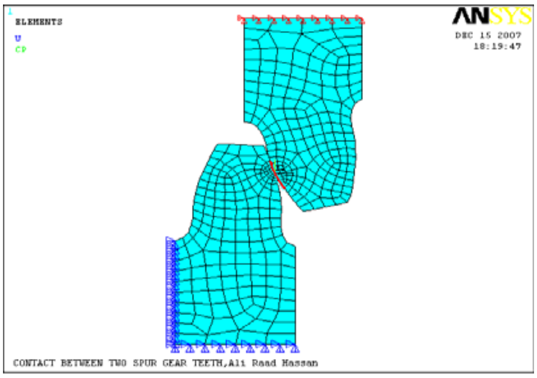


Fig. 11 Model boundary conditions.

VII. RESULTS

The maximum contact stress value for this model was 587MPa which obtained by AGMA, the well known theoretical calculations method. The stress analysis has been done by ANSYS software, where the results have been presented by contours and numerical values.

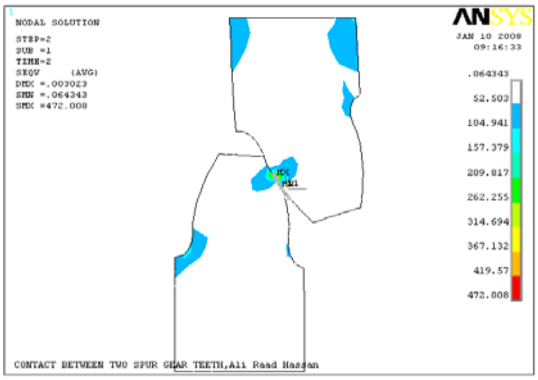


Fig. 12 von Mises contour of case 3

Figures 12 and 14 show the von Mises stress contours for contact cases 3 and 9 respectively, while figures 13 and 15 show magnified contact regions for these cases. Table 1 gives the Maximum von Mises contact stresses for each selected cases of contact and Fig. 16 show the relationship form these stress results along with the prorogation of contact point location from the first to the last.

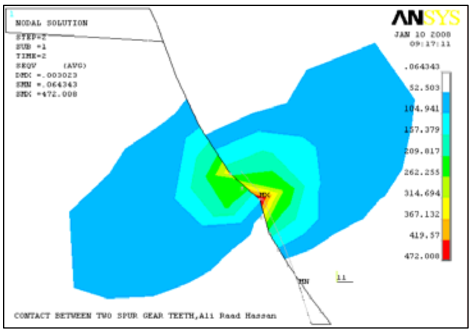


Fig. 13 Magnified of contour of case 3

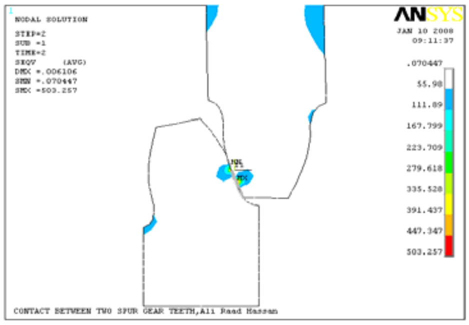


Fig. 14 von Mises contour of case 9

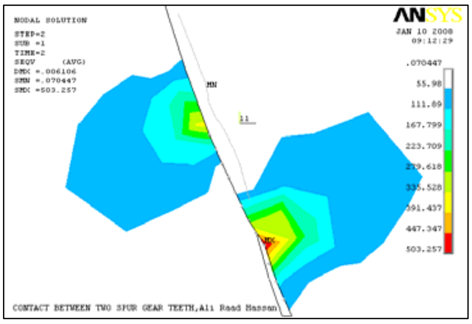


Fig. 15 Magnified of contour of case 9

TABLE 1
MAXIMUM VON MISES STRESSES FOR THE SELECTED CONTACT CASES

Case	Angular location (Degree)	Max. von Mises stress (MPa)
1	3	606
2	6	483.7
3	9	472
4	12	580
5	15	595
6	18	582
7	21	530.6
8	24	470
9	27	503
10	30	570.5

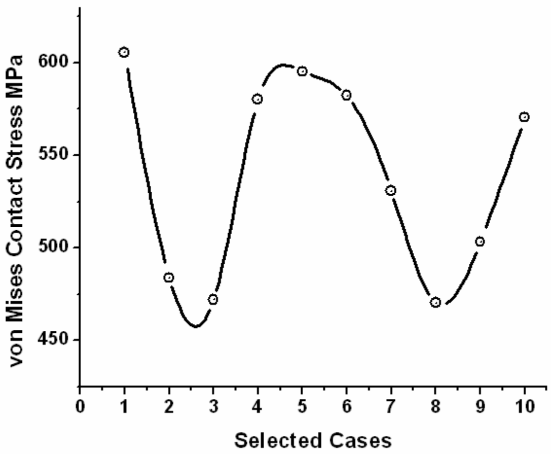


Fig. 16 Curve of maximum von Mises stress for the selected cases

VIII. DISCUSSION

The presentation dealt with contact stress, considering contact ratio, approach angle, recess angle, contact and length of contact. The stress was more than the correct value of contact stress obtaining from approximating tools. This search was certainly not easy and cannot be carried out without the use of finite element analysis. To apply finite element method in contact stress a special technique was used rather the regular elements, to distinguish between the contact regions which were in two parts. One was the first body named target region and the other body was named contact region. For target region, target elements were used and in contact region contact elements were used. ANSYS software presents a significant technique for this purpose which was used here. A computer program was developed to plot one pair of teeth in contact at different positions of contact depending on the formulation as shown in section 5. The selected angular interval value was 3°, the progress of contact was studied at each 3° interval, which means that there were 10 cases of contact under consideration. These 10 cases were used to build 10 finite element contact models and contact finite element analysis was done under the load and material conditions were named. The maximum stress result obtained from AGMA stress calculation method was 587MPa while the maximum contact stress obtained from the finite element contact analysis was 595MPa (case five) under the same conditions, it is clear that the agreement is good. The ultimate stress of steel (C45) is 630-710MPa and the yield stress is 360MPa, therefore, the analysis is at elastic–plastic range, but by considering the very small area of contact region, it will be very natural to obtain high contact stress, which will be for very limited period of time. The results shows a high value of contact stress in the beginning of the contact, and then it starts to reduce until it reaches the location of single tooth contact, here it increased to the maximum value of the contact, but exactly after exceeding this single contact region it was reduced. At the end of the contact, the stress increased suddenly to a high value almost close to the maximum value, at this stage a sliding was occurred in the contact region at the maximum stress points as shown in Fig. 15.

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