World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:6, No:12, 2012

Motion Planning and Control of Autonomous Robots in a Two-dimensional Plane

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Abstract**—**This paper proposes a solution to the motion planning and control problem of a point-mass robot which is required to move safely to a designated target in a priori known workspace cluttered with fixed elliptical obstacles of arbitrary position and sizes. A tailored and unique algorithm for target convergence and obstacle avoidance is proposed that will work for any number of fixed obstacles. The control laws proposed in this paper also ensures that the equilibrium point of the given system is asymptotically stable. Computer simulations with the proposed technique and applications to a planar (RP) manipulator will be presented.

Keywords**—**Point-mass Robot, Asymptotic stability, Motion planning, Planar Robot Arm.

I. Introduction

MOTION planning and control of autonomous robots has been an active research area for more than two decades now. The literature is inundated with algorithms, strategies and methods that addresses the motion planning and control problem of various different robotic systems [1], [2], [3], [4], [5], [6]. The design and development of autonomous robots must ensure that the robot is be able to safely navigate to its goal position while satisfying the cost and time constraints tagged to the system [6]. The presence of obstacles in the robots workspace adds another difficult dimension to the motion planning problem of autonomous robots. According to [7], if a workspace is cluttered with obstacles, an optimal collision-free trajectory is desired, as a solution of motion planning problem that can lead the robot to its goal position.

Researchers, over the years, have produced numerous algorithms for tackling the motion planning and control problem of autonomous robots. The three basic algorithms are: physical analogy-based method, graph search technique and neural networks. The reader refer to [8] for more information on these types of algorithms. With continuous time-invariant feedback control laws it is possible to achieve a collision-free trajectory of the robot that can guarantee stability of the system [7, 8]. However, to show asymptotic stability with smooth controllers, in obstacle-ridden workspace, is still a challenging problem. An asymptotic stable system ensures that all trajectories starting in the neighbourhood of the equilibrium point converges to the equilibrium point where as in a stable system, some trajectories starting in the neighbourhood of the equilibrium point may lead to *traps* outside the equilibrium point [8]. These traps are due to the local minima in the energy function [8].

Some researchers have developed useful techniques to solve the problem of local minima via the use of some special functions. The work of Koditschek [9] and Tanner et al. [10] are noteworthy. Koditschek [9] used Potential Functions to study the problem of local minima while Tanner et al. [10] used Dipolar Inverse Lyapunov Functions. Other methods found in literature include techniques such as, executing a random robot motion [11], temporarily relocating the goal [12] and constructing a potential field based on superquadrics [13]. More recently, Vanualailai et al. in [14] proposed an asymptotically stable point-mass system. The method was based on looking for initial conditions that does not lead to a local minimum. The authors considered a planar point-mass robot moving from its initial position to the desired goal whilst avoiding a static obstacle. Since the system was proven to be asymptotically stable, its Lyapunov function, which produces artificial potential fields around the goal and the obstacle, had no local minima other than the goal.

In this paper, we will control the motion of a point-mass robot in an obstacle-ridden workspace. We will consider elliptical obstacles randomly distributed in a priori-known workspace. The main aim of this paper is to design time-invariant continuous control laws that should ensure asymptotic stability of the system. An algorithm for target convergence and obstacle avoidance will be proposed that will work for any number fixed obstacles of various size and positions. We have a step-by-step method of constructing the control laws. Moreover, the solution proposed in this paper for a point-mass robot can easily be applied to other planar robots such as planar robot arms, carlike robots, to name a few. As an illustration, we have considered the motion of a planar (RP) robot in Section VII.

The rest of the paper is organized as follows. In Section II, we give the definition of a point-mass robot and derive its kinematic model. The main objective of this paper is given in Section III. The motion planning and control problem of the point mass in obstacle free workspace is described in Section IV, together with introducing a tailored velocity algorithm. In Section V, we propose a collision-free algorithm where the point mass robot moves safely from an initial position to its target position in a workspace cluttered with fixed elliptical obstacles. Stability analysis is carried out in Section VI while Section VII contains the applications of the control laws on a

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planar robot (RP) arm. Finally, in Section VIII concluding remarks on the contributions and future work are made.

II. Modeling The Point-Mass Robot

Let P be a *point-mass robot in* the z_1z_2 plane, positioned at (x, y) with a circular protective region of radius $r_P \geq 0$. Precisely, P can be represented as

$$P = \left\{(z_1, z_2) \in \mathbf{R}^2 : (z_1 - x)^2 + (z_2 - y)^2 \leq r_P^2\right\}.$$

According to [8], the disk-representation strategically aids in the construction of the path planning algorithms. Let P be moving with a velocity of v in the z_1z_2 plane. Suppose u_1 and u_2 are the z_1 and z_2 components, respectively, of v, then the kinematic model of P can be expressed as

$$\left.\begin{aligned} \dot{x} = u_1, \quad \dot{y} = u_2 \\ x_0 = x(0), \quad y_0 = y(0) \end{aligned}\right\} \tag{1}$$

System (1) is a description of the instantaneous velocities of the point-mass where u_1 and u_2 are classified as the controllers. Hereafter, we shall use the vector notation $\mathbf{x}(t) = (x, y)$ to refer to the position of P.

III. Main Objective: Problem Statement

Let P be the point-mass robot moving in the workspace W. Suppose $FO_1, FO_2, \ldots, FO_q$ are stationary elliptical obstacles randomly distributed in W. Assume that the position and size of P and $FO_1, FO_2, \ldots, FO_q$ is a *prior* known. The problem statement is:

> *Given any initial position and orientation of P in W, design the controllers u_1 and u_2 so that P can converge to a goal position whilst avoiding collisions with the stationary obstacles.*

IV. Motion Planning

In our motion planning problem, we want the point-mass robot P to start from an initial position, move towards a target and finally converge at the centre of the target. We therefore, require a predefined target fixed for P. This target is defined as follows [8]:

Definition 1: The target for P is a disk of centre (p_1, p_2) and radius r_T. Precisely, it is a set

$$T = \left\{(z_1, z_2) \in \mathbf{R}^2 : (z_1 - p_1)^2 + (z_2 - p_2)^2 \leq r_T^2\right\}.$$

Now, we look for an appropriate form of $v(t)$, which can move P from its initial position to the target position and stop there. That is, we want a velocity which should be depended on the initial and final positions of the robot. The velocity algorithm proposed is

$$v = \frac{|v_0| \left\|\mathbf{x}(t) - \mathbf{e}\right\|}{\left\|\mathbf{x}(0) - \mathbf{e}\right\|}, \tag{2}$$

where $|v_0|$ is the initial velocity of P at $t = 0$ and $\mathbf{e} = (p_1, p_2) \neq \mathbf{x}(0)$ is an equilibrium point of system (1). Note that $v(t)$ is defined, continuous and positive over the domain

$$D = \left\{\mathbf{x} \in \mathbf{R}^2 : (x(0),\, y(0) \neq (p_1,\, p_2)\right\}.$$

We further $\xi(t)$ as the angular position of the target center relative to the current position of P at time t. The angle $\xi(t)$ is defined implicitly

$$\tan \xi(t) = \begin{cases} \dfrac{p_2 - y(t)}{p_1 - x(t)}, & \text{if } \mathbf{x}(t) \neq \mathbf{e} \\ \tan \xi(t-1), & \text{if } \mathbf{x}(t) = \mathbf{e} \end{cases}$$

Since u_1 and u_2 are the z_1 and z_2 components, respectively, of v, we see that

$$\left.\begin{aligned} u_1 = \frac{|v_0|\,(p_1 - x)}{\left\|(\mathbf{x}(0) - \mathbf{e})\right\|}, \\ u_2 = \frac{|v_0|\,(p_2 - y)}{\left\|(\mathbf{x}(0) - \mathbf{e})\right\|}. \end{aligned}\right\} \tag{3}$$

with this definition of u_1 and u_2, we have the following theorem:

Theorem 1: Let u_1 and u_2 be as defined by equation (3). Then the point e is the only equilibrium point of system (1) and is globally asymptotically stable.

Proof. Note that $(\dot{x}, \dot{y}) = (0, 0)$ only if $u_1 = u_2 = 0$ which implies that $\mathbf{x}(t) = \mathbf{e}$. Thus $\mathbf{e}$ is the only equilibrium point of system (1).

To prove global asymptotic stability, consider the Lyapunov function of the form

$$L(\mathbf{x}) = \frac{1}{2}\left\|\mathbf{x}(t) - \mathbf{e}\right\|^2,$$

which is defined, continuous, positive and radially unbounded over the domain

$$D = \left\{\mathbf{x} \in \mathbf{R}^2 : (x(0),\, y(0) \neq (p_1,\, p_2)\right\}.$$

Clearly, $L(\mathbf{x})$ has continuous first partial derivatives in the region D of the neighborhood of the equilibrium point $\mathbf{e}$ of system (1). Moreover, in the region D, we see that $L(\mathbf{e}) = 0$ and $L(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{e}$.

Now, the time-derivative of $L(\mathbf{x})$ along a trajectory of system (1) is given by

$$\dot{L}(\mathbf{x}) = -\sqrt{u_1^2 + u_2^2}\,\|\mathbf{x}(t) - \mathbf{e}\|.$$

Again, it is clear that in the region D, $\dot{L}(\mathbf{e}) = 0$ and $\dot{L}(\mathbf{x}) < 0$ for all $\mathbf{x} \neq \mathbf{e}$. Hence it can be concluded that $\mathbf{e}$ is a global asymptotic stable equilibrium point of system (1).

V. Motion Control in the Presence of Stationary Obstacles

Now we will consider a situation where there are stationary objects in the working space that the point-mass P has to avoid. The lth stationary obstacle is defined below.

Definition 2: The lth stationary obstacle is an elliptical-shaped obstacle with center (o_{l1}, o_{l2}). Precisely, the lth stationary obstacle is the set

$$FO_l = \left\{ (z_1,\, z_2) \in \mathbf{R}^2 : \frac{(z_1 - o_{l1})^2}{a_l^2} + \frac{(z_2 - o_{l2})^2}{b_l^2} \le 1 \right\}$$

for $l = 1,2,\ldots,q$.

Assumption 1: There is sufficient free-space between any two stationary obstacles for the point-mass P to steer through if warranted.

Remark 1: Assumption 1 is inline with the work of Sharma et.al in [8]. This is to ensure that P could fit into the free-space between two obstacles, in case, one desires to steer the robot in between the two obstacles.

In order for the point-mass robot P to avoid the stationary obstacles, we redefine the controllers u_1 and u_2 as:

$$\left.\begin{aligned} u_1 &= v\cos(\xi + \varepsilon), \\ u_2 &= v\sin(\xi + \varepsilon). \end{aligned}\right\} \qquad (4)$$

where ε determines the direction in which the P will turn to avoid the obstacles. If $\varepsilon > 0$, then the point-mass will turn left; if $\varepsilon < 0$, it will turn right; and if $\varepsilon = 0$, it will move straight towards the target. Thus controlling the value of ε will enable the robot to avoid obstacles and reach its target safely.

Definition 3: Let $d_{\max} > 0$ be a predefined scalar. The set $\mathcal{S}$ defined by

$$\mathcal{S} = \bigcup_{l=1}^{q} \left\{ (z_1,\, z_2) \in \mathbf{R}^2 : \frac{(z_1 - o_{l1})^2}{(a_l + d_{\max})^2} + \frac{(z_2 - o_{l2})^2}{(b_l + d_{\max})^2} \le 1 \right\}$$

is called the sensing zone.

Next, we define the following:

$$f_l = (x - o_{l1})(p_2 - y) - (y - o_{l2})(p_1 - x)$$

$$R_l = \frac{(x - o_{l1})^2}{a_l^{\,2}} + \frac{(y - o_{l2})^2}{b_l^{\,2}} - 1$$

$$D_l = \frac{(x - o_{l1})^2}{(a_l + d_{\max})^2} + \frac{(y - o_{l2})^2}{(b_l + d_{\max})^2}$$

$$\alpha_l = \begin{cases} 0, & \text{if } D_l \ge 1 \\ 1 - D_l, & \text{if } D_l < 1 \end{cases}$$

$$\beta_l = \begin{cases} 1, & \text{if } f_l \le 0 \\ -1 & \text{if } f_l > 0 \end{cases}$$

Note that the size of the sensing zone is determined by $d_{\max}$. If $d_{\max}$ is large, then P will avoid the fixed obstacle from a greater distance. Thus $d_{\max}$ is regarded as the *control* parameter in this paper.

Normally seen in literature [7], [8], [15], [16] for effective obstacle avoidance, the measure of the distance from the robot to an obstacle, appears in the denominator of repulsive functions. Thus in our case, we propose the following form of ε:

$$\varepsilon = \tan^{-1}\left(\sum_{l=1}^{q} \frac{\alpha_l \beta_l}{R_l} \right) \qquad (5)$$

Remark 3: The function α_l ensures that the output ε will be a continuous function. Thus the controller derived will be continuous everywhere along the trajectory of the system. The parameter β_l is an *indicator function*. It indicates the direction P should turn in the sensing zone to ensure that an overall shortest path is achieved.

Remark 4: With the form of ε given in equation (5), we see that as P comes closer to FO_l, then the quantity R_l will decrease. This will increase $|\varepsilon|$ since R_l appears in the denominator. Hence an increase in $|\varepsilon|$ will force P will move away from the obstacle.

Substituting (5) into (4) and simplifying, we see that the controllers are

$$\left.\begin{aligned} u_1 &= \frac{|v_0|\left[(p_1 - x) - (p_2 - y)\displaystyle\sum_{l=1}^{q} \frac{\alpha_l \beta_l}{R_l} \right]}{\|\mathbf{x}(0) - \mathbf{e}\| \sqrt{1 + \left(\displaystyle\sum_{l=1}^{q} \frac{\alpha_l \beta_l}{R_l} \right)^2}}, \\ u_2 &= \frac{|v_0|\left[(p_2 - y) - (p_1 - x)\displaystyle\sum_{l=1}^{q} \frac{\alpha_l \beta_l}{R_l} \right]}{\|\mathbf{x}(0) - \mathbf{e}\| \sqrt{1 + \left(\displaystyle\sum_{l=1}^{q} \frac{\alpha_l \beta_l}{R_l} \right)^2}}. \end{aligned}\right\}$$

which are bounded and continuous at every point over the domain

$$D = \left\{ \mathbf{x} \in \mathbf{R}^2 : \mathbf{x}(0) \neq \mathbf{e} \cap \frac{(x - o_{l1})^2}{a_l^2} + \frac{(y - o_{l2})^2}{b_l^2} > 1 \text{ for } l = 1, 2, \ldots, q \right\}$$

VI. Simulation

To demonstrate the effectiveness of our method, we have simulated the trajectory of a point-mass robot navigating in a workspace cluttered with stationary elliptical obstacles. We have considered a simple setup where the robot maneuvers from an initial to a final configuration, whilst avoiding all the fixed obstacles placed randomly in the workspace. This is shown in Fig. 1 and Fig. 2. The nonlinear controllers u_1 and u_2 were simulated to generate feasible robot trajectories. For the numerical integration of system (1), a fourth order Runge-Kutta method was used.

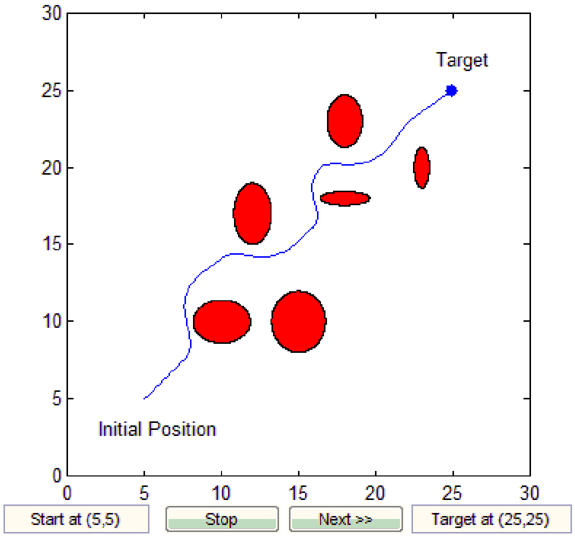

Fig. 1 Trajectory of the point-mass robot with initial position (5, 5) and the target placed at (25, 25)

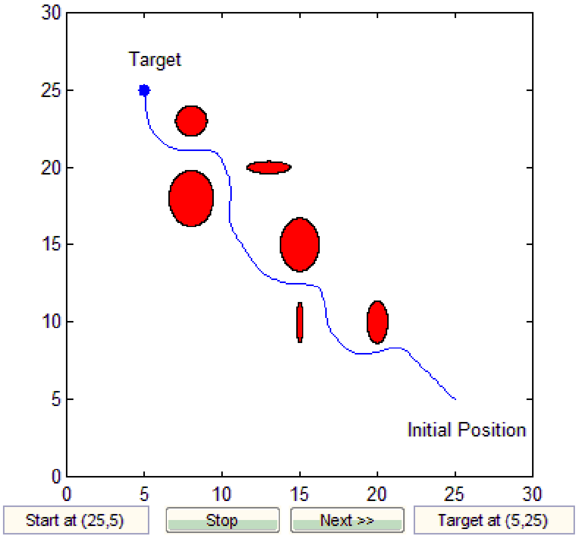

Fig. 2 Trajectory of the point-mass robot with initial position (25,5) and the target placed at (5, 25)

VII. Stability Analysis

We utilize the Direct method of Lyapunov to carry out the stability analysis of system (1).

Theorem 2: The point $\mathbf{e}$ is a global asymptotic stable equilibrium point of system (1).
Proof. Consider the Lyapunov function

$$L(\mathbf{x}) = \frac{1}{2} \left\| \mathbf{x}(t) - \mathbf{e} \right\|^2,$$

which is defined, continuous, positive and radially unbounded over the domain

$$D = \left\{ \mathbf{x} \in \mathbf{R}^2 : \mathbf{x}(0) \neq \mathbf{e} \cap \frac{(x - o_{l1})^2}{a_l^2} + \frac{(y - o_{l2})^2}{b_l^2} > 1 \text{ for } l = 1, 2, \ldots, q \right\}$$

The function, $L(\mathbf{x})$ has continuous first partial derivatives in the region D of the neighborhood of the equilibrium point $\mathbf{e}$ of system (1). Moreover, in the region D, we see that $L(\mathbf{e}) = 0$ and $L(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{e}$.

Now, the time-derivative of $L(\mathbf{x})$ along a trajectory of system (1) is given by

$$\dot{L}(\mathbf{x}) = -\frac{\sqrt{u_1^2 + u_2^2} \left\| \mathbf{x}(t) - \mathbf{e} \right\|}{\sqrt{1 + \left(\sum_{l=0}^{q} \frac{\alpha_l \beta_l}{R_l} \right)^2}}.$$

Again, it is clear that in the region D, $\dot{L}(\mathbf{e}) = 0$ and $\dot{L}(\mathbf{x}) < 0$ for all $\mathbf{x} \neq \mathbf{e}$. Hence it can be concluded that $\mathbf{e}$ is a global asymptotic stable equilibrium point of system (1).

VIII. Application: A Planar Robot Arm

In this section, we apply our approach to a planar robot arm that has a translational joint and a rotational joint in the $z_1 z_2$ plane as shown in Fig. 3. The arm consists of two links made up of uniform slender rods; the revolute (R) first link with fixed length and the prismatic (P) second link of varying length. Hereafter, we will use the abbreviation (RP) to refer to the joint type of the planar robot.

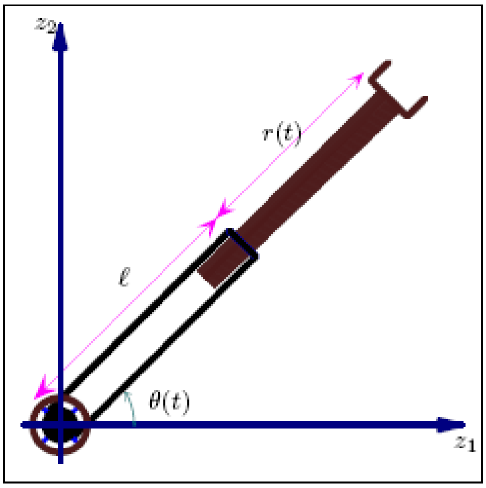

Fig. 3 A planar (RP) manipulator in the z_1z_2 plane (adopted from [6])

With reference to Fig. 3, we assume that:

- the planar (RP) manipulator is anchored at the origin;
- the first link has a fixed length of ℓ;
- the second link has a varying length of $r(t)$ at time t;
- the manipulator has angular position $\theta(t)$ at time t;
- the coordinate of the gripper is $(x(t), y(t))$.

Remark 5: It can easily be observed that we can express the position of the end-effector completely in terms of $r(t)$ and $\theta(t)$ as:

$$x(t) = (\ell + r(t))\cos\theta(t),$$
$$y(t) = (\ell + r(t))\sin\theta(t).$$

We consider the kinematic model of this planar (RP) robot manipulator in order to model $r(t)$ and $\theta(t)$. Suppose that the end-effector is moving with a velocity of u_1 in the z_1 direction and u_2 in the z_2 direction. We see that $r(t) = \sqrt{x^2 + y^2} - \ell$ and

$$\dot{r}(t) = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} = u_1\cos\theta + u_2\sin\theta.$$

Similarly, $\tan\theta = y/x$ so that

$$\dot{\theta}(t) = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} = \frac{u_2\cos\theta - u_1\sin\theta}{\ell + r}.$$

Thus the kinematic equations for the planar robot arm is

$$\left.\begin{aligned} &\dot{r}(t) = u_1\cos\theta + u_2\sin\theta, \\ &\dot{\theta}(t) = \frac{u_2\cos\theta - u_1\sin\theta}{\ell + r} \\ &r(0) = \sqrt{x(0)^2 + y(0)^2} - \ell, \\ &\theta(0) = \operatorname{atan2}(y(0), x(0)). \end{aligned}\right\} \qquad (6)$$

System (6) is a description of the instantaneous velocities of the planar robot arm. Here u_1 and u_2 are classified as the controllers. We shall use the vector notation $\mathbf{x} = (r, \theta)$ to refer to the position of the planar robot arm in the z_1z_2 -plane.

A. Convergence to the Target

For the end-effector, we have a designated target T with center (p_1, p_2) and radius r_T. We want the end-effector of the robot arm to start from an initial configuration, move towards T and converge to the center of the target. To achieve this, we will modify the velocity algorithm described in Section IV as:

$$v = \frac{|v_0|\left\|\left((\ell + r(t))\cos\theta(t) - p_1, (\ell + r(t))\sin\theta(t) - p_2\right)\right\|}{\left\|\left((\ell + r(0))\cos\theta(0) - p_1, (\ell + r(0))\sin\theta(0) - p_2\right)\right\|},$$

where $|v_0|$ is the initial velocity of the end-effector at $t = 0$.

B. Mechanical Singularities

In reality, the motion of the end-effector is restricted in the sense that the end-effector of the prismatic 2-link manipulator cannot go inside the first link [7]. Thus the circular region with the origin (0, 0) as its center that encloses the first link is treated as an *artificial obstacle* for the end-effector (see Fig. 4). For the avoidance of the mechanical singularity, we first define:

$$f_0 = p_2\cos\theta - p_1\sin\theta,$$
$$\alpha_0 = \begin{cases} 0, & \text{if } r \geq d_{\max} \\ d_{\max} - r, & \text{if } r < d_{\max} \end{cases}$$
$$\beta_0 = \begin{cases} 1, & \text{if } f_0 \leq 0 \\ -1 & \text{if } f_0 > 0 \end{cases}$$

We then adopt the controllers u_1 and u_2 from Section V and simplify to get:

$$\left.\begin{aligned} u_1 &= \frac{|v_0|\left[(p_1 - x)r - (p_2 - y)\alpha_0\beta_0\right]}{\left\|(x(0) - p_1, y(0) - p_2)\right\|\sqrt{\alpha_0^2 + r^2}}, \\ u_2 &= \frac{|v_0|\left[(p_2 - y)r - (p_1 - x)\alpha_0\beta_0\right]}{\left\|(x(0) - p_1, y(0) - p_2)\right\|\sqrt{\alpha_0^2 + r^2}}. \end{aligned}\right\} \qquad (7)$$

We note that the controllers given in (7) are bounded and continuous at every point over the domain

$$D = \left\{\mathbf{x} \in \mathbf{R}^2 : (x(0), y(0) \neq (p_1, p_2) \cap r > 0\right\}.$$

C. Simulation

Fig. 4 shows an interesting simulation of the manipulator arm starting from an initial configuration and converging to a target configuration whilst avoiding the artificial obstacle along its path. In this example, we notice that the end-effector encounters the artificial obstacle, formed by the mechanical singularity of the system, along its path. The initial and final position of end-effector are shown in Fig. 4.

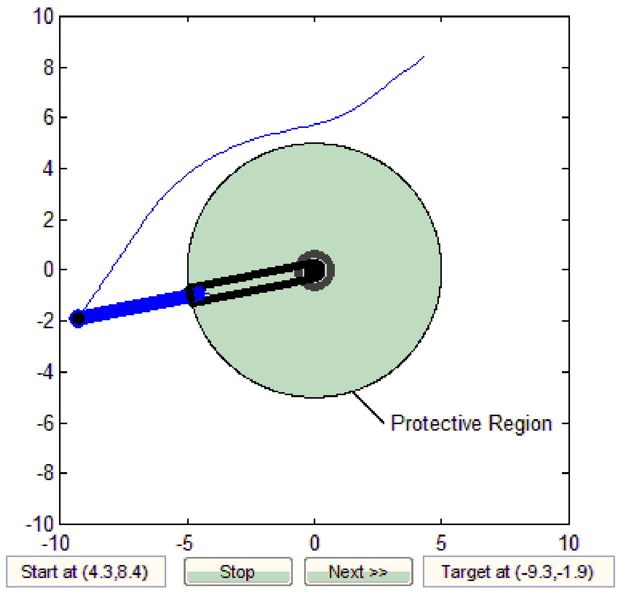

Fig. 4 Trajectory of the end-effector of the robot arm with initial position (4.3,8.4) and the target placed at (−9.3,-1.9)

IX. Conclusion

This paper presents a simple and unique approach for solving the motion planning and control of autonomous robots in a two-dimensional plane. Firstly, a unique velocity algorithm, which vanishes at the goal position, is used to move the robot from an initial position to a goal position. Secondly, in the presence of elliptical obstacles, a turning angle is introduced which helps the robot to deviate away from an obstacle along its path.

The control laws presented in this paper ensures safe and smooth system trajectory and works for any number of fixed elliptical obstacles. This has been verified through computer simulations.

By using the Direct Method of Lyapunov, we see that the system's equilibrium point is asymptotically stable. Moreover, the method proposed in this paper can easily be applied to other robotic systems. As an example, we have applied the idea to an anchored 2-link (RP) manipulator.

Future work will involve fixed and moving obstacles of various other shapes such as line and arc obstacles. The result obtained in this paper can also be generalized to three-dimensional motion planning.

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