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Trajectory Tracking using Artificial Potential Fields

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Abstract**—**In this paper, the trajectory tracking problem for car-like mobile robots have been studied. The system comprises of a leader and a follower robot. The purpose is to control the follower so that the leader's trajectory is tracked with arbitrary desired clearance to avoid inter-robot collision while navigating in a terrain with obstacles. A set of artificial potential field functions is proposed using the Direct Method of Lyapunov for the avoidance of obstacles and attraction to their designated targets. Simulation results prove the efficiency of our control technique.

Keywords**—**Control, Trajectory Tracking, Lyapunov.

I. Introduction

CONTROL problems for nonholonomic systems have been a topic of interest in recent years. The challenges that arise in the development of powerful methods for motion control of autonomous vehicles are due to the nonholonomic constraints, to which many wheeled robots are subjected. The rapid progress in this field has been the interplay of systems, theories and problems [1].

The problems of motion control of mechanical systems with nonholonomic constraints addressed in literature can be roughly classified into three groups [2]:

- point stabilization, where the goal is to stabilize the vehicle at a desired robot posture,
- trajectory tracking, where the vehicle is required to track a time-parameterized reference, and
- path following, where the vehicle is required to converge to and follow a desired path, without temporal specifications.

Point stabilization is very different from problems of trajectory tracking and path following. The subsequent challenge to control systems designers here is when the vehicle has nonholonomic constraints, since according to Brockett [3], asymptotic stabilization of the equilibrium point cannot be achieved using continuous constant state-feedback control laws, although it is controllable in a nonlinear sense.

Path following problems are more flexible than *trajectory tracking* and is primarily concerned with the design of control laws that drive an object to reach and follow a geometric path [4]. Trajectory tracking is more natural for mobile robots where the robot reaches the desired position at a specific time. The reference trajectory is obtained by using a reference robot [5].

Many control algorithms have been proposed in the trajectory tracking framework, such as PID [6], Lyapunov-based controllers [7-9], adaptive controllers [10], model-based predictive controllers [11], fuzzy controllers [12-15], etc. Particular focus is on methods that allow multiple robots to track a reference trajectory without collisions with obstacles or other robots. Recent applications have been evident in fields of exploration, surveillance and even military applications [16].

An extension to our previous work in [17], where we considered the formation control of multiple autonomous robots, this paper presents a set of control laws using a leader-following approach, to ensure a collision-free tracking control strategy for a group of two mobile robots. A robot is assigned the responsibility of a leader, while the follower robot positionsitself relative to the leader so that the trajectory of the leader robot is tracked with arbitrary desired clearance by the follower robot. The scheme uses Cartesian coordinate's representation to avoid any singular points as encountered when using polar coordinates.

Based on artificial potential fields, the Direct Method of Lyapunov is then used to derive continuous acceleration-based controllers which render our system stable. The control algorithm used merges together the control problems of trajectory tracking and obstacle collision avoidance as a single motion control algorithm.

This paper is organized as follows: in Section II the robot model is defined and the proposed scheme to control the mobile robots is discussed; in Sections III and IV the artificial potential field functions are defined; in Section V the dynamic constraints are defined; in Section VI the acceleration-based control laws are derived; in Section VII we illustrate the effectiveness of the proposed controllers via simulations. Conclusions and descriptions of future work are given in Section VIII.

II. Vehicle Model

In this section, we derive a new kinematic model for the leader-following based formation control of two car-like mobile robots.

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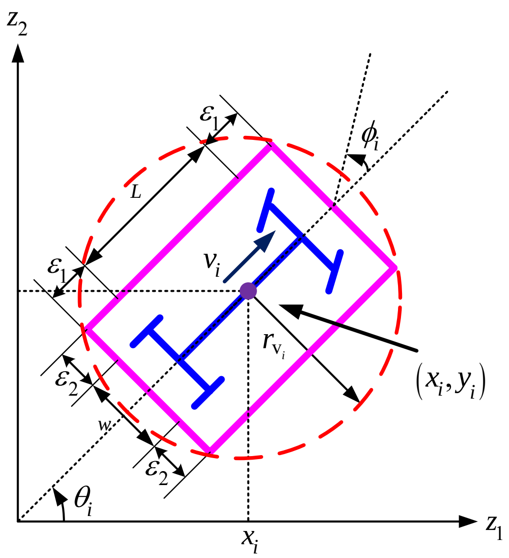


Fig. 1 Kinematic model of the car-like mobile robot

With reference to Fig. 1, $(x_i, y_i) \in \mathbb{R}^2$, for $i = 1,2$, represents the Cartesian coordinates and gives the reference point of each mobile robot, and θ_i gives the orientation of the ith car with respect to the z_1 axis, while ϕ_i gives the ith robot's steering angle with respect to its longitudinal axis. For simplicity, the dimensions of the two mobile robots are kept the same. Therefore, L is the distance between the center of the rear and front axles of the ith car and w is the length of each axle.

Next, to ensure that each robot safely steers past an obstacle, we adopt the nomenclature of [18] and construct circular regions that protect the robot. With reference to Fig. 1, given the clearance parameters, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, we enclose the vehicle by a protective circular region centered at (x_i, y_i) with radius $r_{v_i} := r_v = \sqrt{\frac{(L+2\varepsilon_1)^2 + (w+2\varepsilon_2)^2}{4}}$.

We next assign a Cartesian coordinate system, X-Y, fixed on the leader's body, as shown in Fig. 2. When the leader rotates, we have a rotation of the X-Y axes. Thus, given the leader's position and its orientation, as long as (r_{12}, α) is fixed, the follower's position will be unique. This gives then the polar coordinates representation of the follower's relative position with respect to the leader. However, such representations using polar coordinates lead to certain singularities in the controller.

To eliminate such singular points, we consider the position of the follower by considering the relative distances of the follower from the leader along the X and Y directions.

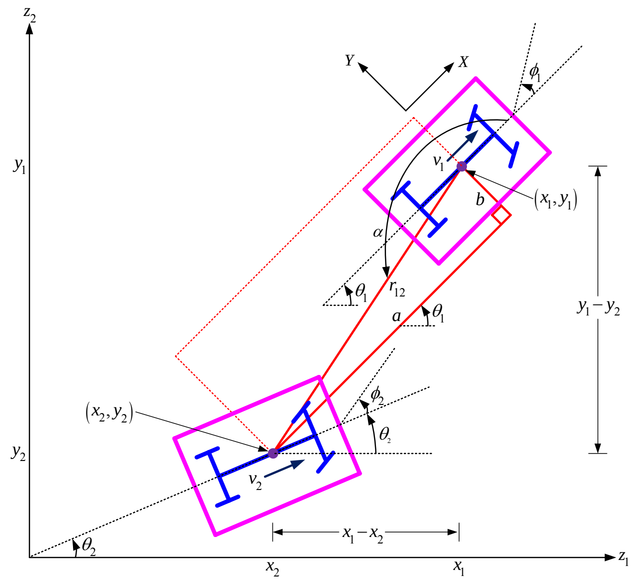


Fig. 2 The proposed scheme utilizing a rotation of axeswith the axes fixed at the leader

Thus, we have:

$$A_X = -(x_1 - x_2)\cos\theta_1 - (y_1 - y_2)\sin\theta_1,$$
$$B_Y = (x_1 - x_2)\sin\theta_1 - (y_1 - y_2)\cos\theta_1,$$

where A_X and B_Y are the followers relative position with respect to the X-Y coordinate system. If A_X and B_Y are known and fixed, the follower's position will be unique. Thus, for the follower robot to track the leader's trajectory with an arbitrary desired clearance, one needs to know distances a and b, the desired relative positions along the X-Y directions, such that the control problem would be to achieve $A_X \to a$ and $B_Y \to b$, that is, $r_{12} \to r_{12}^{\ d}$, where $r_{12}^{\ d} = \sqrt{a^2 + b^2}$. The kinematic model of the system, adopted from [17] is:

$$\left.\begin{aligned} \dot{x}_i &= v_i \cos\theta_i - \tfrac{L}{2}\omega_i \sin\theta_i\ , \\ \dot{y}_i &= v_i \sin\theta_i + \tfrac{L}{2}\omega_i \cos\theta_i\ , \\ \dot{\theta}_i &= \omega_i, \\ \dot{v}_i &= \sigma_{i1}, \\ \dot{\omega}_i &= \sigma_{i2}, \end{aligned}\right\} \tag{1}$$

where v_i and ω_i are, respectively, the instantaneous translational and rotational velocities, while σ_{i1} and σ_{i2} are the instantaneous translational and rotational accelerations of the ith vehicle. Without loss of generality, we assume $\phi_i = \theta_i$. The state of the ith mobile robot is then described by $\mathbf{x}_i := (x_i, y_i, \theta_i, v_i, \omega_i)$, where $i = 1$ represents the leader and $i = 2$ the follower.

III. Attractive Potential Field Functions

This section formulates collision free trajectories of the robot system under kinodynamic constraints in a fixed and bounded workspace. It is assumed that the car-like robots have *apriori* knowledge of the whole workspace. We want to design the acceleration controllers, σ_{i1} and σ_{i2}, so that the pair moves safely towards the leader's target while maintaining a desired formation.

A. Attraction to Target

A target is assigned to the leader. While the leader moves towards its defined target, the follower robot tracks its trajectory. For the leader, we define a target:

$$T = \left\{(z_1, z_2) \in \mathbb{R}^2 : (x_1 - t_1)^2 + (y_1 - t_2)^2 \le rt_1^{\,2}\right\},$$

with center (t_1, t_2) and radius rt_1. For the leader to be attracted to its designated target, we consider thefollowing attractive potential function

$$V_1(\mathbf{x}) = \tfrac{1}{2}\left[(x_1 - t_1)^2 + (y_1 - t_2)^2 + v_1^2 + \omega_1^2\right]. \quad (2)$$

To ensure that the follower robot tracks the leader's trajectory with an arbitrary desired clearance, we utilize the following potential function:

$$V_2(\mathbf{x}) = \tfrac{1}{2}\left[(A_X - a)^2 + (B_Y - b)^2 + v_2^2 + \omega_2^2\right]. \quad (3)$$

B. Auxiliary Function

To guarantee the convergence of the mobile robots to their designated goals, we design auxiliary functions defined as:

$$G_1(\mathbf{x}) = \tfrac{1}{2}\left[(x_1 - t_1)^2 + (y_1 - t_2)^2 + (\theta_1 - t_3)^2\right], \quad (4)$$

and

$$G_2(\mathbf{x}) = \tfrac{1}{2}\left[(A_X - a)^2 + (B_Y - b)^2 + (\theta_2 - t_4)^2\right], \quad (5)$$

where t_3 and t_4 represent the desired final orientations of the leader and follower, respectively. These potential functions are then multiplied to the repulsive potential functions to be designed in the following sections.

IV. Repulsive Potential Field Functions

We desire the mobile robots to avoid all stationary obstacles intersecting their paths. For this, we construct the obstacle avoidance functions that merely measure the Euclidean distances between each robot and the obstacles in the workspace. To obtain the desired avoidance, these potential functions appear in the denominator of the repulsive potential field functions. This creates a repulsive field around the obstacles.

Let us fix q solid obstacles within the workspace and assume that the lth obstacle is circular with center (o_{l1}, o_{l2}) and radius ro_l. For the ith robot with a circular avoidance region of radius r_v to avoid the lth obstacle, we adopt

$$FO_{il}(\mathbf{x}) = \frac{1}{2}\left[(x_i - o_{l1})^2 + (y_i - o_{l2})^2 - (ro_l + r_v)^2\right], \quad (6)$$

for $i = 1,\ 2$ and $l = 1, 2, \ldots, q$.

V. Dynamic Constraints

Practically, the steering and bending angles of the ith mobile robot are limited due to mechanical singularities while the translational speed is restricted due to safety reasons. Subsequently, we have; **(i)** $|v_i| \le v_{max}$, where v_{max} is the maximal speed of the ith robot; **(ii)** $|\phi_i| \le \phi_{max} < \pi/2$, where ϕ_{max} is the maximal steering angle. Considering these constraints as artificial obstacles, we have the following potential field functions:

$$U_{i1}(\mathbf{x}) = \tfrac{1}{2}\left[(v_{\max} - v_i)(v_{\max} + v_i)\right], \quad (7)$$

$$U_{i2}(\mathbf{x}) = \tfrac{1}{2}\left[\left(\frac{v_{\max}}{|\rho_{\min}|} - \omega_i\right)\left(\frac{v_{\max}}{|\rho_{\min}|} + \omega_i\right)\right], \quad (8)$$

for $i = 1,\ 2$. These potential functions guarantee the adherence to the above restrictions placed upon the translational velocity v_i, and the steering angle ϕ_i.

VI. Control Laws

Combining all the potential functions $(2-8)$, and introducing constants, denoted as the control parameters, $\alpha_{il} > 0$ and $\beta_{is} > 0$, for $i = 1, 2$, $l = 1, 2, \ldots, q$, $q \in \mathbb{N}$ and $s = 1, 2$, we define a candidate Lyapunov function as:

$$L(\mathbf{x}) = \sum_{i=1}^{2}\left\{V_i(\mathbf{x}) + G_i(\mathbf{x})\left[\sum_{l=1}^{q}\frac{\alpha_{il}}{FO_{il}(\mathbf{x})} + \sum_{s=1}^{2}\frac{\beta_{is}}{U_{is}(\mathbf{x})}\right]\right\} \quad (9)$$

Clearly, $L(\mathbf{x})$ is locally positive and continuous on the domain $D(L) = \left\{\mathbf{x} \in \mathbb{R}^{5\times 2} : FO_{il}(\mathbf{x}) > 0,\ U_{is}(\mathbf{x}) > 0\right\}$. We define $\mathbf{x}_e := (t_1, t_2, t_3, 0, 0)$ as an equilibrium point of system (1). Thus, we have $L(\mathbf{x}_e) = 0$.

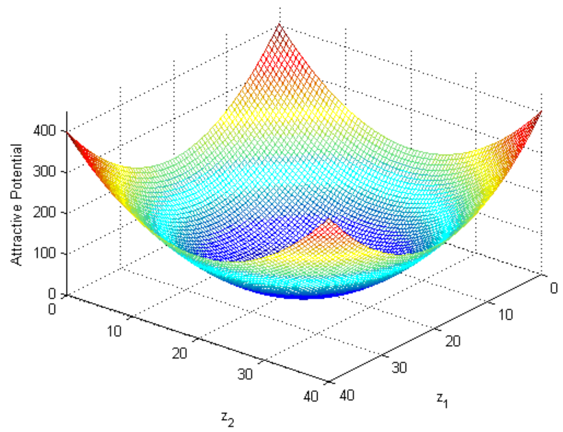


Fig. 3 A three-dimensional view of the attractive potential

The attractive potential, as shown in Fig. 3 and the corresponding contour plot in Fig. 4, are generated for target attraction. For better visualization, the target of the leader is located at $(t_1, t_2) = (20, 20)$.

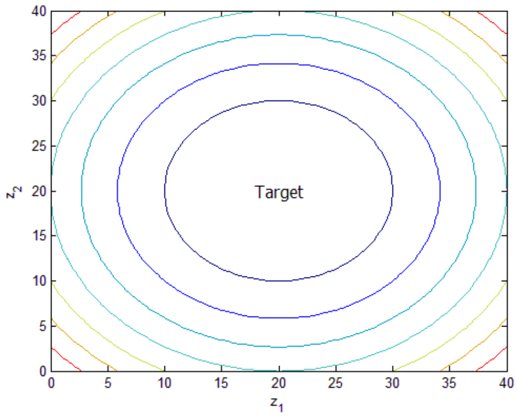


Fig. 4 The corresponding contour plot of the attractive potential

To extract the control laws, we differentiate the various components of $L(\mathbf{x})$ separately and carry out the necessary substitutions from (1). The nonlinear control laws for system (1) will be designed using Lyapunov's Direct Method. The process begins with the following theorem:

Theorem: *Consider a pair of car-like mobile robots whose motion is governed by the ODE's described in system (1).The principal goal is to establish and facilitate maneuvers of the robots within a constrained environment and reach the target configuration while ensuring that the follower robot tracks the trajectory of the lead robot. The subtasks include; restrictions placed on the workspace, convergence to predefined targets, and consideration of kinodynamic constraints. Utilizing the attractive and repulsive potential field functions, the following continuous time-invariant acceleration control laws can be generated, that intrinsically guarantees stability,in the sense of Lyapunov, of system (1) as well:*

$$\sigma_{i1} = -\tfrac{1}{g_{i1}}\left[\delta_{i1} v_i + f_{i1}\cos\theta_i + f_{i2}\sin\theta_i\right], \tag{10}$$

and

$$\sigma_{i2} = -\tfrac{1}{g_{i2}}\left[\delta_{i2}\omega_i + \tfrac{L}{2}\left(f_{i2}\cos\theta_i + f_{i1}\sin\theta_i\right) + f_{i3}\right], \tag{11}$$

for $i = 1,\ 2$.

Proof: The time derivative of our Lyapunov function $L(\mathbf{x})$ along a particular trajectory of system (1) is:

$\dot{L}_{(1)}(\mathbf{x}) = -\sum_{i=1}^{n}\left(\delta_{i1}v_i^2 + \delta_{i2}\omega_i^2\right) \le 0$ for all $\mathbf{x} \in D(L)$, and $\dot{L}_{(1)}(\mathbf{x}_e) = 0$,

where the functions f_{ik} to g_{ij}, for $i, j = 1,\ 2$ and $k = 1,..,3$ are defined as (upon suppressing $\mathbf{x}$):

$$f_{11} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](x_1 - t_1) - G_1\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}^2}(x_1 - o_{l1}),$$

$$f_{12} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](y_1 - t_2) - G_1\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}^2}(y_1 - o_{l2}),$$

$$f_{13} = \left[\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](\theta_1 - t_3),$$

$$g_{11} = 1 + G_1\frac{\beta_{11}}{U_{11}^2},\ g_{12} = 1 + G_1\frac{\beta_{12}}{U_{12}^2},$$

$$f_{21} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right]\left[(-A_X - a)\cos\theta_1 + (B_Y - b)\sin\theta_1\right] - G_2\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}^2}(x_2 - o_{l1}),$$

$$f_{22} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right]\left[(-A_X - a)\sin\theta_1 - (B_Y - b)\cos\theta_1\right] - G_2\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}^2}(y_2 - o_{l2}),$$

$$f_{23} = \left[\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right](\theta_2 - t_4),$$

$$g_{21} = 1 + G_2\frac{\beta_{21}}{U_{21}^2},\ g_{22} = 1 + G_2\frac{\beta_{22}}{U_{22}^2}.$$

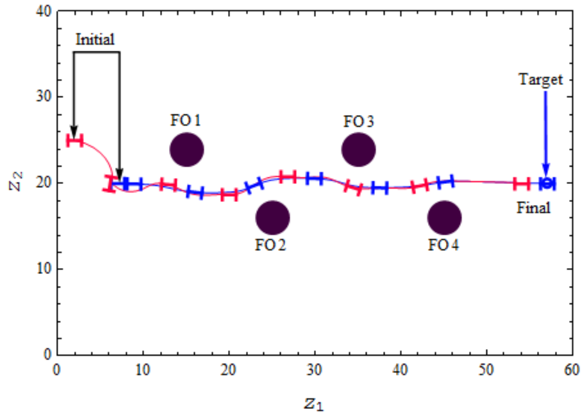
A careful scrutiny of the properties of our scalar function reveals that $\mathbf{x}_e$ is an equilibrium point of system (1) in the sense of Lyapunov and $L(\mathbf{x})$ is a legitimate Lyapunov function guaranteeing stability. This is in no contradiction with Brockett's result [3], as we have not proven asymptotic stability.

VII. Simulations

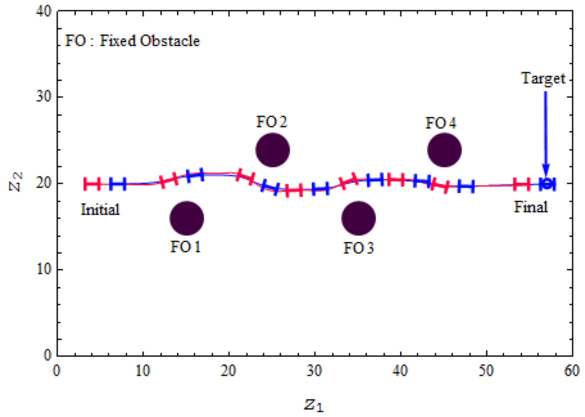
To illustrate the effectiveness of the proposed controllers, we present two car-like mobile robots in Fig. 5. The follower robot is assigned a unique position relative to the leader robot. As such, while the leader moves towards its intended target, the follower tracks the leader's trajectory. Upon encountering an obstacle, the formation of the articulated robots does not change, but the robots are able to move around the obstacle.

In Scenario 1, given a desired clearance distance between the leader and follower robots, we purposely placed the follower robot away from the desired position. This was done to view the effectiveness of the proposed scheme. It is evident that while the follower robot is positioned away from the leader, the scheme guarantees convergence to the desiredcoordinates, to enable the follower robot to track the trajectory of the leader. Scenario 2 had the follower placed at the desired relative position and had smooth convergence.

Fig. 6 shows convergence of the relative positions, (A_X, B_Y) of the follower robot to the desired relative positions, (a, b) in Scenario 1.



(a) Scenario 1



(b) Scenario 2

Fig. 5 Follower robot tracking the leader robot's trajectory

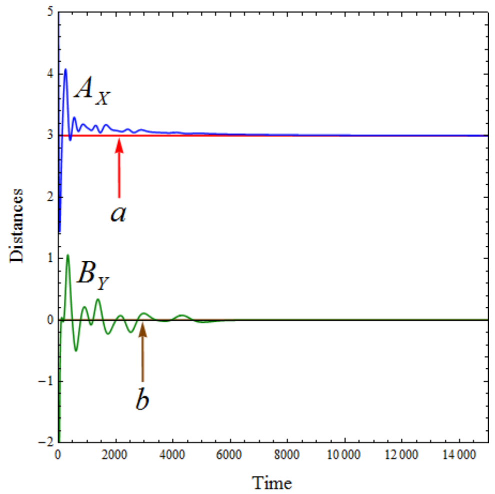


Fig. 6 Relative distances of the follower robot to the leader

Fig. 8 shows the translational and rotations velocities of the leader and follower. The corresponding initial and final states and other details for the simulation are listed in Table I (assuming that appropriate units have been taken into account).

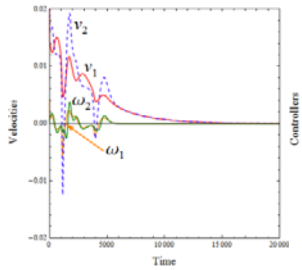


Fig. 7 Velocities v_i, ω_i

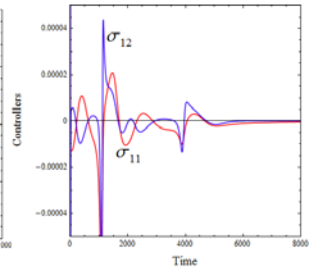


Fig. 8 Accelerations σ_{11}, σ_{12}

TABLE I
NUMERICAL VALUES OF INITIAL STATES, CONSTRAINTS AND PARAMETERS FOR SCENARIOS 1 AND 2

		Initial Conditions
Rectangular positions		$(x_1, y_1) = (7, 20)$
Angular Positions and velocities		$v_1 = 0.5$, $\omega_i = 0$, $\theta_i = 0$ for $i = 1,2$
		Constraints and Parameters
Final Orientations		$t_3 = t_4 = 0$
Leader Target		$(t_1, t_2) = (57, 20)$, $rt_1 = 0.5$
Dimensions of Robots		$L = 1.6$, $w = 1.2$
Desired clearance		$(a, b) = (3, 0)$
Relative Positions	Scenario 1	$(x_2, y_2) = (2, 25)$;
	Scenario 2	$(x_2, y_2) = (4, 20)$
Fixed Obstacles (o_{l1}, o_{l2})	Scenario 1	$(o_{11}, o_{12}) = (15, 24)$, $(o_{21}, o_{22}) = (25, 16)$; $(o_{31}, o_{32}) = (35, 24)$, $(o_{41}, o_{42}) = (45, 16)$; $ro_1 = ro_2 = ro_3 = ro_4 = 2$
	Scenario 2	$(o_{11}, o_{12}) = (15, 16)$, $(o_{21}, o_{22}) = (25, 24)$; $(o_{31}, o_{32}) = (35, 16)$, $(o_{41}, o_{42}) = (45, 24)$; $ro_1 = ro_2 = ro_3 = ro_4 = 2$
Max. translational speed		$v_{\max} = 5$
Min. turning radius		$\rho_{\min} = 0.14$
Clearance parameters		$\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.05$
		Control and Convergence Parameters
Obstacle avoidance		$\alpha_{il} = 0.1$ for $i = 1,2$ and $l = 1$ to 4
Dynamics constraints		$\beta_{is} = 0.001$ for $i = 1,2$ and $s = 1,2$
Convergence		$\delta_{i1} = 500$, $\delta_{i2} = 50$ for $i = 1,2$

VIII. CONCLUSION

This paper presents a set of artificial field functions derived using Lyapunov's Direct Method, for the control problem of trajectory tracking of mobile robots, using a leader-following strategy. By using Cartesian coordinates to uniquely assign a position to a follower, we can achieve a desired convergence with bounded distance error and heading angle. The derived controllers produced feasible trajectories and ensured a nice convergence of the system to its equilibrium state while satisfying the necessary kinematic and dynamic constraints. We note here that convergence is only guaranteed from a number of initial states of the system.

Future research will address more general applications with more than two mobile robots, and tractor trailer systems.

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