Speed Sensorless Control with a Linearization by State Feedback of Asynchronous Machine Using a Model Reference Adaptive System

A. Larabi, and M. S. Boucherit

Abstract—In this paper, we show that the association of the PI regulators for the speed and stator currents with a control strategy using the linearization by state feedback for an induction machine without speed sensor, and with an adaptation of the rotor resistance. The rotor speed is estimated by using the model reference adaptive system approach (MRAS). This method consists of using two models: The first is the reference model and the second is an adjustable one in which two components of the stator flux, obtained from the measurement of the currents and stator voltages are estimated. The estimated rotor speed is then obtained by canceling the difference between stator-flux of the reference model and those of the adjustable one. Satisfactory results of simulation are obtained and discussed in this paper to highlight the proposed approach.

Keywords—Asynchronous actuator, PI Regulator, adaptive method with reference model, Vector control.

I. INTRODUCTION

WITH the advent of automated production, the control became the operation consisting in creating a strong dependency between an articulated system and its command. As usual, asynchronous actuators influence the articular variables of a robot; the position control is realized through a suitable action on their command.

The vector control allows having a decoupling between torque and flux of the machine and consequently dynamic performances similar to those of a DC machine are achieved [1], [2]. Its implementation based on a linear approximation of the machine dynamics around the operation point does not allow having a good compensation of the machine non-linear model, in particular, for the disturbed and the transitory modes. In order to solve some specific problems related to vector control and ensure a decoupling whatever the machine operation mode, the usage of nonlinear commands become unavoidable.

Among these nonlinear techniques, we can quote the control using the linearization strategy by state feedback. However, from literature studies, PI regulators strongly depend on the machine parameters in particular the rotor time-constant (T_r). A bad identification of these parameters or a bad estimation of the variables to be controlled will lead to a deterioration of the adjustment performances especially when

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the machine is loaded. Moreover, the presence of mechanical sensor increases the volume and overall system cost. In addition, the installation of this sensor is difficult and reduces the reliability of the system especially for small machines.

Considering this major inconvenience, a solution is obtained by the association of the PI regulators for the speed and stator currents with a control strategy using the linearization by state feedback for an induction machine without speed sensor, and with an adaptation of the rotor resistance [3], [4], [5], [6]. The rotor speed is estimated by using the model reference adaptive system approach (MRAS). This method consists of using two models. The first one is the reference model and the second is an adjustable one in which two components of the stator flux, obtained from the measurement of the currents and stator voltages are estimated [7] [8].

The estimated rotor speed is then obtained by canceling the difference between stator-flux of the model of reference and those of the adjustable model. The adaptation mechanism is obtained from the hyperstability theory. Satisfactory results of simulation are presented at the end of this paper to highlight the proposed approach.

II. PRINCIPLE OF THE LINEARIZATION BY STATE FEEDBACK

The control of the system is governed by the following state equation: [3]

$$\frac{dx}{dt} = f(x) + g(x)\,u \,,\qquad(1)$$

Where:
- $f(x)$ and $g(x)$ are nonlinear functions of the state variable x;
- u represents the command vector applicable to the system.

Each output x_i is derived several times until one of the components of the control vector appears. Thus, the following equation is obtained:

$$\frac{d^{r_1} x_1}{dt^{r_1}},$$

$$\frac{d^{r_2} x_2}{dt^{r_2}} = B(x) + A(x)\,u,\qquad(2)$$

$$\frac{d^{r_n} x_n}{dt^{r_n}}.$$

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If the matrix *A(x)* is invertible, it becomes possible to adopt any dynamic behaviour on the outputs of the system:

$$\frac{d^{r_1} x_1}{dt^{r_1}} = v_1,$$

$$\frac{d^{r_2} x_2}{dt^{r_2}} = v_2, \qquad (3)$$

$$\frac{d^{r_n} x_n}{dt^{r_n}} = v_n.$$

By giving the following command:

$$u = A^{-1}(x).[v - B(x)]. \qquad (4)$$

The flowchart of figure (1) is obtained by the linearization by state feedback, which transforms the original system into a linear and decoupled equivalent system with the condition that the decoupling matrix is non-singular.
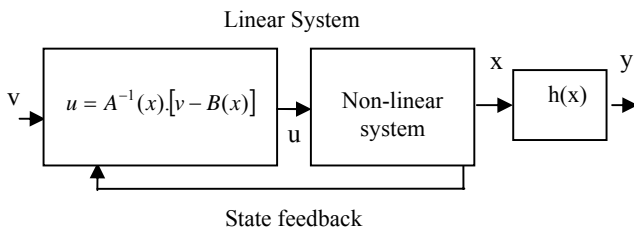

Fig. 1 Flowchart of the linearization by state feedback

A. Application to the Asynchronous Actuator

The equations of the electric part of the asynchronous actuator can be written in the *Park d-q* configuration as follows:

$$\frac{di_{ds}}{dt} = \frac{1}{\sigma L_s}(V_{ds} - R_s i_{ds} - \frac{M}{L_r}\frac{d\psi_{dr}}{dt}) + \omega\, i_{qs}, \qquad (5.a)$$

$$\frac{di_{qs}}{dt} = \frac{1}{\sigma L_s}(V_{qs} - R_s i_{qs} - \frac{\omega M}{L_r}\psi_{dr}) - \omega\, i_{ds}, \qquad (5.b)$$

$$\frac{d\psi_{dr}}{dt} = \frac{R_r}{L_r}(M\, i_{ds} - \psi_{dr}). \qquad (5.c)$$

The position of the reference frame is selected in command to obtain $\psi_{qr} = 0$, rotor flux is then entirely represented by ψ_{dr}.

By introducing the equation (5.c) in (5.a) we obtain the following equations:

$$\sigma L_s \frac{di_{ds}}{dt} + R_s\, i_{ds} = V_{ds} + \sigma L_s \omega\, i_{qs} - \frac{L_s(1-\sigma)}{T_r}(i_{ds} - \frac{\phi_r}{M})$$

$$\sigma L_s \frac{di_{qs}}{dt} + R_s i_{qs} = V_{qs} - \omega\,(\sigma L_s\, i_{ds} + \frac{L_s(1-\sigma)}{M}\phi_r) \qquad (6)$$

$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

The equilibrium equation between the actuator torque and the load torque generated by the mechanical part of the system has the following form:

$$J\frac{d\omega_m}{dt} = C_{em} - f\,\omega_m - C_r \qquad (7)$$

$$\omega_m = \omega_r / \mathrm{p}$$

Where:

- i_{ds} and i_{qs}, v_{ds} and v_{qs} are respectively the *d* and *q* axes stator currents and voltages;
- ψ_r is the rotor flux;
- ω is the angular speed of the d,q reference frame with respect to the actuator stator and θ its position;
- ω_{sr} is the difference between the angular speed of the reference frame and the electrical speed ω_r of the rotor (the electrical speed of the rotor is equal to *p* times its mechanical speed ω_m, p being the pole pairs number) $\omega_{sr} = \omega - \omega_r$;
- T_s and T_r are respectively the stator and rotor *d* and *q* windings time constants;
- σ is the leakage coefficient of the machine;
- M is the mutual inductance between rotor and stator *d-q* windings;
- L_r is the rotor *d-q* windings inductance;
- R_s and R_r are respectively the stator and rotor *d-q* windings resistances;
- C_{em} is the electromagnetic torque;
- *J* is inertia of actuator;
- *f* is Viscous friction coefficient.

In our case we take:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}, \ [U] = \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix}$$

Therefore:

$$B(x) = \begin{bmatrix} \dfrac{-1}{\sigma L_s}\left[\ (R_s + \dfrac{M^2 R_r}{L_r^2})\,i_{ds} - \dfrac{MR_r}{L^2_r}\,\psi_{dr}\ \right] + \omega\ i_{qs} \\[3mm] \dfrac{-1}{\sigma L_s}\left[\ R_s i_{qs} + \dfrac{\omega M}{L_r}\,\psi_{dr}\ \right] - \omega\, i_{ds} \end{bmatrix}$$

$$A(x) = \frac{1}{\sigma L_S}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let us remind that *A(x)* is an invertible matrix.

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B. Nonlinear Law of Control

Since the system is linearisable, it is possible to impose on it the desired dynamics. In our case we considered the control law given by equation 8.

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = A^{-1}\left[-B(x) + \begin{pmatrix} v1 \\ v2 \end{pmatrix}\right] \qquad (8)$$

Where v_1 *et* v_2 are the outputs of the PI regulators of the stator currents appearing in the flowchart of the asynchronous machine given by Fig. 3.

III. ADAPTATION OF THE ROTOR RESISTANCE

To estimate the resistance of the rotor we used a method which makes possible to calculate the latter in function to flux (real, reference) and of the electromagnetic torque (real, reference) [4].

To clarify the relation which binds the electromagnetic torque and rotor flux to the variations of the parameters of the machine, we let us proceed as follows:

$$\begin{aligned} L_r &= K_l L_r^* \\ R_r &= K_r R_r^* \end{aligned} \qquad (9)$$

L_r^*, R_r^* Values used in the command.

The actual value of the rotor time-constant:

$$T_r = \frac{K_l}{K_r}T_r^* = KT_r^*$$

The block of decoupling imposes on the command of the inverter the values V$_{ds}$, V$_{qs}$ and ω_{sr}. In permanent mode we have:

$$\begin{aligned} C_e^* &= \frac{PM}{L_r^*}\varphi_r i_{qs} = \frac{PM^2}{L_r^*}i_{ds}i_{qs} \\ \omega_{sr}^* &= \frac{1}{T_r^*}\frac{i_{qs}}{i_{ds}} \\ \varphi_r^* &= M\ i_{ds} \end{aligned} \qquad (10)$$

From the equations of Park of the machine we determine the direct components and in squaring from rotor flux and the real torque of the machine with permanent regime:

$$\varphi_{dr} = M\frac{i_{ds} + \omega_{sr}T_r i_{qs}}{1 + (\omega_{sr}T_r)^2}$$

$$\varphi_{qr} = M\frac{i_{qs} - \omega_{sr}T_r i_{ds}}{1 + (\omega_{sr}T_r)^2}$$

$$C_e = P\frac{K_l^2}{K_r}\frac{M^2}{L_r^*}i_{ds}i_{qs}\frac{1+(\frac{i_{qs}}{i_{ds}})^2}{1+(K\frac{i_{qs}}{i_{ds}})^2} \qquad (11)$$

$$\varphi_r = K_l M\ i_{ds}\sqrt{\frac{1+\frac{(\frac{i_{qs}}{i_{ds}})^2}{i_{ds}}}{1+(K\frac{i_{qs}}{i_{ds}})^2}}$$

The rapports of the real torque and flux on the estimated values are:

$$\frac{\varphi_r}{\varphi_r^*} = K_l\sqrt{\frac{1+(\frac{i_{qs}}{i_{ds}})^2}{1+(K\frac{i_{qs}}{i_{ds}})^2}}$$

$$\frac{C_e}{C_e^*} = \frac{K_l^2}{K_r}\frac{1+(\frac{i_{qs}}{i_{ds}})^2}{1+(K\frac{i_{qs}}{i_{ds}})^2} = \frac{1}{K_r}\left(\frac{\varphi_r}{\varphi_r^*}\right)^2 \qquad (12)$$

$$K_r = \frac{C_e^*}{C_e}\left(\frac{\varphi_r}{\varphi_r^*}\right)^2$$

Therefore, the estimation of the rotor resistance is given by following relation:

$$R_r = K_r R_r^* \qquad (13)$$

IV. SPEED ESTIMATION BY THE MRAS TECHNIQUE

To estimate rotor speed, it is judicious to use a reference frame related to the stator (α, β) given by the following equations. This transformation does not call upon the position of the rotor which we estimate by the model reference method [7],[8].

$$\begin{aligned} V_{\alpha s} &= R_s i_{\alpha s} + \frac{d\varphi_{\alpha s}}{dt} \\ V_{\beta s} &= R_s i_{\beta s} + \frac{d\varphi_{\beta s}}{dt} \\ 0 &= R_r i_{\alpha r} + \frac{d\varphi_{\alpha r}}{dt} + \omega_r \varphi_{\beta r} \\ 0 &= R_r i_{\beta r} + \frac{d\varphi_{\beta r}}{dt} - \omega_r \varphi_{\alpha r} \end{aligned} \qquad (14)$$

$$\begin{aligned} \varphi_{\alpha s} &= L_s i_{\alpha s} + L_m i_{\alpha r} \\ \varphi_{\beta s} &= L_s i_{\beta s} + L_m i_{\beta r} \\ \varphi_{\alpha r} &= L_r i_{\alpha r} + L_m i_{\alpha s} \\ \varphi_{\beta r} &= L_r i_{\beta r} + L_m i_{\beta s} \end{aligned} \qquad (15)$$

While using (15), (14) rewrites itself in the following form:

$$\begin{aligned} 0 &= L_s(R_r + p\sigma L_r)i_{\alpha s} - \sigma L_r L_s \omega_r i_{\beta s} + (R_r + pL_r)\varphi_{\alpha s} + \omega_r L_r \varphi_{\beta s} \\ 0 &= \sigma L_r L_s \omega_r i_{\alpha s} - L_s(R_r + p\sigma L_r)i_{\beta s} - \omega_r L_r \varphi_{\alpha s} + (R_r + pL_r)\varphi_{\beta s} \end{aligned} \qquad (16)$$

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Basing on the dynamic model of the asynchronous machine, formulated in a stator reference frame, and by using measurements of the stator currents and tensions, we build two estimators of stator flux. The first is based on (14) and the second on (16), such as:

$$\varphi_{\alpha s} = \int_0^t (V_{\alpha s} - R_s i_{\alpha s})dt$$
$$\varphi_{\beta s} = \int_0^t (V_{\beta s} - R_s i_{\beta s})dt \qquad (17)$$

$$\hat{\varphi}_{\alpha s} = \frac{T_r}{1+T_r p}(\sigma L_s \omega_r i_{\beta s} + \frac{L_s}{T_r}(1+\sigma T_r p)i_{\alpha s} - \omega_r \hat{\varphi}_{\beta s})$$
$$\hat{\varphi}_{\beta s} = \frac{T_r}{1+T_r p}(\sigma L_s \omega_r i_{\alpha s} + \frac{L_s}{T_r}(1+\sigma T_r p)i_{\beta s} - \omega_r \hat{\varphi}_{\alpha s}) \qquad (18)$$

In the system (17) relating to the stator observations, it is easy to notice that it does not depend on the rotor speed ω_r. This model is retained as a reference. In the rotor estimators relating to the system (18), we notice the existence of the rotor speed ω_r. It is the adjustable model. The system (18) can be written in the following form:

$$p[\hat{\varphi}_s] = [A][\hat{\varphi}_s] + [B][I_s] \qquad (19)$$

$$[\hat{\varphi}_s] = \begin{bmatrix} \hat{\varphi}_{\alpha s} \\ \hat{\varphi}_{\beta s} \end{bmatrix}; \quad [A] = \begin{bmatrix} -\frac{1}{T_r} & -\omega_r \\ \omega_r & -\frac{1}{T_r} \end{bmatrix}; \quad [I_s] = \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix}$$

$$[B] = \begin{bmatrix} (1+\sigma T_r p)\frac{L_s}{T_r} & \sigma L_s \omega_r \\ -\sigma L_s \omega_r & (1+\sigma T_r p)\frac{L_s}{T_r} \end{bmatrix}$$

By using the same inputs (stator currents and voltages) for the two models (reference and adjustable), we define the flux variations by the expressions:

$$\begin{cases} \varepsilon_d = \varphi_{\alpha s} - \hat{\varphi}_{\alpha s} \\ \varepsilon_q = \varphi_{\beta s} - \hat{\varphi}_{\beta s} \end{cases} \qquad (20)$$

These variations are used by the adaptation mechanism to generate the estimated speed and to make it converge towards its actual value. The adaptation mechanism must be designed in order to obtain a fast and a stable time response. The following figure represents the estimation technique principle.
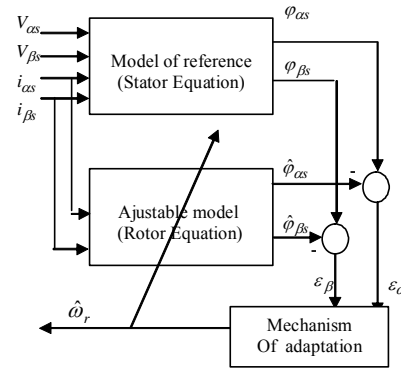

Fig. 2 Model reference estimation structure

The derivative of the components of error (20) is defined by:

$$p\begin{bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_r} & -\omega_r \\ \omega_r & -\frac{1}{T_r} \end{bmatrix}\begin{bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \end{bmatrix} + \begin{bmatrix} -\hat{\varphi}_{\beta s} + \sigma L_s i_{\beta s} \\ \hat{\varphi}_{\alpha s} - \sigma L_s i_{\alpha s} \end{bmatrix}(\omega_r - \hat{\omega}_r)$$

In the form of state

$$p[\varepsilon] = [A][\varepsilon] + [W] \qquad (21)$$

According to the general structure of the adaptive mechanism, the speed estimation is a function of the error. It is given by:

$$p\hat{\omega}_r = A_1 + PA_2 \qquad (22)$$

Functions A_1 and A_2, are calculated starting from the inequality of Popov. Thus, we obtain:

$$A_1 = K_2\left[\varphi_{\beta s}\hat{\varphi}_{\alpha s} - \varphi_{\alpha s}\hat{\varphi}_{\beta s} - (i_{\alpha s}\varepsilon_\beta - i_{\beta s}\varepsilon_\alpha)\sigma L_s\right]$$
$$A_2 = K_1\left[\varphi_{\beta s}\hat{\varphi}_{\alpha s} - \varphi_{\alpha s}\hat{\varphi}_{\beta s} - (i_{\alpha s}\varepsilon_\beta - i_{\beta s}\varepsilon_\alpha)\sigma L_s\right] \qquad (23)$$

Where K_1 and K_2 positive constants are called profits of adaptation.

V. DIGITAL SIMULATION

The flowchart of vector control using the linearization by state feedback without speed sensor of the induction machine fed by PWM inverter, by using PI regulators for the speed and stator currents is given by the Fig. 3.

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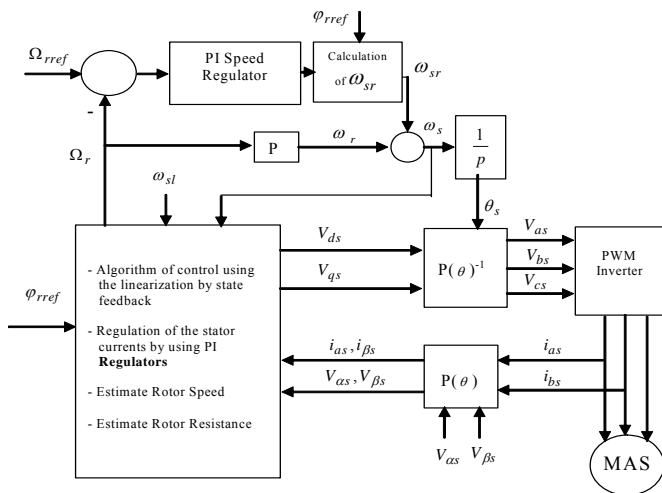Vol:3, No:9, 2009

Fig. 3 Flowchart of control with a linearization by state feedback of the asynchronous machine

Figs. 4 (a), (b), (c), shows the dynamic responses of the adjustment using the linearization by state feedback of the asynchronous machine supplied by PWM inverter. The stator-currents are controlled by PI regulators.

These figures represent the dynamic responses of the real speed, estimated and of reference, the torque and rotor flux at a step speed of 100 rd/s followed by an application of a load equal to the nominal torque between 1 and 1.5 sec, then of an inversion speed from 2 sec in the following cases:

- Without variation of the rotor time-constant T_r
- Reduction of 50% of the rotor time-constant T_r
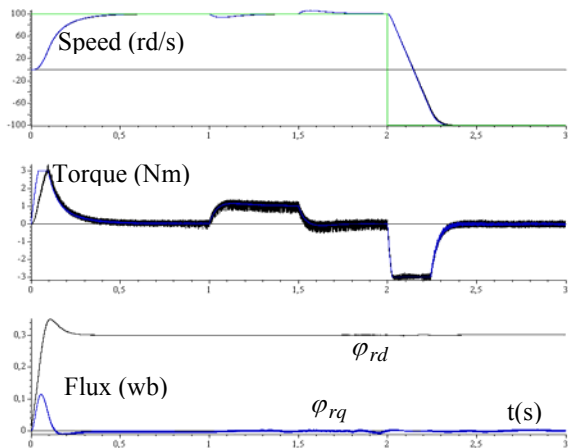

Fig. 4 (a) Simulation of the vector control without speed sensor and without variation of T_r
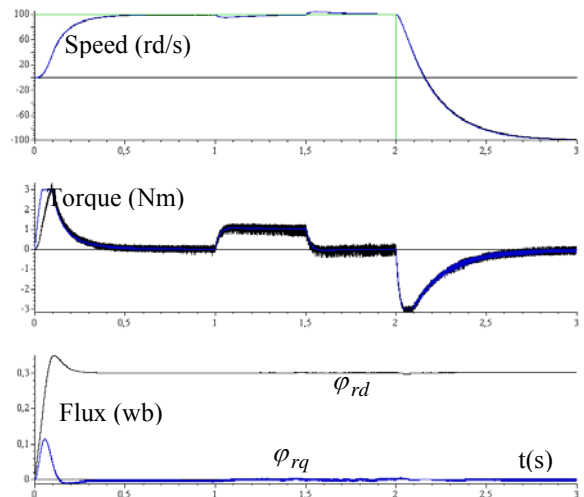

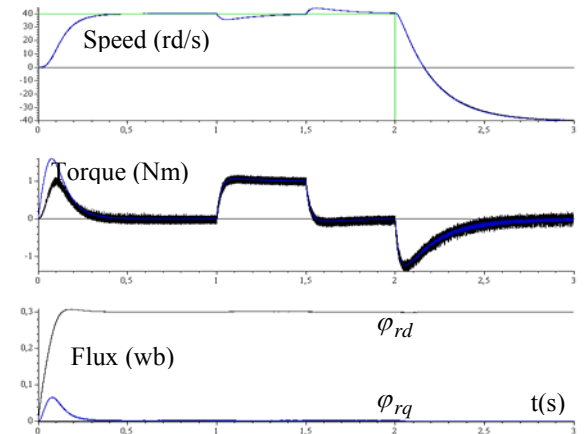Fig. 4 (b) Simulation of the vector control without speed sensor with a 50% reduction of T_r at t=1sec.

Fig. 4 (c) Simulation (for low speed) of the vector control without speed sensor with a 50% reduction of T_r at t=1sec.

VI. CONCLUSION

The results obtained carry out us to conclude that the control strategy used offers good performances like its insensitivity with respect to the disturbances internal and external, such as the level speed, the load torque, in the presence of a variation of the rotor time-constant $\mathbf{T}_r$, which attests robustness of the command used.

This association could be employed for the robotics or the numerical control of the machine tools where the parametric variations frequently occur.

APPENDIX

Machine parameters:

0.25 kW, 110 V, 1800 rpm

The parameters are:

Rr = 1,739 Ω,   Rs = 1.923 Ω,   Lr = 0, 1154 H
M = 0, 1126 H,   Ls = 0, 1157 H
J = 0,004 kgm^2 f =2, 5 Nm/rd/s

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