Convective Heat Transfer of Viscoelastic Flow in a Curved Duct

M. Norouzi, M. H. Kayhani, M. R. H. Nobari, and M. Karimi Demneh

***Abstract*—**In this paper, fully developed flow and heat transfer of viscoelastic materials in curved ducts with square cross section under constant heat flux have been investigated. Here, staggered mesh is used as computational grids and flow and heat transfer parameters have been allocated in this mesh with marker and cell method. Numerical solution of governing equations has being performed with FTCS finite difference method. Furthermore, Criminale-Eriksen-Filbey (CEF) constitutive equation has being used as viscoelastic model. CEF constitutive equation is a suitable model for studying steady shear flow of viscoelastic materials which is able to model both effects of the first and second normal stress differences. Here, it is shown that the first and second normal stresses differences have noticeable and inverse effect on secondary flows intensity and mean Nusselt number which is the main novelty of current research.

***Keywords*—**Viscoelastic, fluid flow, heat convection, CEF model, curved duct, square cross section.

I. Introduction

INVESTIGATION of flow and heat transfer in curved duct is an interesting subject for researchers in the present and the past. This type of flow is an important subject in fluid mechanics that has different applications in industry and medical issues. So far, a lot of researches have been carried out in these fields. The most of these researches related to Newtonian fluids while, a few number of researches have been done about non-Newtonian fluids specially viscoelastic fluids. First done research about flow in curved pipes was carried out by Dean [1]-[2]. He used perturbation method and analyzed Newtonian flow in curved pipe. He found Taylor-Gortler secondary flows through in his research. This method was used with other researchers for studying the flow of viscoelastic materials in curved pipes such as Thomas and Walters [3], Robertson and Muller [4] and Sarin [5]-[6] about Oldroyd-B fluid, Jitchote and Robertson [7], Bowen et al. [8] and Sharma and Prakash [9] about the second order fluid and Imeto [10]-[11] about power law and White-Metzner fluid. Some of researchers studied this type of flows by using numerical methods that can be mentioned the researches of Phan-Thien and Zheng [12], Fan et al. [13] and Zhang et al. [14] about Oldroyd-B fluid and Helin et al. [15] and Boutabaa et al. [16] about PTT fluid. According to these researches, the first normal stress difference cause to increasing secondary flows intensity and decreasing flow rate. About forced heat transfer of non-Newtonian fluids flow in curved ducts, a few numbers of researches have been carried out. Zhang et al. [17] and Shen et al. [18] studied forced convective heat transfer of Oldroyd-B fluid in curved ducts.

In this paper, fully developed flow and heat transfer of viscoelastic materials in curved square ducts under constant heat flux have been investigated. Fig. 1 shows geometry of this flow. According to this figure, cylindrical coordinate system was used. This flow is symmetric toward central radius of curvature, so in this article, governing equations have solved in half of duct's cross section. In this research, CEF constitutive equation has been used for modeling viscoelastic behavior. CEF constitutive equation is a suitable model for studying steady shear flow of viscoelastic materials that is able to model both effects of the first and second normal stress differences.

According to the best knowledge of authors, only two researches [17]-[18] about heat transfer of viscoelastic materials in curved duct have been carried out. These researches have been done by using Oldroyd-B model. Oldroyd-B constitutive equation is not able to model the second normal stress difference, but the experimental results [19]-[20]-[21] shows considerable effect of negative second normal stress difference on this kind of flow. So, current paper is the first research that focuses on reverse effect of the first and second normal stress differences on force convection of viscoelastic fluid flow in curved ducts. Furthermore, in this article, effect of elasticity property on secondary flow intensity and mean Nusselt number has been studied.

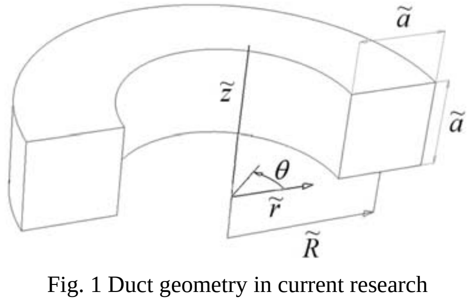

Fig. 1 Duct geometry in current research

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II. Materials and Methods

A. Governing Equations

Governing equations in this fluid flow consist of continuity, momentum and heat transfer equations:

$$\nabla.\tilde{V} = 0 \qquad (1\text{-}1)$$

$$\rho\tilde{V}.\nabla\tilde{V} = -\nabla\tilde{P} + \nabla\tilde{\tau} \qquad (1\text{-}2)$$

$$\rho C_p \tilde{V}.\nabla\tilde{T} = k\nabla^2\tilde{T} \qquad (1\text{-}3)$$

In these relations, $\tilde{V}$ is velocity vector $(m.s^{-1})$, $\tilde{P}$ is static pressure (pa), $\tilde{\tau}$ is stress tensor (pa), ρ is density $(kg.m^{-3})$, $\tilde{T}$ is temperature (K), C_p is specific heat capacity $(J.kg^{-1}.K^{-1})$ and k is heat conduction coefficient $(W.m^{-1}.K^{-1})$.

It should be mentioned that in this article, forced convection of incompressible viscoelastic fluid in curved duct has been investigated. Therefore, solving the heat transfer equation is independent of continuity and momentum equations. So, after solving set of equations that includes continuity and momentum equations, heat transfer equation will be solved by using obtained flow field. Non-dimensional parameters in this article are as below:

$$\begin{array}{lll}
x_i = \dfrac{\tilde{x}}{\tilde{a}} & v_i = \dfrac{\tilde{v}_i}{W_0} & \delta = \dfrac{\tilde{a}}{2\tilde{R}} \\
P = \dfrac{\tilde{P}\tilde{a}}{\eta W_0} & \tau = \dfrac{\tilde{\tau}\tilde{a}}{\eta W_0} & T = \dfrac{\tilde{T}-\tilde{T}_m}{q''a/k} \\
\mathrm{Re} = \dfrac{\rho W_0 \tilde{a}}{\eta} & Dn = \mathrm{Re}\,\delta^{1/2} & Nu = \dfrac{ha}{k} \\
\mathrm{Pr} = \dfrac{\eta}{\rho\alpha} & \gamma_{(1)} = \tilde{\gamma}_{(1)}\dfrac{\tilde{a}}{W_0} & \gamma_{(2)} = \tilde{\gamma}_{(2)}(\dfrac{\tilde{a}}{W_0})^2 \\
\alpha = \dfrac{k}{\rho C_p} & \Psi_1 = \dfrac{\tilde{\Psi}_1 W_0}{\eta\tilde{a}} & \Psi_2 = \dfrac{\tilde{\Psi}_2 W_0}{\eta\tilde{a}}
\end{array} \qquad (2)$$

In this relation, ~ in header of each parameter mentions to a dimensional parameter. Also, in relation (2), $\tilde{x}_i$ is component of coordinate system, $\tilde{a}$ is dimension of duct cross section, $\tilde{R}$ is pitch radius of duct curvature, W_0 is reference velocity, η is viscosity, $\tilde{v}_i$ is velocity component in cylindrical coordinate system, $\tilde{\gamma}_{(1)}$ and $\tilde{\gamma}_{(2)}$ are the first and second order shear rate tensor, $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$ are the first and second normal stress difference coefficients, $\tilde{T}$ is temperature, $\tilde{T}_m$ is mean temperature of flow, q'' is heat flux at walls, h is heat convection coefficient, δ is curvature ratio, Re is Reynolds number, Dn is Dean number and Pr is Prandtle number. In fully developed fluid flow, derivative of all flow parameters except static pressure compare to path curvature angle (θ) is zero. Thus, below relation is valid for static pressure [4]:

$$\frac{\partial\tilde{P}}{\partial\theta} = constant < 0 \qquad (3)$$

Generally, pressure gradient of fully developed fluid flow in curved ducts is defined base on axial pressure gradient:

$$\frac{1}{\tilde{R}}\frac{\partial\tilde{P}}{\partial\theta} = -G \qquad (4)$$

In relation (4), G is constant parameter that defines absolute of axial pressure drop. For creating non-dimensional governing equations, maximum velocity of fully developed flow of Newtonian fluid in straight circular pipe with the similar amount of pressure gradient, viscosity and hydraulic diameter has been used as a reference velocity [4]:

$$W_0 = \frac{G\tilde{a}^2}{16\eta} \qquad (5)$$

Also in Newtonian fully developed fluid flow in straight pipe, non-dimensional pressure gradient is constant and is obtained from below relation [4]:

$$\frac{\partial P}{\partial s} = -16 \qquad (6)$$

In governing equations, reference velocity is defined base on equality of pressure gradient in curved duct with pressure gradient of Newtonian fluid flow in straight pipe. So, from relations (2), (4) and (6) below relation for non-dimensional pressure gradient of fully developed flow in curved ducts is obtained:

$$\frac{\partial P}{\partial\theta} = -16R = -C \qquad (7)$$

In this relation, C is a positive constant. Therefore, for steady fully developed fluid flow in a curved duct; non-dimensional forms of governing equations are as below [22]:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \qquad (8\text{-}1)$$

$$v_r\frac{\partial v_\theta}{\partial r} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} = \frac{1}{\mathrm{Re}}\left[\frac{C}{r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) + \frac{\partial\tau_{z\theta}}{\partial z}\right] \qquad (8\text{-}2)$$

$$v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^{\,2}}{r} = \frac{1}{\mathrm{Re}}\left[-\frac{\partial P}{\partial r} + \frac{\partial\tau_{rr}}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\tau_{rr}-\tau_{\theta\theta}}{r}\right] \qquad (8\text{-}3)$$

$$v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} = \frac{1}{\mathrm{Re}}\left[-\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right) + \frac{\partial\tau_{zz}}{\partial z}\right] \qquad (8\text{-}4)$$

No slip condition is valid for velocity components at walls of duct. Due to symmetrical condition of flow; governing equations have been solved in half of cross section and homogenized Newman condition has been used on symmetry boundary. Here, staggered grid has been used so there is no require to implement boundary condition for static pressure.

In this research, fully developed forced heat convection of viscoelastic flow has been studied and it is assumed that there is a constant heat flux on walls of duct. In fully developed heat convection in a curved duct, following relation is valid [23]:

$$\frac{\partial}{\partial\theta}\left(\frac{\tilde{T}_w-\tilde{T}}{\tilde{T}_w-\tilde{T}_m}\right) = 0 \qquad (9)$$

Where, $\tilde{T}_w$ is wall temperature and $\tilde{T}_m$ is mean temperature of fluid flow which obtained from following relation [23]:

$$\tilde{T}_m = \frac{1}{\tilde{U}A}\int_A \tilde{v}_\theta \tilde{T} dA \tag{10}$$

In this relation, $\tilde{U}$ is bulk velocity of fluid flow. With supposing constant heat flux condition, the heat transfer equation in fully developed condition is as bellow:

$$v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} + \Gamma \frac{v_\theta}{r} = \frac{1}{Pe}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right\} \tag{11}$$

In this relation, Γ is a non-dimensional constant that obtained from following relation:

$$\Gamma = \frac{4}{\mathrm{Re}_b \Pr \delta} \tag{12}$$

Where, Re_b is Reynolds number base on bulk velocity. In this condition, Nusselt number at walls is obtained from following relation:

$$Nu = \frac{1}{T} \tag{13}$$

Also, the mean Nusselt number is defined:

$$Nu_m = \frac{1}{p}\int_p Nu dp \tag{14}$$

Where, p is the non-dimensional form of the duct perimeter. Boundary conditions in this equation consist of constant heat flux on walls and homogenized Newman boundary condition on symmetry boundary. Non-dimensional boundary condition at walls is as below:

$$\frac{\partial T}{\partial n} = -1 \tag{15}$$

In this relation, *n* is perpendicular direction at walls.

Also in this research, CEF constitutive equation has been used as viscoelastic model [22]:

$$\tilde{\tau} = \eta\tilde{\gamma}_{(1)} - \frac{1}{2}\tilde{\Psi}_1\tilde{\gamma}_{(2)} + \tilde{\Psi}_2\left\{\tilde{\gamma}_{(1)}.\tilde{\gamma}_{(1)}\right\} \tag{16}$$

The first and second order shear rate tensors for fully developed fluid flow is defined as below [22]:

$$\tilde{\gamma}_{(1)} = \nabla\tilde{V} + \left(\nabla\tilde{V}\right)^{\mathrm{T}} \tag{17-1}$$

$$\tilde{\gamma}_{(2)} = \tilde{V}.\nabla\tilde{\gamma}_{(1)} - \left\{\left(\nabla\tilde{V}\right)^{\mathrm{T}}\cdot\tilde{\gamma}_{(1)} + \tilde{\gamma}_{(1)}\cdot\left(\nabla\tilde{V}\right)\right\} \tag{17-2}$$

CEF constitutive equation is suitable for modeling steady shear flow of viscoelastic materials (viscometric region of Pipkin diagram) and it is usual for industrial calculation [24]. By using relation (2) in relation (16), non-dimensional form of CEF model is obtained:

$$\tau = \gamma_{(1)} - \frac{1}{2}\Psi_1\gamma_{(2)} + \Psi_2\left[\gamma_{(1)}.\gamma_{(1)}\right] \tag{18}$$

B. Numerical Method

Flow analysis by quasi unsteady assumption is one of the CFD methods for solving steady state problems. In this method, the term of time derivatives are not eliminated from the governing equations and flow analysis is accomplished like unsteady flow until answers converge toward steady state solution. Here, time has an iterative rule and it does not have a physical worth [25]. Though only solutions steady with respect to duct are of interest, initial condition were needed because the unsteady form of the conservation equations was used. Here, homogenized dirichlet condition is used as the initial condition for all of parameters. From the governing equations, velocity and temperature quantities have time derivative term, and static pressure is only quantity that does not have time term. In this research, for creating pressure derivative time term, artificial compressibility method is used. This method was demonstrated by Chorin [26]. According to Chorin [26] theory, pressure time derivative is added to continuity equation.

$$\frac{\partial P}{\partial t} + c^2\frac{\partial U_i}{\partial x_i} = 0 \tag{19}$$

When the flow is being steady, then $\partial P / \partial t = 0$, and continuity equation will be satisfied. In the current research, mesh is generated by staggered mesh method [25]. In this method of mesh generation, grid is displaced along each cell diameter by half of its diameter. Also marker and cell method is used for allocating the flow parameter to grids [25]. This method brings about possibility of variables coupling and improves stability.

Here, governing equations are formulated explicitly, and forward first order approximation is used for time derivation and central second order approximation is used for space derivation (FTCS).

III. Results and Discussion

In the current research, 80×40 square grids is used as calculation mesh. For the CEF fluid flow in a curved duct with $\mathrm{Re} = 1000$, $\delta = 0.1$, $\Psi_1 = 1$ and $\Psi_2 / \Psi_1 = -5\%$, the mean error of axial velocity distribution in 80×40 grid compare to 120×60 grid is around 0.23%. Therefore, it confirmed that the numerical solution of 80×40 grid is grid independent for this condition. Hence, the 80×40 grid has been used as the reference in this research. Here, we have been validated the numerical result of flow field in four ways:

- With assumption of a too small curvature ratio (large radius of curvature), numerical solution converges to the flow and heat transfer in a straight duct. For fully developed Newtonian fluid flow in a straight duct, the mean error of axial velocity's profile obtained from numerical results to analytical solution is 0.27% at Reynolds number of 25.
- In creeping flow in a curved duct, the mean error of axial velocity obtained from numerical results to analytical solution is around 0.49% at $\delta = 0.1$. Here,

creeping flow results is obtained with supposing an infinitesimal value for Reynolds number.

- In the CEF fluid flow in a straight duct with the rheological properties, $\Psi_1 = 0.5$, $\Psi_2 / \Psi_1 = -10\%$ and in Reynolds number of 30, the mean error of axial velocity profile obtained from numerical solution to result of Twonsend et al. [27] is about 1.15%.
- In the fully developed Newtonian fluid flow in a curved duct and Dn_b=125, the mean error of axial velocity's profile obtained from numerical method to experimental result of Bara [28] is nearly 1.07%.

Additionally, to evaluate the results of numerical solution of heat transfer equation, the obtained Nusselt numbers have been compared to the results of Shah and London [29] which focused on the forced heat transfer of Newtonian fluid flow in a straight duct with rectangular cross section. They used the numerical integration method to calculate the Nusselt number for constant heat flux condition. Also, they assumed that the duct's surface temperature is constant, too. In this research, for validating the heat transfer results, we assumed that the duct's surface temperature is constant. For converging the numerical results of heat transfer equation to temperature distribution of flow in a straight duct, we assumed that the curvature ratio is infinitesimal (very large radius of curvature). Table I present Nusselt numbers which obtained in current research and Shah and London's [29] research at different value of aspect ratio. According to this table, the current research's results have a good agreement with Shah and London's [29] results.

TABLE I
NUSSENT NUMBER AT DIFFERENT VALUE OF ASPECT RATIO IN CURRENT RESEARCH AND SHAH AND LONDON'S [29] RESEARCH

Aspect Ratio	Shah and London's [29]	Current research
1.0	3.61	3.6154
1.43	3.73	3.7369
2.0	4.12	4.1274
3.0	4.79	4.7983
4.0	5.33	5.3402

It is important to knowing that the assumption of constant temperature at walls for constant heat flux situation is an approximation for calculation of Nusselt number. It will be mentioned that this assumption is just used in this section to validate the accuracy of numerical results and is not applicable in other results of this paper. At the continuance, the effect of centrifugal force on the flow is studied. Here, the Newtonian fluid flow is used to study the effect of centrifugal force that there are no normal stress differences. In this flow, the presence of centrifugal force due to curvature will lead to significant radial pressure gradient in flow core region. However, in the proximity of curved duct's inner and outer walls, the axial velocity and as a result the centrifugal force will approach toward zero. In this situation, for preserving the balance, momentum mechanisms act and Taylor-Gortler secondary flows will be appeared. Fig. 2 shows the secondary flows in Dn_b =125. Also according to the figure, the axial velocity distribution's tendency is toward the outer wall of the duct due to the centrifugal force. Generally, in Newtonian fluid flow in curved ducts with square cross section, Taylor's secondary flows will be appeared as a one pair of vortices at Dean number less than 125. Though, it will be changed to two pair of vortices for larger Dean number, gradually. This phenomenon is due to the high radial pressure gradient and substantial tendency of the main flow toward outer wall which lead the flow to be changed. Fig. 2 shows the secondary flows in Dn_b=137, too. In this situation, as it has been seen in Bara [28] research experimentally, the presence of two pair of vortices is obvious. Also, axial velocity distribution and pressure have been changed which gain the maximum velocity in two concurrent points. Referring to the figure, temperature's distribution has been shown non-dimensional. In Dn_b=137 fluid's temperature in inner wall's proximity is higher than of Dn_b=125. For Dn_b=125 and Dn_b=137, the mean Nuselt number will be 9.48 and 16.16, respectively. The phenomenon is due to changing the secondary flows from two vortices to four vortices rather than increasing of the axial velocity and Reynolds number, which have been resulted in the increasing flow mixing and also increasing the heat transfer.

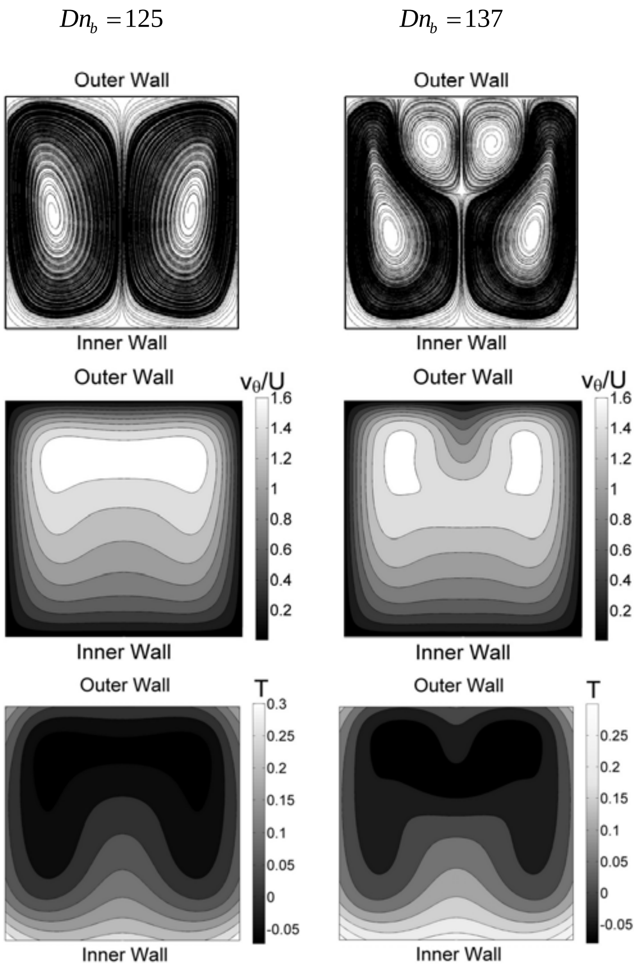

Fig. 2 Streamlines of secondary flows, main velocity and temperature distribution for Newtonian fluid flow at different Dean Numbers

Fig. 3 illustrates the streamlines of secondary flows and temperature distribution at Dn_b=125 for different values of first and second normal stress differences. The above mentioned situation has been chosen because of the secondary flows form is changed in the vicinity of Dn_b=125, (see Fig. 2). According to the Figure, while keeping the Dn_b=125 and providing the first normal stress difference, secondary flows intensity will be increased and the number of vortices is changed from one pair to two pairs. Furthermore, while providing the negative second normal stress difference, secondary flows intensity will be decreased and the vortices that located in upper side of cross section is weaken. According to this Figure, the upper secondary flows is disappeared in $\Psi_2/\Psi_1 = -10\%$. It is important to knowing that the second normal stress difference in the most of viscoelastic materials is negative.

Summarily, the effect of first and second normal stress difference on this flow is completely opposite. The first normal stress difference leads to increase and negative second normal stress difference decrease the secondary flows intensity.

Fig. 3 illustrates the non-dimensional temperature's distribution which is influenced by secondary flow's intensity and shape, too. For CEF fluid with rheological properties, $\Psi_1 = 0.6$ and $\Psi_2/\Psi_1 = 0.0$, there are two pair of vortices with high intensity. So, the flow mixing is high and the area with high temperature is larger than other cases. In this condition, the mean Nusselt number is 17.04 and heat transfer comparing to Newtonian fluid with Nusselt number of 9.48, is almost 1.8 times greater. The increasing of negative second normal stress difference leads to decreasing in secondary flows intensity and Nusselt number in CEF fluid flow which the mean Nusselt number decreases to 11.00 and 10.03 for the $\Psi_2/\Psi_1 = -7\%$ and $\Psi_2/\Psi_1 = -10\%$, respectively. According to the figure, temperature's distribution and secondary flows' shape in CEF flow with rheological properties $\Psi_1 = 0.6$ and $\Psi_2/\Psi_1 = -10\%$ is too similar to Newtonian fluid.

Using elastic mumber to consider the effect of first normal stress difference in flow of viscoelastic materials in curved duct, is usual. For CEF flow, elastic number is defined by dividing the non-dimensional first normal stress difference to Reynolds number:

$$En = \frac{\Psi_1}{\mathrm{Re}} = \frac{\tilde{\Psi}_1}{\rho a^2} \tag{20}$$

In fact, the elastic number is the effects of elasticity on the inertia of flow and according to equation (20) for a specific geometry is a function of fluid properties only.

In the following, the mechanism which heat transfer is influenced by rheological properties is considered. In Fig. 4, hoop stress distribution $(\tau_{\vartheta\theta})$ and stress parameters τ_{rr} and τ_{rz} are illustrated. Here, the numerical simulation for CEF fluid flow with properties $En = 0.0075$, $\mathrm{Re} = 100$ and $\delta = 0.15$ are implemented. It is noticeable that these stress components are insignificant and in order of ε in fully developed Newtonian fluid flow in a curved duct but these stresses are appeared in CEF fluid flow. According to the figure, when the effect of second normal stress difference is zero ($\Psi_2/\Psi_1 = 0.0$), the effect of first normal stress difference causes the strong hoop stress $(\tau_{\vartheta\theta})$. In the core area, the radial pressure gradient is balanced with the hoop stress and centrifugal force.

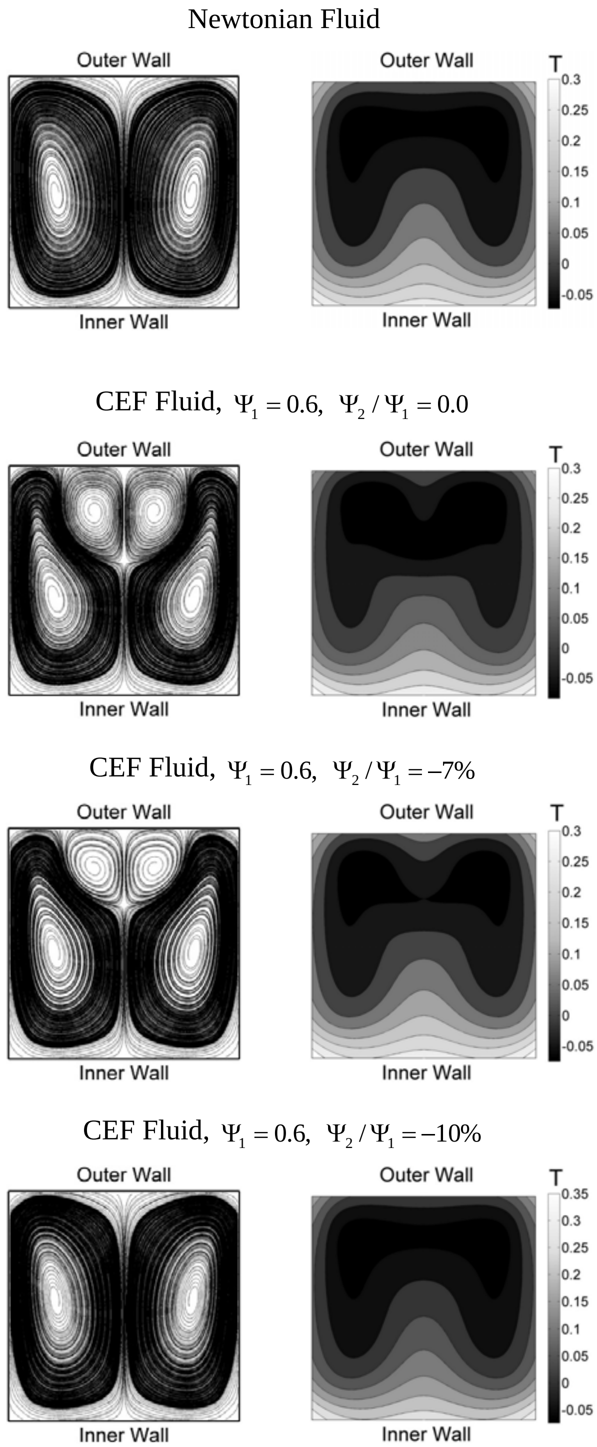

Fig. 3 Streamlines of secondary flows and temperature distributions of Newtonian and CEF fluid at $Dn_b = 125$ and different value of rheological properties

Due to small value of centrifugal force and large value of hoop stress near the outer wall, the imbalanced is appeared in this region. For keeping the balance, momentum mechanism acts and produces secondary flows at the direction of Taylor-Gortler vortices. Therefore, the mixing of the fluids and heat transfer of flows is increase. On the other hand, producing negative second normal stress difference leads to decreasing hoop stress and changing the distribution of stress parameters τ_{rr} and τ_{rz} in the flow which both factors play a substantial role in secondary flows intensity's decrease and consequently decreasing in heat transfer.

In this reserach, S_{max} is used as the ratio of secondary flows' maximum velocity to the main flow's mean velocity, to consider the secondary flows intensity. In Fig. 5, the effect of first normal stress difference in terms of elastic number on secondary flow's intensity and mean Nusselt number is shown. Here, we assume that the second normal stress difference equal to zero. Regarding to the figure, increasing the Reynolds number leads to raise the secondary flows intensity and mean Nusselt number for all kind of fluids. Moreover, an increase in elastic number (first normal stress difference) brings higher secondary flows intensity and higher Nusselt number.

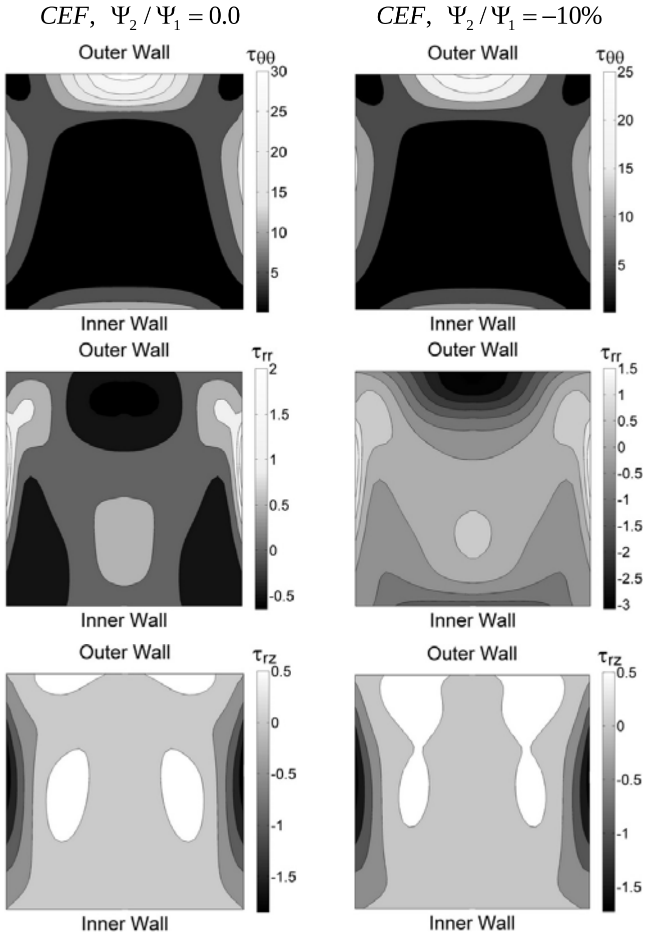

Fig. 4 Distribution of stress components which are effective in radial pressure gradient at $En = 0.0075$, $\text{Re} = 100$ and $\delta = 0.15$

The effect of elastic number on secondary flows intensity and mean Nusselt number is much more for large Reynolds number. This subject is related to the second order dependence of elastic stress to shear rate. In Fig. 6, the effect of second normal stress difference on secondary flows intensity (S_{max}) and mean Nusselt number has been presented. As it shows, an increase in second normal stress difference leads to a decrease in secondary flows intensity and mean Nusselt number; as for Reynolds number more than 120, secondary flows intensity and Nusselt number is even less than Newtonian fluid flow. The significant effect of second normal stress difference on secondary flows intensity decrement in viscoelastic fluids flow in curved duct is a remarkable phenomenon which has been proven in experimental observations [19]-[20]-[21].

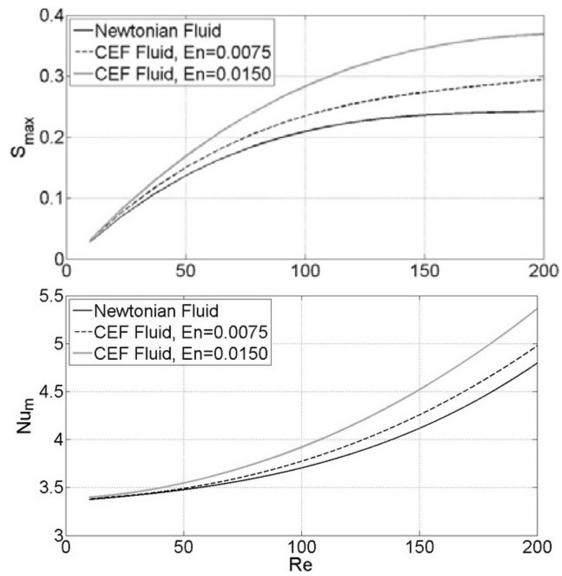

Fig. 5 Effect of the first normal stress difference on secondary flows intensity and mean Nusselt number at $\delta = 0.15$ and $\Psi_2 = 0.0$

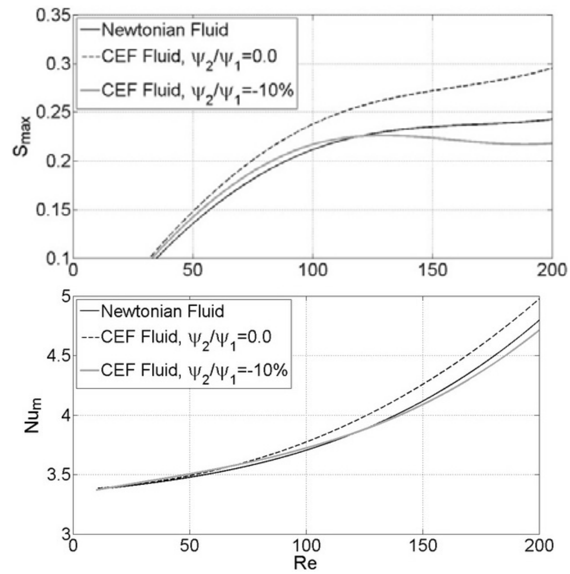

Fig. 6 Effect of the second normal stress difference on secondary flows intensity and mean Nusselt number at $\delta = 0.15$ and $En = 0.0075$

IV. Conclusion

Inverse effects of the first and second normal stress difference on flow and heat transfer of viscoelastic materials in curved duct with square cross section have being proven in this research and it has been shown that:

- On viscoelastic fluids flow in curved duct, with a rise in first normal stress difference, intensity of Taylor-Gortler vortices and mean Nusselt number are increased. The effect of negative second normal stress difference on flow and heat transfer is on the contrary of first normal stress difference. For viscoelastic fluids flow with large negative second normal stress difference, mean Nusselt number may be less than Newtonian fluid flow.
- The mechanism of first normal stress difference's on the flow is producing a large axial normal stress near the outer wall which leads to amplify the Taylor-Gortler vortices. On the other hand, negative second normal stress difference results a decrement in axial normal stress and amplifying the effect of stress components τ_{rr} and τ_{rz}, which both factors reduce the radial pressure gradient and both of consequently the secondary flows intensity and Nusselt number.

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