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Development of New Control Techniques for Vibration Isolation of Structures using Smart Materials

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Abstract**—**In this paper, the effects of the restoring force device on the response of a space frame structure resting on sliding type of bearing with a restoring force device is studied. The NS component of the El - Centro earthquake and harmonic ground acceleration is considered for earthquake excitation. The structure is modeled by considering six-degrees of freedom (three translations and three rotations) at each node. The sliding support is modeled as a fictitious spring with two horizontal degrees of freedom. The response quantities considered for the study are the top floor acceleration, base shear, bending moment and base displacement. It is concluded from the study that the displacement of the structure reduces by the use of the restoring force device. Also, the peak values of acceleration, bending moment and base shear also decreases. The simulation results show the effectiveness of the developed and proposed method.

Keywords**—**DOF, Space structures, Acceleration, Excitation, Smart structure, Vibration, Isolation, Earthquakes.

I. INTRODUCTION

EARTHQUAKE is the result of a sudden release of energy in the Earth's crust that creates seismic waves. Earthquakes are recorded with a seismometer, also known as a seismograph. The moment magnitude of an earthquake is conventionally reported, or the related and mostly obsolete Richter magnitude, with magnitude 3 or lower earthquakes being mostly imperceptible and magnitude 7 causing serious damage over large areas. Intensity of shaking is measured on the modified Mercalli scale. At the Earth's surface, earthquakes manifest themselves by shaking and sometimes displacing the ground.

When a large earthquake epicenter is located offshore, the seabed sometimes suffers sufficient displacement to cause a tsunami. The shaking in earthquakes can also trigger landslides and occasionally volcanic activity. In its most generic sense, the word earthquake is used to describe any seismic event - whether a natural phenomenon or an event caused by humans - that generates seismic waves. Earthquakes are caused mostly by rupture of geological faults, but also by volcanic activity, landslides, nuclear experiments and mine blasts. An earthquake's point of initial rupture is called its focus or hypocenter [1].

The term epicenter refers to the point at ground level directly above this. This earthquake is one of the most frightening and destructive phenomena of nature and its terrible aftereffects can't be imaginable. An earthquake is a sudden movement of the earth caused by the abrupt release of strain that has accumulated over a long time. If the earthquake occurs in a populated area it may cause many deaths and injuries and extensive property damage. Earthquakes not only cause loss of life and property but also shake the morale of the people.

In recent years, structural engineers have developed new concepts to absorb the earthquake forces so that the force transferred to the structure can be minimized. One of the concepts to protect a building from the damaging effects of an earthquake is by introducing some type of support that isolates it from the shaking the ground using specially designed materials called as isolators [2].

A purely sliding system is the earliest and simplest isolation system proposed using pure sliding in 1909 by J.A. Calantarients, a medical doctor. He suggested separating the structure from foundation by a layer of talc. Isolation was first considered as a seismic-resistant design strategy by the Italian government after the great messimo-reggio earthquake of 1908. The commission set up by the government proposed two approaches to earthquake resistant design.

The first approach isolated the building from the ground by interposing a sand layer in its foundation and the other using rollers under columns to allow the building to move horizontally. In the severe Indian earthquake of Bihar in 1934, it was observed that small masonry buildings that slide on their foundation survived the earthquake, while similar buildings fixed at the base were destroyed.

Mostaghel N. and M. Khodaverdian have proposed the model for dynamics of friction base isolator. A sliding system was proposed by Arya *et.al.* and a considerable research on

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this approach using a shock type shake table was carried out and demonstrated the effectiveness of this approach. Bhasker and Jangid also analyzed the structure resting on sliding type of bearing assuming different equations for non-sliding and sliding phase. James Kelly discusses the theory and application of base isolation in greater detail [3].

Abe, *et. al.* performed tests on various bearing materials to determine their properties such as stiffness and multi-directional behavior. The major findings from the above literature survey could be summarized as follows. It was observed that the acceleration and displacement is maximum when natural frequency of the structure is equal to the excitation frequency for the structure fixed at base, where as, for the structure isolated at base, the acceleration will not change much with excitation frequency and it is independent of the excitation frequency.

Also, the acceleration of the isolated structure is considerably less than the acceleration of the fixed base structure. But, the sliding displacement of the structure isolated at base is considerably large when the frequency of excitation is in the range of 0 - 1 rad / sec. Thus, the isolation reduces the acceleration and forces in the structure, but it increases the sliding displacement when the excitation frequency is small. Hence, the isolation of structure with sliding bearing has not become still popular.

Seismic isolation is an old design idea, proposing the decoupling of a structure or part of it, or even the equipment placed in the structure, from the damaging effects of ground accelerations. One of the goals of the seismic isolation is to shift the fundamental frequency of a structure away from the dominant frequencies of earthquake ground motion and fundamental frequency of the fixed base superstructure.

The other purpose of an isolation system is to provide an additional means of energy dissipation, thereby reducing the transmitted acceleration into the superstructure. This innovative design approach aims mainly at the isolation of a structure from the supporting ground, generally in the horizontal direction, in order to reduce the transmission of the earthquake motion to the structure.

There are two types of isolation devices and they are rubber bearings and sliding bearings. The sliding type of bearing uses rollers or sliders between foundation and the base of the structure. The various types of sliding material used for sliding bearing are dry sand, wet sand and graphite powder, which have sufficient coefficient of friction. It is observed from the Figs. 1 and 2 that the acceleration and displacement is maximum when natural frequency of the structure is equal to the excitation frequency for the structure fixed at base, where as, for the structure isolated at base, the acceleration will not change much with excitation frequency and it is independent of the excitation frequency.

Also, the acceleration of the isolated structure is considerably less than the acceleration of the fixed base structure. But, the sliding displacement of the structure isolated at base is considerably large when the frequency of excitation is in the range of 0 - 1 radians / sec. Thus, the isolation reduces the acceleration and forces in the structure but it increases the sliding displacement when the excitation frequency is small. Hence, the isolation of structure with sliding bearing has not become still popular [4].

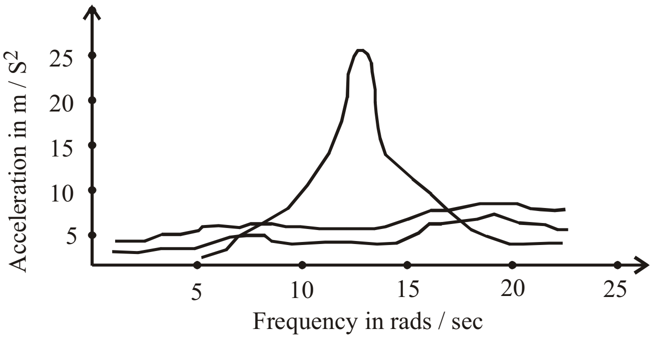

Fig. 1 Variation of response with excitation frequency

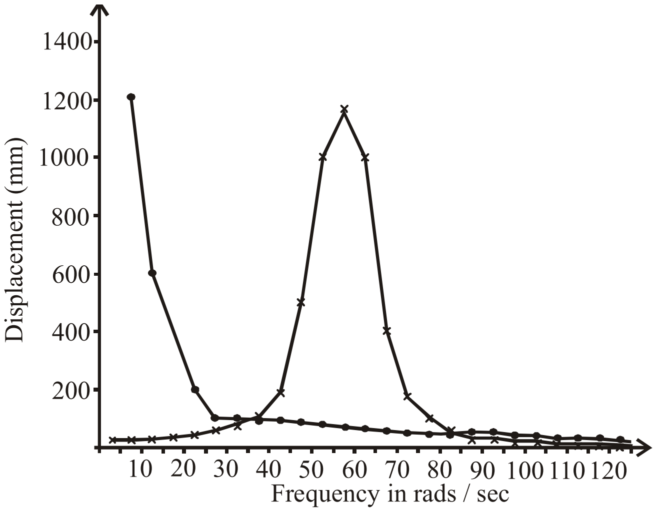

Fig. 2 Variation of response with excitation frequency

Finally, to overcome the problem of isolating the structure from vibrations, it is planned to provide some mechanism of sliding bearing to reduce both the acceleration as well as sliding displacement at all excitation frequencies.

The paper is organized in the following sequence. A brief introduction / literature survey about the vibration suppression, the smart structures, the existing study of the various control techniques to curb down the vibrations in engineering structures were herewith dealt with in greater detail in section I. Section II deals with the analytical modelling of the considered structure. Mathematical model of the same is also explained here in this context. Section III deals with the control aspects of the same followed by the results & discussions in section IV. Conclusions are presented in the last section followed by the references.

II. ANALYTICAL MODELLING

A space frame structure resting on sliding bearing subjected to earthquake ground acceleration is analyzed. The details of analysis are as follows. The structure is divided into number of elements consisting of beams and columns connected at each node as shown in the Fig. 3. Each element is modeled using two noded frame element with six degrees of freedom at each node. The space frame structure is similar to a 4 storeyed building structure with the ground floor and the next 3 floors [5].

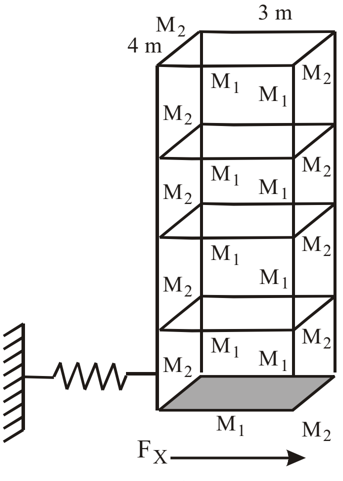

Fig. 3 Structure divided into no. of nodes & beams with restoring force F_x

The structure is divided into number of elements consisting of beams and columns connected at nodes. Each element is modeled using two noded frame element with six degrees of freedom at each node i.e., three translations along X, Y and Z axes and three rotations about these axes. For each element, the stiffness matrix, [*k*], consistent mass matrix [*m*] and transformation matrix [T] are obtained and the mass matrix and the stiffness matrix from local direction are transformed to global direction as proposed by Paz. The mass matrix and stiffness matrix of each element are assembled by direct stiffness method to get the overall mass matrix [M] and stiffness matrix [K] for the entire structure. This is done as follows.

For each element stiffness matrix [*k*], mass matrix [*m*] and the transformation matrix [*T*] is obtained. Mass matrix [*m*] and the stiffness matrix [*k*] is then transformed from local direction to the global direction. The mass matrix and the stiffness matrix of each element are assembled by direct stiffness method to obtain the overall mass matrix [M] and stiffness matrix [K] for the entire structure.

The overall dynamic equation of equilibrium for the entire structure can be expressed in matrix notations as

$$\left[M\right]\left\{\ddot{u}\right\}+\left[C\right]\left\{\dot{u}\right\}+K\left\{u\right\}=F(t), \tag{1}$$

where $\left\{\ddot{u}\right\},\left\{\dot{u}\right\},\left\{u\right\}$ are the relative acceleration, velocity and displacement vectors at nodes with respect to ground.

The damping of super structure is assumed to as Rayleigh type and the damping matrix [C] is determined using the equation [C] = α [M] + β[K], where α and β are the Rayleigh constants & {F(t)} is the nodal load vector and is calculated by using the equation [6]

$$\left\{F(t)\right\}=-\left[M\right]\left\{1\right\}\ddot{u}_g(t) \tag{2}$$

The structure resting on sliding bearing passes through two types of phases, viz., stick phase & sliding phase. In the stick phase, the frictional resistance between the base of the structure and the foundation raft is greater than the base shear of the system. The acceleration of the base is equal to zero and the displacement at base remains constant during this phase.

The stiffness of the spring at the bottom of each column is considered as very high. When the mobilized frictional force is equal to or more than the frictional resistance the structure starts sliding at the base and this phase is known as the sliding phase.

A. *Change of Phases*

The change of phases can be carried out in 6 steps, which are best described as follows one after the other.

1. In the present analysis, sliding bearing is modeled as fictitious spring connected to the base of each column. The conditions for sliding and non-sliding phase are duly checked at the end of each time step.
2. When the structure is in non-sliding phase the stiffness of the spring is assigned a very high value to prevent the movement of the structure at the base.
3. When the structure is in the sliding phase the value of spring is made equal to zero to allow the movement of the structure at base.
4. During the non-sliding phase, the relative acceleration and relative velocity of the base is equal to zero and the relative displacement at the base is constant. The stiffness of the spring at base of each column are considered as very large and the dynamic equation of motion for this phase is same as that given in Eq. (1).
5. During the sliding phase, the stiffness of the spring at base of each column is considered as zero and the mobilized frictional force under each column is equal to F_s and remains constant.
6. The dynamic equation of motion of the structure during this phase is given by

   $$\left[M\right]\left\{\ddot{u}\right\}+\left[C\right]\left\{\dot{u}\right\}+K\left\{u\right\}=\left\{F(t)\right\}-\left\{F_{max}\right\} \quad (3),$$

   where, $\{F_{max}\}$ is the vector with zeros at all locations except those corresponding to the horizontal degree of freedom at base of structure. at these degrees of freedom this vector will have values equal to F_s and direction opposite to the velocity vector $\dot{u}_b$.

The dynamic Eqs. (1) - (3) are solved for displacement, acceleration and velocity by using the Newmark's method as explained below.

1. Frictional force mobilized in the sliding system is a nonlinear function and hence the response of the isolated

structure is obtained in a incremental form using Newmark's method [16].

2. In this method, from the response at time t the response at (t + Δt) is determined.
3. Constant acceleration scheme is adopted.

Eqn. (1) in incremental form can be written as

$$\mathrm{M}\ \Delta\ddot{\mathrm{u}}_i + \mathrm{C}\ \Delta\dot{\mathrm{u}}_i + \mathrm{K}\ \Delta\mathrm{u}_i = \Delta\mathrm{F}_i \tag{4}$$

Δ denotes the variations of each parameters from time *t* to time (*t* +Δ*t*) and the index i indicates the i^{th} time step

$$\Delta\dot{\mathrm{u}}_i = \left(\frac{2}{\Delta\mathrm{t}}\right)\Delta\mathrm{u}_i - 2\dot{\mathrm{u}}_i \tag{5}$$

$$\Delta\ddot{\mathrm{u}}_i = \left(\frac{4}{\Delta\mathrm{t}}\right)^2 \Delta\mathrm{u}_i - 4\left(\frac{4}{\Delta\mathrm{t}}\right)\dot{\mathrm{u}}_i - 2\,\ddot{\mathrm{u}}_i \tag{6}$$

Substituting Eqs. (4) and (5) in (3) yields

$$\hat{\mathrm{K}}_i\ \Delta\mathrm{u}_i = \Delta\hat{\mathrm{F}}_i \tag{7}$$

$$\hat{\mathrm{K}}_i = \mathrm{K}_i + \left(\frac{2}{\Delta\mathrm{t}}\right)\mathrm{C} + \left(\frac{4}{\Delta\mathrm{t}}\right)^2\mathrm{M} \tag{8}$$

$$\Delta\hat{\mathrm{F}}_i = \Delta\mathrm{F}_i + \left[\left(\frac{4}{\Delta\mathrm{t}}\right)\mathrm{M} + 2\ \mathrm{C}\right]\dot{\mathrm{u}}_i + 2\mathrm{M}\,\ddot{\mathrm{u}}_i \tag{9}$$

By solving Eq. (6), $\Delta\mathrm{u}_i$ is determined and the subsequent values of displacement and velocity at the beginning of step (*i* + 1) are calculated using the following two equations given below [7]

$$\mathrm{u}_{i+1} = \mathrm{u}_i + \Delta\mathrm{u}_i \tag{10}$$

$$\dot{\mathrm{u}}_{i+1} = \dot{\mathrm{u}}_i + \Delta\dot{\mathrm{u}}_i \tag{11}$$

Accelerations are calculated based on Eq. (1) to increase the accuracy and stability of the solutions. In the present method, a time step of $\Delta\mathrm{t} = 0.0004$ sec is used [15].

III. CONTROL ASPECTS

Active vibration control is an important problem in structures. One of the ways to tackle this problem is to make the structure smart, adaptive and self-controlling. The objective of active vibration control is to reduce the vibration of a system by automatic modification of the system's structural response. The objective of active vibration control is to reduce the vibration of a system by automatic modification of the system's structural response [14].

To study the response of the structure and effectiveness of sliding bearing for the structure subjected to earthquake, a four story three-dimensional structure shown in Fig. 4 is analyzed. The structure is subjected to sinusoidal ground acceleration of varying intensity and to El Centro earthquake. The geometric and material properties considered for the analysis is as given below [8].

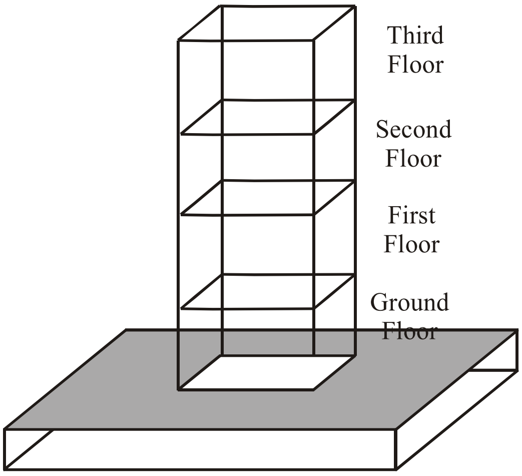

Fig. 4 A four storeyed building

TABLE I ANALYSIS OF THE RESULTS

Frequency rad / sec	Present Techniques (Variable stiffness) Displacement (mm)	With Restoring force device (stiffness = 1500) Displacement (mm)	Present Techniques (Variable stiffness) Acceleration (m/sec^2)	With Restoring force device (stiffness =1500) Acceleration (m/sec^2)
1	9.32322	52.216	2.422	2.61
2	10.1345	64.42	2.54	2.99
3	12.3593	81.73	2.87	3.866
4	12.6359	122.22	3.2	4.78
5	12.89	222.74	3.26	8.87
6	12.92	226.27	3.97	10.47
7	14.4	103.44	3.94	5.83
8	28.31	66.42	3.24	3.456
9	23.11	49.98	2.77	2.51
10	18.459	38.077	3.41	1.85
11	17.69	28.9654	3.2	1.581
12	18.91	25.64	2.81	1.72
13	18.61	22.68	2.53	1.94
14	17.73	20.04	2.311	2.3654
15	18.71	17.73	2.47	2.832
16	20.2	15.71	2.35	2.3573
17	23.34	13.94	2.34	1.81
18	26.43	12.39	2.381	1.55
19	23.99	11.04	2.41	1.335
20	18.451	9.86	2.456	1.14
21	15.15	8.83	2.4891	1.16
22	12.417	7.93	2.515	1.09
23	10.59	7.15	2.5328	1.07
24	8.76	6.46	2.54	1.04
25	7.56	5.86	2.55	2.55

Column Size $= 0.6\text{ m} \times 0.6\text{ m}$
Beam Size $= 0.3\text{ m} \times 0.6\text{ m}$
Mass on beam $= 3\text{ kN / m}$
$E = 2.2 \times 10^{-7}\text{ kN / m}^2$

The structure is subjected to a sinusoidal ground acceleration (similar to a sinusoidal signal) of intensity $2\sin(\omega t)$ for various values of excitation frequency ω. The variation of response of the structure with excitation frequency ω is shown in Fig. 5. The variation of response of the structure with excitation frequency ω for the structure fixed at base is also shown in the same Fig. 5.

As observed from the Figs. 5(a) & 5(b), the acceleration of the structure fixed at base varies with excitation frequency ω and shows a peak value when the frequency of excitation is equal to the natural frequency of the structure $(\omega/\omega_n = 1)$ whereas, for the structure isolated at base, the acceleration, will not vary much with variations in excitation frequency. The result of the analysis is shown in the table I.

Thus, the observed response of the structure from these figures is as reported in literature and they are [9]

1. The acceleration of the isolated structure is considerably less than the acceleration of the fixed base structure [13].
2. The sliding displacement of the structure isolated at base is considerably large when the frequency of excitation is in the range of 0 to 1 rad/sec.
3. Thus, the isolation reduces the acceleration and forces in the structure but it increases the sliding displacement when the excitation frequency is small.

IV. SIMULATION RESULTS

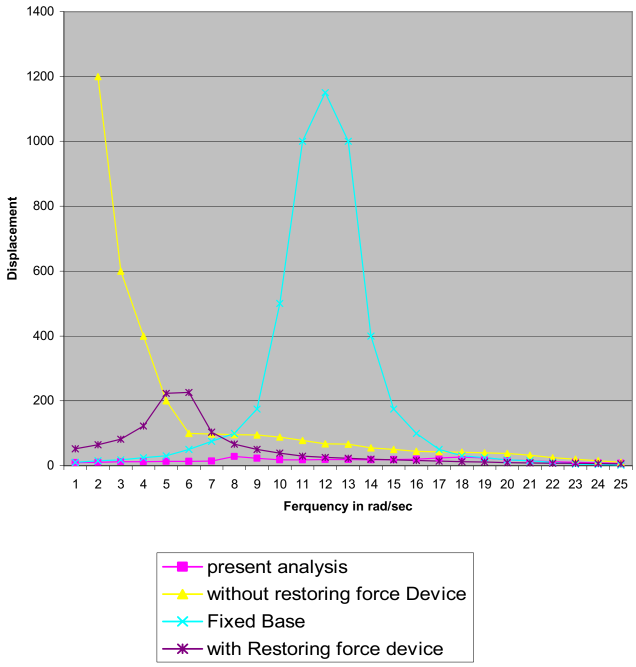

Fig. 5(a) Variation of displacement with frequency

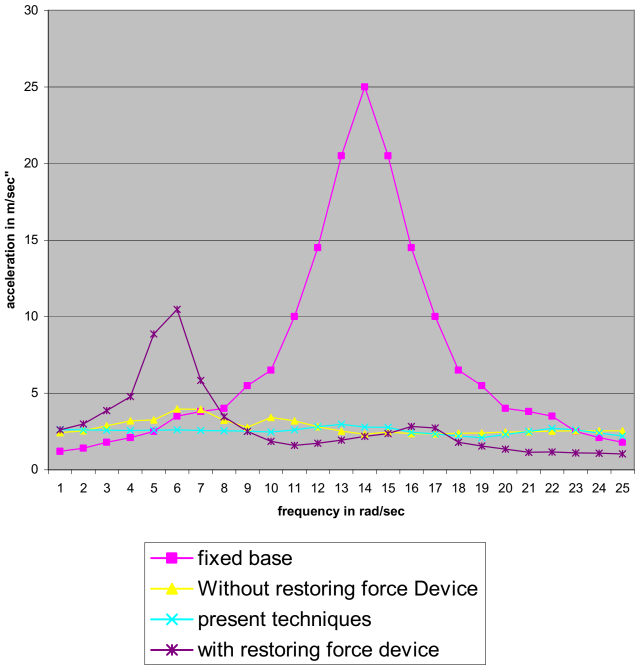

Fig. 5(b) Variation of acceleration with frequency

The following techniques are adopted as a first step to reduce the displacement at all excitation frequency without increasing the acceleration and force in the structure [10].

1. A restoring force F_x device is added as shown in Fig. 3.
2. The frequency of excitation is obtained at each 0.1 sec interval. This is obtained by counting the number of cycles/sec.
3. The stiffness of the spring is adjusted by some mechanism (piezo electric sensor, MR fluid)
4. The effect of stiffness of restoring force device is analyzed at various frequencies and same checked for different cases.
5. Stiffness optimization is done by using the formula $F_s = \left(\omega^2 m + m\sqrt{\frac{\omega^4 + 8 \times \xi^2 \omega^4}{2}}\right)$, where m is the mass of the structure , ω is the frequency of excitation, ξ is the damping ratio [11].

Case1 :
T > 0.0 && T < 0.2 = 23.0
T > 0.2 && T < 0.4 = 10.0
T > 0.4 && T < 1.0 = 16.0
T > 1.0 && T < 2.4 = 12.0
T > 2.4 && T < 3.0 = 3.0
T > 3.0 && T < 3.8 = 17.0
T > 3.8 && T < 4.2 = 9.0
T > 4.2 && T < 4.8 = 1.0
T > 4.8 && T < 5.0 = 15.0

Discussion on the above data :

	Displacement	Acceleration
Present techniques	49.097	2.8567
$S_{force} = 0.0$	97.97	2.77
$S_{force} = 1500$	52.36	3.82
$S_{force} = \text{high}$	0.0000	9.075

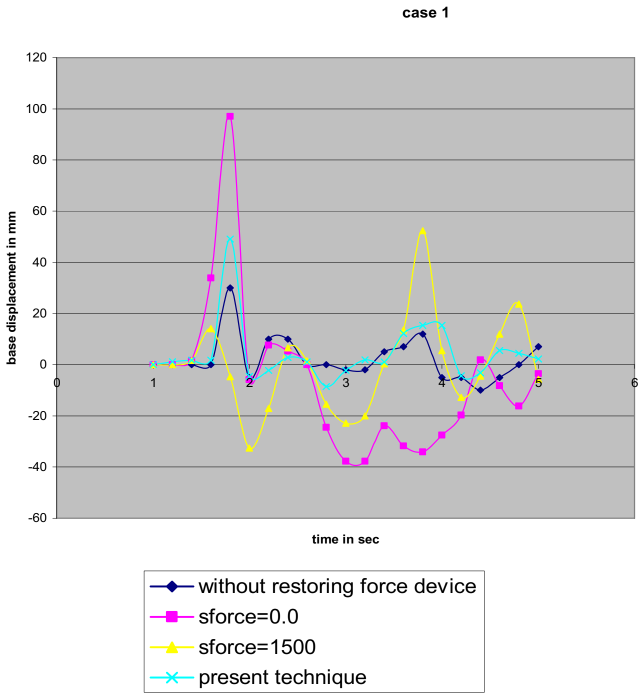

Fig. 5(a) Variation of displacement with frequency

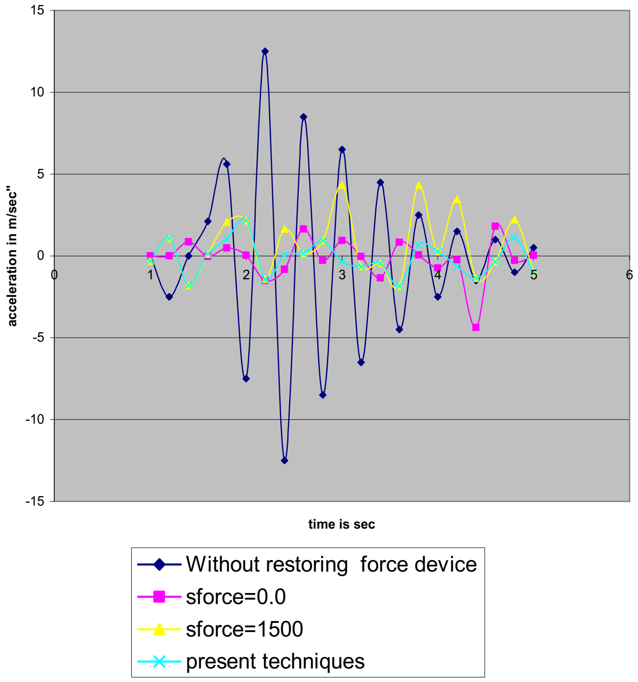

Fig. 5(b) Variation of acceleration with frequency

Case 2 :

T > 0.0 && T < 0.4 = 16.0
T > 0.4 && T < 1.0 = 12.0
T > 1.0 && T < 2.0 = 8.0
T > 2.0 && T < 2.4 = 5.0
T > 2.4 && T < 2.8 = 19.0
T > 2.8 && T < 3.2 = 3.0
T > 3.2 && T < 3.8 = 23.0
T > 3.8 && T < 4.2 = 15.0
T > 4.2 && T < 5.0 = 11.0

Discussion on the above data [12]

	Displacement	Acceleration
Present Techniques	39.99	2.82
$S_{force} = 0.0$	114.97	2.877
$S_{force} = 1500$	50.173860	3.04
$S_{force} = \text{high}$	0.0000	11.187

Case 3 :

T > 0.1 && T < 0.3 = 13.0
T > 0.3 && T < 0.7 = 10.0
T > 0.7 && T < 1.0 = 17.0
T > 1.0 && T < 1.3 = 9.0
T > 1.3 && T < 1.8 = 10.0
T > 1.8 && T < 2.0 = 15.0
T > 2.0 && T < 2.4 = 7.0
T > 2.4 && T < 2.8 = 1.0
T > 2.8 && T < 3.2 = 5.0
T > 3.2 && T < 4.0 = 25.0
T > 4.0 && T < 4.2 = 12.0
T > 4.2 && T < 4.8 = 18.0
T > 4.8 && T < 5.0 = 3.0

Discussion on above data

	Displacement	Acceleration
Present Techniques	57.05	3.8567
$S_{force} = 0.0$	155.89	2.764
$S_{force} = 1500$	52.54	5.12
$S_{force} = \text{high}$	0.0000	4.9075

Case 4 :

T > 0.0 && T < 0.8 = 13.0
T > 0.8 && T < 1.4 = 10.0
T > 1.4 && T < 1.8 = 12.0
T > 1.8 && T < 2.0 = 0.0
T > 2.0 && T < 2.6 = 23.0
T > 2.6 && T < 3.4 = 17.0
T > 3.4 && T < 3.8 = 9.0
T > 3.8 && T < 4.1 = 10.0
T > 4.1 && T < 4.6 = 12.0
T > 4.6 && T < 5.0 = 5.0

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Discussion on above data

	Displacement	Acceleration
Present Techniques	50.59	4.1198
$S_{force} = 0.0$	65.56	2.775
$S_{force} = 1500$	51.023	4.23
$S_{force} = high$	0.0000	8.2419

Case 5 :

T > 0.0 && T < 0.6 = 3.0
T > 0.6 && T < 1.2 = 8.0
T > 1.2 && T < 1.9 = 12.0
T > 1.9 && T < 2.4 = 16.0
T > 2.4 && T < 3.2 = 14.0
T > 3.2 && T < 3.6 = 9.0
T > 3.6 && T < 4.2 = 6.0
T > 4.2 && T < 5.0 = 13.0

Discussion on above data

	Displacement	Acceleration
Present Techniques	59.49	4.24
$S_{force} = 0.0$	369.64	1.67
$S_{force} = 1500$	71.84	1.67
$S_{force} = high$	0.0000	15.6138

V.CONCLUSIONS

From the above it is clear that base isolation protects structures from vibration. The effects of the restoring force device on the response of a space frame structure resting on sliding type of bearing with a restoring force device are herewith studied in this paper. The NS component of the El - Centro earthquake and harmonic ground acceleration was considered for earthquake excitation. A 4 storeyed building was considered for the simulation & experimentation purposes. The structure was modeled by considering 6 DOF at each node. The sliding support was modeled as a fictitious spring with 2 horizontal DOF. The simulation results of acceleration, base shear, bending moment and base displacement were obtained. It is concluded from the study that the displacement of the structure reduces by the use of the restoring force device. Also, the peak values of acceleration, bending moment and base shear also decreases. The simulation results show the effectiveness of the developed and proposed method. Various case studies were also dealt with in this paper. From the above discussions, it is clear that base isolation protects the engineering structures from vibration by the use of smart structures.

REFERENCES

[1] Abe. M., Yoshida J. and Fujino Y., "Multiaxial behaviours of laminated rubber bearings and their modeling I : Experimental study", *Journal of Structural Engineering*, *ASCE*, Vol. 130, pp. 1119 - 1132, 2004.

[2] Abe. M., Yoshida J. and Fujino Y., "Multiaxial behaviours of laminated rubber bearings and their modeling II : Experimental study", *Journal of Structural Engineering, ASCE*, Vol. 130, pp. 1133 - 1144, 2004.

[3] Arya A.S, Chandra B. and Qamaruddin M., "A new building system for improved earthquake performance", *Proc. of the Sixth Symposium of Earthquake Engineering*, University of Roorkee, India, 1978.

[4] Bhasker P. and Jangid R.S., "Experimental study of base isolated structures", *Journal of Earthquake Technology*, *ISET*, Vol. 38, pp 1-15, 2001.

[5] Chalhoub, M.S., and Kelly J.M., "Sliders and tension controlled reinforced bearings combined for earthquake isolation system", *Earthquake Engineering and Structural Dynamics*, Vol. 19, 1990, pp. 333 - 358.

[6] Chopra. A.K., "Dynamic of structures : theory and applications to earthquake engineering Second edition., Prentice Hall, Upper saddle river, New Jersey, 2001.

[7] Jain S.K and Thakkar S.K., "Response control of buildings with rubber bearings", *International Journal of Structures*, Vol. 18, No. 2, pp. 109 - 125, 1998.

[8] Jangid R.S. and Londhe Y.B., "Effectiveness of elliptical rolling rods for base isolation", *Journal of Structural Engineering, ASCE*, Vol. 124, pp. 469 - 472, 1998.

[9] Jangid R.S., "Stochastic seismic response of structures isolated by rolling rods", *Journal of Engineering Structures*, Vol. 22, pp. 937 - 946, 2000.

[10] Krishnamoorthy. A., "Effects of Damping Ratio of Restoring force Device on Response of a Structure Resting on Sliding Supports with Restoring Force Device", Electronics Journal of Structural Engineering, eJSE Journal, pp. 55 - 68, Vol. 5, 2005.

[11] Krishnamoorthy. A., and Saumil P., "In plane response of a symmetric space frame with sliding supports", *International Journal of Applied Science and Engineering,* Vol. 3, pp. 1-11, 2005.

[12] Paz, M., "Structural Dynamics - Theory and Computation", 1991, Van Nostrand Reinhold, New York, USA.

[13] Qamaruddin M., Arya A.S. and Chandra B.,"Seismic response of brick buildings with sliding substructures", *Journal of Structural Engineering*, *ASCE*, Vol. 112, pp. 558-572, 1986.

[14] Vafai. A., Hamidi M., and Ahmadi., "Numerical modeling of MDOF structures with sliding supports using rigid plastic link", *Earthquake Engineering and Structural Dynamics,* Vol. 30, 2001, pp. 27 - 42.

[15] Yang, Y.B., Lee T.Y., and Tsai I.C., "Response of multi - DOF structures with sliding supports", *Earthquake engineering and Structural Dynamics*, Vol. 19, 1990, pp. 739 - 752.

[16] Zayas. V.A., Low. S.S., and Mahin. S.A., "A simple pendulum technique for achieving seismic isolation", *Earthquake Spectra,* Vol. 6, 1990, pp. 317 - 333.