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Stability Analysis of Impulsive BAM Fuzzy Cellular Neural Networks with Distributed Delays and Reaction-diffusion Terms

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Abstract—In this paper, a class of impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms is formulated and investigated. By employing the delay differential inequality and inequality technique developed by Xu et al., some sufficient conditions ensuring the existence, uniqueness and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms are obtained. In particular, the estimate of the exponential convergence rate is also provided, which depends on system parameters, diffusion effect and impulsive disturbed intention. It is believed that these results are significant and useful for the design and applications of BAM fuzzy cellular neural networks. An example is given to show the effectiveness of the results obtained here.

Keywords—Bi-directional associative memory; fuzzy cellular neural networks; reaction-diffusion; delays; impulses; global exponential stability.

I. Introduction

THE reaction-diffusion neural networks were firstly introduced by L. O. Chua in order to study passivity and complexity [1]. M. Itoh and L. O. Chua had further investigated the complexity of reaction-diffusion CNN with various boundary conditions such as the Neumann boundary conditions, the Dirichlet boundary conditions and the periodic boundary conditions [2]. In 2003, Wang and Xu proposed a class of reaction-diffusion Hopfield neural networks with the Neumann boundary conditions and studied the global exponential stability of this class of networks. Since than, many researchers have done extensive works on this subject, there exist some results on global asymptotical stability, global exponential stability and periodic solutions for the reaction-diffusion neural networks with various delays and the Neumann boundary conditions, for example, see [4]-[16] and references therein. On the other hand, besides delay and diffusion effect, impulsive effect likewise exists in the neural network system. As artificial electronic systems, neural networks such as CNNs, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems just as time delays. Therefore, it is necessary to consider impulsive effect, diffusion effect and delay effect on the stability of neural networks [21]-[25], [28]. However, we note that, in [3]-[16], [21]-[23], [25], [28], all criteria on stability and periodicity for reaction-diffusion neural networks with the Neumann boundary conditions were independent of the diffusion effect. In other words, in those results of Ref. [3]-[16], [21]-[23], [25], [28], we do not know the reaction-diffusion phenomenon how to affect stability of neural networks. Recently, Lu et al. proposed some reaction-diffusion delayed recurrent neural networks with Dirichlet boundary conditions and obtained some diffusion-dependent criteria on stability and periodicity for the formulated neural networks [17]-[19], these criteria show that the diffusion phenomenon is beneficial to the stabilization of neural systems.

The bidirectional associative memory (BAM) neural network is first introduced by Kosko [29]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers, the X-layer and Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer. Through iterations of forward and backward information flows between the two layer, it performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer autoassociative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks possesses good application prospects in some fields such as pattern recognition, signal and image process, artificial intelligence. To the best of our knowledge, few authors have considered BAM fuzzy cellular neural networks. To the best of our knowledge, few authors have investigated impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms.

Motivated by the above discussions, the objective of this paper is to formulate and study impulsive BAM fuzzy neural networks with distributed delays and diffusion. Under quite general conditions, by employing delay differential inequality and inequality technique developed by Xu et al. [27], [28], some sufficient conditions ensuring the existence, uniqueness and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with distributed delays and diffusion are obtained.

The paper is organized as follows. In Section 2, the new neural network model is formulated, and the necessary knowledge is provided. Main results are given in Section 3. In Section 4, An example is given to show the effectiveness of the results obtained here. Finally, we give the conclusion in Section 5.

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II. Model description and preliminaries

In this section, we will consider the model of impulsive BAM fuzzy neural networks with distributed delays and reaction-diffusion terms, it is described by the following functional differential equations:

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial u_i(t,x)}{\partial x_r}) - a_i u_i(t,x) \\ \qquad + \sum\limits_{j=1}^{m} a_{ij} g_j(v_j(t,x)) + \sum\limits_{j=1}^{m} \tilde{a}_{ij} w_j \\ \qquad + \bigwedge\limits_{j=1}^{m} \alpha_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \qquad + \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \qquad + \bigwedge\limits_{j=1}^{m} T_{ij} w_j + \bigvee\limits_{j=1}^{m} H_{ij} w_j + I_i, \quad t \ge 0, t \ne t_k, \\ u_i(t^+,x) = u_i(t^-,x) + P_{ik}(u_i(t^-,x)), \quad t = t_k, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(\bar{D}_{jr}\frac{\partial v_j(t,x)}{\partial x_r}) - b_j v_j(t,x) \\ \qquad + \sum\limits_{i=1}^{n} b_{ji} f_i(u_i(t,x)) + \sum\limits_{i=1}^{n} \tilde{b}_{ji} \tilde{w}_i \\ \qquad + \bigwedge\limits_{i=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \qquad + \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \qquad + \bigwedge\limits_{i=1}^{n} \tilde{T}_{ji} \tilde{w}_i + \bigvee\limits_{i=1}^{n} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \ge 0, t \ne t_k, \\ v_j(t^+,x) = v_j(t^-,x) + Q_{ik}(v_j(t^-,x)), \quad t = t_k \end{cases} \tag{1}$$

for $i \in \mathscr{I} \triangleq \{1,2,\cdots,n\}$, $j \in \mathscr{J} \triangleq \{1,2,\cdots,m\}$, $k \in N \triangleq \{1,2,\cdots\}$, where $x = (x_1,x_2,\cdots,x_l)^T \in \Omega \subset R^l$, and $\Omega = \{x = (x_1,x_2,\cdots,x_l)^T, \ |x_r| < l_r, r = 1,2,\cdots,l\}$ is a bounded compact set with smooth boundary and $mes\Omega > 0$ in space $R^l; u = (u_1,u_2,\cdots,u_n)^T \in R^n, v = (v_1,v_2,\cdots,v_m)^T \in R^m$; $u_i(t,x)$ and $v_j(t,x)$ are the states of the ith neuron and the jth neuron at time t and in space x, respectively; $D_{ik} \ge 0$ and $\bar{D}_{jk} \ge 0$ correspond to the transmission diffusion coefficient along the ith neuron and the jth neuron, respectively; f_i and g_j denote the signal functions of the ith neuron and the jth neuron at time t and in space x, respectively; $\tilde{w}_i$ and w_j denote inputs of the ith neuron and the jth neuron at the time t, respectively; and I_i and J_j denote bias of the ith neuron and the jth neuron at the time t, respectively; $a_i > 0, b_j > 0, a_{ij}, \tilde{a}_{ij}, \alpha_{ij}, \tilde{\alpha}_{ij}, b_{ji}, \tilde{b}_{ji}, \beta_{ji}, \tilde{\beta}_{ji}$ are constants, a_i and b_j represent the rate with which the ith neuron and the jth neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; a_{ij}, b_{ji} and $\tilde{a}_{ij}, \tilde{b}_{ji}$ denote connection weights of feedback template and feed-forward template, respectively; α_{ij}, β_{ji} and $\tilde{\alpha}_{ij}, \tilde{\beta}_{ji}$ denote connection weights of the delays fuzzy feedback MIN template and the delays fuzzy feedback MAX template, respectively; $T_{ij}, \tilde{T}_{ji}$ and $H_{ij}, \tilde{H}_{ji}$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; t_k is called impulsive moment, and satisfies $0 < t_1 < t_2 < \cdots$, $\lim\limits_{k \to +\infty} t_k = +\infty$; $u_i(t_k^-,x)$, $v_j(t_k^-,x)$ and $u_i(t_k^+,x)$, $v_j(t_k^+,x)$ denote the left-hand and right-hand limits at t_k, respectively; P_{ik} and Q_{jk} show impulsive perturbation of the ith neuron and jth neuron at time t_k, respectively. We always assume $u_i(t_k^+,x) = u_i(t_k,x)$ and $v_j(t_k^+,x) = v_j(t_k,x)$, $k \in N$.

The Dirichlet boundary conditions and initial conditions are respectively given by

$$\begin{cases} u_i(t,x) = 0, & t \ge 0, \quad x \in \partial\Omega, \quad i \in \mathscr{I}, \\ v_j(t,x) = 0, & t \ge 0, \quad x \in \partial\Omega, \quad j \in \mathscr{J}, \end{cases} \tag{2}$$

and

$$\begin{cases} u_i(s,x) = \phi_{ui}(s,x), & -\infty < s \le 0, \\ v_j(s,x) = \phi_{vj}(s,x), & -\infty < s \le 0, \end{cases} \tag{3}$$

where $\phi_{ui}(s,x), \phi_{vj}(s,x)$ $(i \in \mathscr{I}, j \in \mathscr{J})$ are bounded and continuous on $(-\infty,0] \times \Omega$, respectively.

If the impulsive operators $P_{ik}(u_i) = 0$, $Q_{jk}(v_j) = 0$, $i \in \mathscr{I}, j \in \mathscr{J}, k \in N$, then system (1) may reduce to the following model:

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial u_i(t,x)}{\partial x_r}) - a_i u_i(t,x) \\ \qquad + \sum\limits_{j=1}^{m} a_{ij} g_j(v_j(t,x)) + \sum\limits_{j=1}^{m} \tilde{a}_{ij} w_j \\ \qquad + \bigwedge\limits_{j=1}^{m} \alpha_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \qquad + \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \qquad + \bigwedge\limits_{j=1}^{m} T_{ij} w_j + \bigvee\limits_{j=1}^{m} H_{ij} w_j + I_i, \quad t \ge 0, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(\bar{D}_{jr}\frac{\partial v_j(t,x)}{\partial x_r}) - b_j v_j(t,x) \\ \qquad + \sum\limits_{i=1}^{n} b_{ji} f_i(u_i(t,x)) + \sum\limits_{i=1}^{n} \tilde{b}_{ji} \tilde{w}_i \\ \qquad + \bigwedge\limits_{i=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \qquad + \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \qquad + \bigwedge\limits_{i=1}^{n} \tilde{T}_{ji} \tilde{w}_i + \bigvee\limits_{i=1}^{n} \tilde{H}_{ji} \tilde{w}_i + J_j, \quad t \ge 0, \end{cases} \tag{4}$$

where the Dirichlet boundary conditions and initial conditions are given by (2) and (3), respectively. System (4) is called the continuous system of model (1).

Throughout this paper, we make the following assumptions:

(H1) For neuron activation functions f_i and g_j $(i \in \mathscr{I}; j \in \mathscr{J})$, there exist two positive diagonal matrices $F = \mathrm{diag}(F_1,F_2,\cdots,F_n)$ and $G = \mathrm{diag}(G_1,G_2,\cdots,G_m)$ such that

$$F_i = \sup_{u \ne v} \left| \frac{f_i(u) - f_i(v)}{u - v} \right|, \quad G_j = \sup_{u \ne v} \left| \frac{g_j(u) - g_j(v)}{u - v} \right|$$

for all $u, v \in R$ $(u \ne v)$.

(H2) The delay kernels $N_{ij}, K_{ji} : [0,+\infty) \to [0,+\infty)$, $(i \in \mathscr{I}; j \in \mathscr{J})$ are real-valued non-negative continuous, and there exists a constant $\delta > 0$ such that functions

$$n_{ij}(\lambda) = \int_0^{+\infty} e^{\lambda s} N_{ij}(s) ds$$

and

$$k_{ji}(\lambda) = \int_0^{+\infty} e^{\lambda s} K_{ji}(s) ds$$

are continuous for $\lambda \in [0, \delta)$ $(i \in \mathscr{I}; j \in \mathscr{J})$.

(H3) Let $\bar{P}_k(u) = u + P_k(u)$ and $\bar{Q}_k(v) = v + Q_k(v)$ be Lipschitz continuous in R^n and R^m, respectively, that is, there exist nonnegative diagnose matrices $\Gamma_k = \mathrm{diag}(\gamma_{1k}, \gamma_{2k}, \cdots, \gamma_{nk})$ and $\bar{\Gamma}_k = \mathrm{diag}(\bar{\gamma}_{1k}, \bar{\gamma}_{2k}, \cdots, \bar{\gamma}_{mk})$ such that

$$|\bar{P}_k(u) - \bar{P}_k(v)| \leq \Gamma_k |u - v|, \quad for\ all\ u, v \in R^n,\ k \in N,$$

$$|\bar{Q}_k(u) - \bar{Q}_k(v)| \leq \bar{\Gamma}_k |u - v|, \ for\ all\ u, v \in R^m,\ k \in N,$$

where $\bar{P}_k(u) = (\bar{P}_{1k}(u_1), \bar{P}_{2k}(u_2), \cdots, \bar{P}_{nk}(u_n))^T$, $\bar{Q}_k(v) = (\bar{Q}_{1k}(v_1), \bar{Q}_{2k}(v_2), \cdots, \bar{Q}_{mk}(v_m))^T$, $P_k(x) = (P_{1k}(u_1), P_{2k}(u_2), \cdots, P_{nk}(u_n))^T$, $Q_k(v) = (Q_{1k}(v_1), Q_{2k}(v_2), \cdots, Q_{mk}(v_m))^T$.

To begin with, we introduce some notation and recall some basic definitions.

$PC[J \times \Omega, R^\iota] \triangleq \{w(t,x) : J \times \Omega \to R^\iota \,|\, w(t,x)$ is continuous at $t \neq t_k$, $w(t_k^+, x) = u(t_k, x)$ and $w(t_k^-, x)$ exists for $t,\ t_k \in J,\ k \in N\}$, where $J \subset R$ is an interval.

$PC[J, R^\iota] \triangleq \{w(t) : J \to R^\iota \,|\, w(t)$ is continuous at $t \neq t_k$, $w(t_k^+) = w(t_k)$ and $w(t_k^-)$ exists for $t,\ t_k \in J,\ k \in N\}$, where $J \subset R$ is an interval.

$PC(\Omega) \triangleq \{\varphi : (-\infty, 0] \times \Omega \to R^\iota |\ \varphi(s^+, x) = \phi(s, x)$ for $s \in (-\infty, 0)$, $\varphi(s^-, x)$ exists for $s \in (-\infty, 0]$, $\varphi(s^-, x) = \varphi(s, x)$ for all but at most a finite number of points $s \in (-\infty, 0]\}$.

$PC_n \triangleq \{\phi : (-\infty, 0] \to R^n |\ \phi(s^+) = \phi(s)$ for $s \in (-\infty, 0)$, $\phi(s^-)$ exists for $s \in (-\infty, 0]$, $\phi(s^-) = \phi(s)$ for all but at most a finite number of points $s \in (-\infty, 0]\}$.

$PC_m \triangleq \{\phi : (-\infty, 0] \to R^m |\ \phi(s^+) = \phi(s)$ for $s \in (-\infty, 0)$, $\phi(s^-)$ exists for $s \in (-\infty, 0]$, $\phi(s^-) = \phi(s)$ for all but at most a finite number of points $s \in (-\infty, 0]\}$.

For $w(t,x) = (w_1(t,x), w_2(t,x), \cdots, w_\iota(t,x))^T \in R^\iota$, we define $\|w_i(t,x)\|_2 = \left[\int_\Omega |w_i(t,x)|^2 dx\right]^{\frac{1}{2}}$, $i = 1, 2, \cdots, \iota$, and for any $\phi(s,x) = (\phi_1(s,x), \phi_2(s,x), \cdots, \phi_\iota(s,x))^T \in PC(\Omega)$, the norm on $PC(\Omega)$ is defined by

$$\|\phi\|_2 = \sup_{-\infty \leq s \leq 0} \sum_{i=1}^{\iota} \|\phi_i(s,x)\|_2,$$

then it can be proved that $PC(\Omega)$ is a Banach space [19].

Definition 1: A function $(u(t,x), (v(t,x))^T$ $(u : (-\infty, +\infty) \times \Omega \to R^n,\ v : (-\infty, +\infty) \times \Omega \to R^m)$ is said to be the solution of impulsive system (1), if the following two conditions are satisfied

(i) For t, $u(t,x)$ and $v(t,x)$ are piecewise continuous with first kind discontinuity at the points $t_k,\ k \in N$. Moreover, $u(t,x)$ and $v(t,x)$ is right continuous at each discontinuity point.

(ii) $u(t,x)$ and $v(t,x)$ satisfy (1) and (2) for $t \geq 0$, and $u(t,x)$ and $v(t,x)$ satisfy (3) for $t \leq 0$.

Especially, a point $(u^*, v^*)^T$ $(u^* \in R^n, v^* \in R^m)$ is called an equilibrium point of system (1), if $(u(t,x), v(t))^T = (u^*, v^*)^T$ is a solution of system (1).

Throughout this paper, we always assume that the impulsive jumps P_k and Q_k satisfy (referring to[14])

$$P_k(u^*) = 0 \ and\ Q_k(v^*) = 0, \quad k \in N,$$

i.e.,

$$\bar{P}_k(u^*) = u^* \ and\ \bar{Q}_k(v^*) = v^*, \quad k \in N, \tag{7}$$

where $(u^*, v^*)^T$ is the equilibrium point of continuous systems (4). That is, if $(u^*, v^*)^T$ is an equilibrium point of continuous system (4), then $(u^*, v^*)^T$ is also the equilibrium of impulsive system (1).

Definition 2: The equilibrium point $(u^*, v^*)^T$ of system (1) is said to be globally exponentially stable, if there exist constants $\lambda > 0$ and $M \geq 1$ such that

$$\begin{aligned} &\sum_{i=1}^{n} \| u_i(t,x) - u_i^* \|_2 + \sum_{j=1}^{m} \| v_j(t,x) - v_j^* \|_2 \\ \leq\ & M(\| \phi_u - u^* \|_2 + \| \phi_v - v^* \|_2) e^{-\lambda t} \end{aligned} \tag{8}$$

for all $t \geq 0$, where

$$(u_1(t,x), \cdots, u_n(t,x), v_1(t,x), \cdots, v_m(t,x))^T$$

is any solution of system (1) with the initial condition $(\phi_{u1}, \phi_{u2}, \cdots, \phi_{un}, \phi_{v1}, \phi_{v2}, \cdots, \phi_{vm})^T$, and $\phi_u = (\phi_{u1}, \phi_{u2}, \cdots, \phi_{un})^T$, $\phi_v = (\phi_{v1}, \phi_{v2}, \cdots, \phi_{vm})^T$.

Lemma 1: [26] For any positive integer n, let $h_j : R \to R$ be a function $(j = 1, 2, \cdots, n)$, then we have

$$\Big| \bigwedge_{j=1}^{n} \alpha_j h_j(u_j) - \bigwedge_{j=1}^{n} \alpha_j h_j(v_j) \Big| \leq \sum_{j=1}^{n} \Big|\alpha_j\Big| \cdot \Big|h_j(u_j) - h_j(v_j)\Big|,$$

$$\Big| \bigvee_{j=1}^{n} \alpha_j h_j(u_j) - \bigvee_{j=1}^{n} \alpha_j h_j(v_j) \Big| \leq \sum_{j=1}^{n} \Big|\alpha_j\Big| \cdot \Big|h_j(u_j) - h_j(v_j)\Big|$$

for all $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)^T$, $u = (u_1, u_2, \cdots, u_n)^T$, $v = (v_1, v_2, \cdots, v_n)^T \in R^n$.

Lemma 2: [17] Let Ω be a cube $|x_r| < l_r\ (r = 1, 2, \cdots, l)$ and let $h(x)$ be a real-valued function belonging to $C^1(\Omega)$ which vanish on the boundary $\partial\Omega$ of Ω, i.e., $h(x)|_{\partial\Omega} = 0$. Then

$$\int_\Omega h^2(x) dx \leq l_r^2 \int_\Omega \Big| \frac{\partial h}{\partial x_r} \Big|^2 dx.$$

By the methods developed by Xu et al. [27], [28], we can get the following inequality.

Lemma 3: Let $a < b \leq +\infty$, and let $\tilde{u}(t) = (\tilde{u}_1(t), \tilde{u}_2(t), \cdots, \tilde{u}_n(t))^T \in PC[[a,b), R^n]$ and $\tilde{v}(t) = (\tilde{v}_1(t), \tilde{v}_2(t), \cdots, \tilde{v}_m(t))^T \in PC[[a,b), R^m]$ satisfy the following delay differential inequality with the initial condition $\tilde{u}(a+s) \in PC_n$ and $\tilde{v}(a+s) \in PC_m$:

$$\begin{cases} D^+ \tilde{u}_i(t) \leq -r_i \tilde{u}_i(t) + \sum\limits_{j=1}^{m} p_{ij} \tilde{v}_j(t) \\ \qquad + \sum\limits_{j=1}^{m} q_{ij} \int_0^{+\infty} N_{ij}(s) \tilde{v}_j(t-s) ds, \quad i \in \mathscr{I}, \\ D^+ \tilde{v}_j(t) \leq -\bar{r}_j \tilde{v}_j(t) + \sum\limits_{i=1}^{n} \bar{p}_{ji} \tilde{u}_i(t) \\ \qquad + \sum\limits_{i=1}^{n} \bar{q}_{ji} \int_0^{+\infty} K_{ji}(s) \tilde{u}_i(t-s) ds, \quad j \in \mathscr{J}. \end{cases} \tag{8}$$

where $r_i > 0$, $p_{ij} > 0$, $q_{ij} > 0$, $\bar{r}_j > 0$, $\bar{p}_{ji} > 0$, $\bar{q}_{ji} > 0$, $i \in \mathscr{I}$, $j \in \mathscr{J}$. If the initial conditions satisfy

$$\begin{cases} \tilde{u}(s) \le \kappa \xi e^{-\lambda(s-a)}, & s \in [-\infty, a], \\ \tilde{v}(s) \le \kappa \eta e^{-\lambda(s-a)}, & s \in [-\infty, a], \end{cases} \tag{9}$$

in which $\lambda > 0$, $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$ and $\eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ satisfy

$$\begin{cases} (\lambda - r_i)\xi_i + \sum_{j=1}^{m} (p_{ij} + q_{ij} \int_0^{+\infty} N_{ij}(s) e^{\lambda s} ds)\eta_j < 0, \\ (\lambda - \bar{r}_j)\eta_j + \sum_{i=1}^{n} (\bar{p}_{ji} + \bar{q}_{ji} \int_0^{+\infty} K_{ji}(s) e^{\lambda s} ds))\xi_i < 0, \end{cases} \tag{10}$$

for $i \in \mathscr{I}$, $j \in \mathscr{J}$. Then

$$\begin{cases} \tilde{u}(t) \le \kappa \xi e^{-\lambda(t-a)}, & t \in [a, b), \\ \tilde{v}(t) \le \kappa \eta e^{-\lambda(t-a)}, & t \in [a, b). \end{cases}$$

Proof. For $i \in \mathscr{I}$, $j \in \mathscr{J}$ and arbitrary $\varepsilon > 0$, set $z_i(t) \triangleq (\kappa + \varepsilon)\xi_i e^{-\lambda(t-a)}$, $\bar{z}_j(t) \triangleq (\kappa + \varepsilon)\eta_j e^{-\lambda(t-a)}$, we prove that

$$\begin{cases} \tilde{u}_i(t) \le z_i(t) = (\kappa + \varepsilon)\xi_i e^{-\lambda(t-a)}, & t \in [a, b), \, i \in \mathscr{I}, \\ \tilde{v}_j(t) \le \bar{z}_j(t) = (\kappa + \varepsilon)\eta_j e^{-\lambda(t-a)}, & t \in [a, b), \, j \in \mathscr{J}. \end{cases} \tag{11}$$

If this is not true, no loss of generality, suppose that there exist i_0 and $t^* \in [a, b)$ such that

$$\begin{aligned} &\tilde{u}_{i_0}(t^*) = z_{i_0}(t^*), \quad D^+\tilde{u}_{i_0}(t^*) \ge \dot{z}_{i_0}(t^*), \\ &\tilde{u}_i(t) \le z_i(t), \quad \tilde{v}_j(t) \le \bar{z}_j(t), \quad t \in [a, t^*] \end{aligned} \tag{12}$$

for $i \in \mathscr{I}$, $j \in \mathscr{J}$.

However, from (8), (12) and (11), we get

$$\begin{aligned} D^+\tilde{u}_{i_0}(t^*) &\le -r_{i_0}\tilde{u}_{i_0}(t^*) + \sum_{j=1}^{m} p_{i_0 j}\tilde{v}_j(t^*) \\ &\quad + \sum_{j=1}^{m} q_{i_0 j} \int_0^{+\infty} N_{i_0 j}(s)\tilde{v}_j(t^* - s) ds \\ &\le -r_{i_0}(\kappa + \varepsilon)\xi_{i_0} e^{-\lambda(t^* - a)} \\ &\quad + \sum_{j=1}^{m} p_{i_0 j}\eta_j(\kappa + \varepsilon)\eta_j e^{-\lambda(t^* - a)} \\ &\quad + \sum_{j=1}^{m} q_{i_0 j}(\kappa + \varepsilon)\eta_j e^{-\lambda(t^* - a)} \\ &\quad \times \int_0^{+\infty} e^{\lambda s} N_{i_0 j}(s) ds \\ &= \Big[-r_{i_0}\xi_{i_0} + \sum_{j=1}^{m} \Big(p_{i_0 j} \\ &\quad + q_{i_0 j} \int_0^{+\infty} e^{\lambda s} N_{i_0 j}(s) ds \Big) \\ &\quad \times \eta_j \Big] (\kappa + \varepsilon) e^{-\lambda(t^* - a)}. \end{aligned}$$

Since (10) holds, it follows that $-r_{i_0}\xi_{i_0} + \sum_{j=1}^{m}(p_{i_0 j} + q_{i_0 j} \int_0^{+\infty} e^{\lambda s} N_{i_0 j}(s) ds)\eta_j < -\lambda \xi_{i_0} < 0$. Therefore, we have

$$D^+\tilde{u}_{i_0}(t^*) < -\lambda \xi_{i_0}(\kappa + \varepsilon) e^{-\lambda(t^* - a)} = \dot{z}_{i_0}(t^*),$$

which contradicts the inequality $D^+\tilde{u}_{i_0}(t^*) \ge \dot{z}_{i_0}(t^*)$ in (12). Thus (11) holds for all $t \in [a, b)$. Letting $\varepsilon \to 0$, we have

$$\begin{cases} \tilde{u}_i(t) \le \kappa \xi_i e^{-\lambda(t-a)}, & t \in [a, b), \quad i \in \mathscr{I}, \\ \tilde{v}_j(t) \le \kappa \eta_j e^{-\lambda(t-a)}, & t \in [a, b), \quad j \in \mathscr{J}. \end{cases}$$

The proof is completed.

III. Main results

In this section, we will discuss the existence and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms, and give their proofs.

Theorem 1: Under assumptions (H1), (H2) and (H3), if the following conditions hold:

(C1) there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ such that

$$\begin{cases} (\lambda - a_i - \sum_{r=1}^{l} \frac{D_{ir}}{l_r^2})\xi_i + \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} N_{ij}(s) ds \Big] G_j \eta_j < 0, \quad i \in \mathscr{I} \\ (\lambda - b_j - \sum_{r=1}^{l} \frac{\bar{D}_{jr}}{l_r^2})\eta_j + \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} K_{ji}(s) ds \Big] F_i \, \xi_i < 0, \quad j \in \mathscr{J}; \end{cases}$$

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \le i \le n, 1 \le j \le m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}$, $k \in N$;

then system (1) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\lambda - \mu$.

Proof. We shall prove this theorem in two steps.

Step 1: We prove the existence of the equilibrium point of system (4).

Let $(u(t, x), v(t, x))^T$ and $(\bar{u}(t, x), \bar{v}(t, x))^T$ be arbitrary two solutions of system (4), where $u(t, x) = (u_1(t, x), \cdots, u_n(t, x))^T$, $v(t, x) = (v_1(t, x), \cdots, v_m(t, x))^T$ and $\bar{u}(t, x) = \bar{u}_1(t, x), \cdots, \bar{u}_n(t, x))^T$, $\bar{v}(t, x) = \bar{v}_1(t, x), \cdots, \bar{v}_m(t, x))^T$, then we have

$$\begin{aligned} &\frac{\partial(u_i(t, x) - \bar{u}_i(t, x))}{\partial t} \\ &= \sum_{r=1}^{l} \frac{\partial}{\partial x_r} \Big(D_{ir} \frac{\partial(u_i(t, x) - \bar{u}_i(t, x))}{\partial x_r} \Big) \\ &\quad - a_i(u_i(t, x) - \bar{u}_i(t, x)) \\ &\quad + \sum_{j=1}^{m} a_{ij}[g_j(v_j(t, x)) - g_j(\bar{v}_j(t, x))] \\ &\quad + \bigwedge_{j=1}^{m} \alpha_{ij} \int_{-\infty}^{t} N_{ij}(t - s) g_j(v_j(s, x)) ds \\ &\quad - \bigwedge_{j=1}^{m} \alpha_{ij} \int_{-\infty}^{t} N_{ij}(t - s) g_j(\bar{v}_j(s, x)) ds \\ &\quad + \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} \int_{-\infty}^{t} N_{ij}(t - s) g_j(v_j(s, x)) ds \end{aligned}$$

$$-\bigvee_{j=1}^{m}\tilde{\alpha}_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(\bar{v}_j(s,x))ds \quad (13)$$

and

$$\begin{aligned}&\frac{\partial(v_j(t,x)-\bar{v}_j(t,x))}{\partial t}\\ =\ & \sum_{r=1}^{l}\frac{\partial}{\partial x_r}\Big(\bar{D}_{jr}\frac{\partial(v_j(t,x)-\bar{v}_j(t,x))}{\partial x_r}\Big)\\ &-b_j(v_j(t,x)-\bar{v}_j(t,x))\\ &+\sum_{i=1}^{n}b_{ji}[f_i(u_i(t,x))-f_i(\bar{u}_i(t,x))]\\ &+\bigwedge_{i=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(u_j(s,x))ds\\ &-\bigwedge_{i=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(\bar{u}_j(s,x))ds\\ &+\bigvee_{i=1}^{n}\tilde{\beta}_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(u_j(s,x))ds\\ &-\bigvee_{i=1}^{n}\tilde{\beta}_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(\bar{u}_j(s,x))ds \quad (14)\end{aligned}$$

for $i\in\mathscr{I}, j\in\mathscr{J}$.

Let $y_i(t,x)=u_i(t,x)-\bar{u}_i(t,x)$, $i\in\mathscr{I}$, $z_j(t,x)=v_j(t,x)-\bar{v}_j(t,x)$, $j\in\mathscr{J}$. Multiply both sides of (13) by $y_i(t,x)$ and integrate it, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_{\Omega}(y_i(t,x))^2dx\\ =\ & \int_{\Omega}y_i(t,x)\sum_{r=1}^{l}\frac{\partial}{\partial x_r}\Big(D_{ir}\frac{\partial y_i(t,x)}{\partial x_r}\Big)dx\\ &-a_i\int_{\Omega}(y_i(t,x))^2dx\\ &+\sum_{j=1}^{m}a_{ij}\int_{\Omega}y_i(t,x)\Big[g_j(v_j(t,x))-g_j(v_j(t,x))\Big]dx\\ &+\int_{\Omega}y_i(t,x)\Big[\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\\ &-\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(\bar{v}_j(s,x))ds\Big]dx\\ &+\int_{\Omega}y_i(t,x)\Big[\bigvee_{j=1}^{m}\tilde{\alpha}_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\\ &-\bigvee_{j=1}^{m}\tilde{\alpha}_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(\bar{v}_j(s,x))ds\Big]dx. \quad (15)\end{aligned}$$

From Green's formula and the Dirichlet boundary condition, we have

$$\begin{aligned}&\int_{\Omega}y_i(t,x)\sum_{r=1}^{l}\frac{\partial}{\partial x_r}\Big(D_{ir}\frac{\partial y_i(t,x)}{\partial x_r}\Big)dx\\ =\ & -\sum_{r=1}^{l}\int_{\Omega}D_{ir}\Big(\frac{\partial(y_i(t,x)}{\partial x_r}\Big)^2dx.\end{aligned}$$

By Lemma 2, we can obtain

$$\begin{aligned}&\int_{\Omega}y_i(t,x)\sum_{r=1}^{l}\frac{\partial}{\partial x_r}\Big(D_{ir}\frac{\partial y_i(t,x)}{\partial x_r}\Big)dx\\ \le\ & -\sum_{r=1}^{l}\frac{D_{ir}}{l_r^2}\|y_i(t,x)\|_2^2. \quad (16)\end{aligned}$$

From assumption (H1) and Höder inequality, we have

$$\begin{aligned}&\sum_{j=1}^{m}a_{ij}\int_{\Omega}y_i(t,x)\Big[g_j(v_j(t,x))-g_j(\bar{v}_j(t,x))\Big]dx\\ \le\ & \sum_{j=1}^{m}|a_{ij}|\int_{\Omega}|y_i(t,x)||g_j(v_j(t,x))-g_j(\bar{v}_j(t,x))|dx\\ \le\ & \sum_{j=1}^{m}|a_{ij}|\int_{\Omega}|y_i(t,x)||z_j(t,x)|G_jdx\\ \le\ & \sum_{j=1}^{m}|a_{ij}|G_j\|y_i(t,x)\|_2\|z_j(t,x)\|_2. \quad (17)\end{aligned}$$

By Lemma 1, assumption (H1) and Höder inequality, we have

$$\begin{aligned}&\int_{\Omega}y_i(t,x)\Big[\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\\ &-\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(\bar{v}_j(s,x))ds\Big]dx\\ \le\ & \int_{\Omega}|y_i(t,x)|\Big|\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\\ &-\bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\Big|dx\\ \le\ & \int_{\Omega}|y_i(t,x)|\sum_{j=1}^{m}|\alpha_{ij}|\Big|\int_{-\infty}^{t}N_{ij}(t-s)g_j(v_j(s,x))ds\\ &-\int_{-\infty}^{t}N_{ij}(t-s)g_j(\bar{v}_j(s,x))ds\Big|dx\\ =\ & \sum_{j=1}^{m}|\alpha_{ij}|\int_{\Omega}|y_i(t,x)|\Big|\int_{-\infty}^{t}N_{ij}(t-s)\\ &\times[g_j(v_j(s,x))-g_j(\bar{v}_j(s,x)]ds\Big|dx\\ \le\ & \sum_{j=1}^{m}|\alpha_{ij}|G_j\int_{\Omega}|y_i(t,x)|\Big[\int_{-\infty}^{t}N_{ij}(t-s)\\ &\times|v_j(s,x))-\bar{v}_j(s,x)|ds\Big]dx\\ =\ & \sum_{j=1}^{m}|\alpha_{ij}|G_j\int_{-\infty}^{t}N_{ij}(t-s)\\ &\times\Big[\int_{\Omega}|y_i(t,x)||v_j(s,x))-\bar{v}_j(s,x)|dx\Big]ds\\ \le\ & \sum_{j=1}^{m}|\alpha_{ij}|G_j\|y_i(t,x)\|_2\int_{-\infty}^{t}N_{ij}(t-s)\|z_j(s,x)\|_2ds. \quad (18)\end{aligned}$$

By the same reason, we have

$$\begin{aligned}&\int_\Omega y_i(t,x)\Big[\bigvee_{j=1}^m \tilde{\alpha}_{ij}\int_{-\infty}^t N_{ij}(t-s)g_j(v_j(s,x))ds\\&-\bigvee_{j=1}^m \tilde{\alpha}_{ij}\int_{-\infty}^t N_{ij}(t-s)g_j(v_j(s,x))ds\Big]dx\\ \le\ &\sum_{j=1}^m |\tilde{\alpha}_{ij}|G_j\|y_i(t,x)\|_2\int_{-\infty}^t N_{ij}(t-s)\|z_j(s,x)\|_2 ds.\end{aligned} \tag{19}$$

Substituting inequalities (16)-(19) into (15), we can obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|y_i(t,x)\|_2^2\\ \le\ &-\big(a_i+\sum_{r=1}^l\frac{D_{ir}}{l_r^2}\big)\|y_i(t,x)\|_2^2\\&+\sum_{j=1}^m |a_{ij}|G_j\|y_i(t,x)\|_2\|z_j(t,x)\|_2\\&+\sum_{j=1}^m(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j\|y_i(t,x)\|_2\\&\times\int_{-\infty}^t N_{ij}(t-s)\|z_j(s,x)\|_2 ds,\end{aligned}$$

i.e.

$$\begin{aligned}&D^+\|y_i(t,x)\|_2\\ \le\ &-\big(a_i+\sum_{r=1}^l\frac{D_{ir}}{l_r^2}\big)\|y_i(t,x)\|_2\\&+\sum_{j=1}^m |a_{ij}|G_j\|z_j(t,x)\|_2\\&+\sum_{j=1}^m(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j\\&\times\int_{-\infty}^t N_{ij}(t-s)\|z_j(s,x)\|_2 ds,\quad i\in\mathscr{I}.\end{aligned} \tag{20}$$

By the same consequence, multiply both sides of (14) by $z_j(t,x)$ and integrate it, we can obtain

$$\begin{aligned}&D^+\|z_j(t,x)\|_2\\ \le\ &-\big(b_j+\sum_{r=1}^l\frac{D_{jr}}{l_r^2}\big)\|z_j(t,x)\|_2\\&+\sum_{i=1}^n |b_{ji}|F_i\|y_i(t,x)\|_2\\&+\sum_{i=1}^n(|\beta_{ji}|+|\tilde{\beta}_{ji}|)F_i\\&\times\int_{-\infty}^t K_{ji}(t-s)\|y_i(s,x)\|_2 ds,\quad j\in\mathscr{J}.\end{aligned} \tag{21}$$

Let $\tilde{u}_i(t)=\|y_i(t,x)\|_2$, $\tilde{v}_j(t)=\|z_j(t,x)\|_2$, $i\in\mathscr{I}$, $j\in\mathscr{J}$, and $r_i=a_i+\sum\limits_{r=1}^l\frac{D_{ir}}{l_r^2}$, $p_{ij}=|a_{ij}|G_j$, $q_{ij}=(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)G_j$, $\bar{r}_j=b_j+\sum\limits_{r=1}^l\frac{D_{jr}}{l_r^2}$, $\bar{p}_{ji}=|b_{ji}|F_i$, $\bar{q}_{ji}=(|\beta_{ji}|+|\tilde{\beta}_{ji}|)F_i$ for $i\in\mathscr{I}$, $j\in\mathscr{J}$, where $r_i>0$, $p_{ij}>0$, $q_{ij}>0$, $\bar{r}_j>0$, $\bar{p}_{ji}>0$, $\bar{q}_{ji}>0$, $i\in\mathscr{I}$, $j\in\mathscr{J}$.
from (20), (21), we can obtain

$$\begin{cases} D^+\tilde{u}_i(t)\le -r_i\tilde{u}_i(t)+\sum\limits_{j=1}^m p_{ij}\tilde{v}_j(t)\\ \qquad +\sum\limits_{j=1}^m q_{ij}\int_0^{+\infty}N_{ij}(s)\tilde{v}_j(t-s)ds,\quad i\in\mathscr{I},\\ D^+\tilde{v}_j(t)\le -\bar{r}_j\tilde{v}_j(t)+\sum\limits_{i=1}^n \bar{p}_{ji}\tilde{u}_i(t)\\ \qquad +\sum\limits_{i=1}^n \bar{q}_{ji}\int_0^{+\infty}K_{ji}(s)\tilde{u}_i(t-s)ds,\quad j\in\mathscr{J}.\end{cases} \tag{22}$$

From condition (C1), there exist $\lambda>0$ and $\xi=(\xi_1,\xi_2,\cdots,\xi_n)^T>0,\eta=(\eta_1,\eta_2,\cdots,\eta_m)^T>0$ such that

$$\begin{cases}(\lambda-r_i)\xi_i+\sum\limits_{j=1}^m(p_{ij}+q_{ij}\int_0^{+\infty}N_{ij}(s)e^{\lambda s}ds)\eta_j<0,\\ (\lambda-\bar{r}_j)\eta_j+\sum\limits_{i=1}^n(\bar{p}_{ji}+\bar{q}_{ji}\int_0^{+\infty}K_{ji}(s)e^{\lambda s}ds)\xi_i<0\end{cases} \tag{23}$$

for $i\in\mathscr{I}$, $j\in\mathscr{J}$.

Taking $\kappa=\frac{\|\varphi\|_2}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}=\frac{\|\phi_y\|_2+\|\phi_z\|_2}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}=\frac{\|\phi_u-\phi_{\bar{u}}\|_2+\|\phi_v-\phi_{\bar{v}}\|_2}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}$, it is easy to prove that

$$\begin{cases}\tilde{u}(s)\le\kappa\xi e^{-\lambda s}, & s\in[-\infty,0],\\ \tilde{v}(s)\le\kappa\eta e^{-\lambda s}, & s\in[-\infty,0].\end{cases} \tag{24}$$

Combining (22)-(24) and Lemma 3, we have

$$\begin{cases}\tilde{u}(t)\le\kappa\xi e^{-\lambda t}, & t\ge 0,\\ \tilde{v}(t)\le\kappa\eta e^{-\lambda t}, & t\ge 0.\end{cases} \tag{25}$$

Let $M=\frac{\sum\limits_{i=1}^n\xi_i+\sum\limits_{j=1}^m\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}$, we can obtain

$$\begin{aligned}&\sum_{i=1}^n\|y_i(t,x)\|_2+\sum_{j=1}^m\|z_j(t,x)\|_2\\ \le\ &M\Big(\sup_{-\infty<s\le 0}\sum_{i=1}^n\|y_i(s,x)\|_2\\&+\sup_{-\infty<s\le 0}\sum_{j=1}^m\|z_j(s,x)\|_2\Big)e^{-\lambda t}.\end{aligned} \tag{26}$$

It follows that

$$\begin{aligned}&\|(u(t,x),v(t,x))^T-(\bar{u}(t,x),\bar{v}(t,x))^T\|_2\\ \le\ &Me^{-\lambda t}\|(\phi_u,\phi_v)^T-(\phi_{\bar{u}},\phi_{\bar{v}})^T\|_2\end{aligned} \tag{27}$$

for all $t\ge 0$.

That is

$$\|(u(t,x),v(t,x))^T-(\bar{u}(t,x),\bar{v}(t,x))^T\|_2=O(e^{-\lambda t}). \tag{28}$$

For any fixed $\Delta t>0$, $(u(t-\Delta t,x),v(t-\Delta t,x))^T$ is also a solution of system (4), from (27), one can derive that

$$\begin{aligned}&\Big\|\frac{(u(t,x),v(t,x))^T-(u(t-\Delta t,x),v(t-\Delta t,x))^T}{\Delta t}\Big\|_2\\ =\ &O(e^{-\lambda t}).\end{aligned} \tag{29}$$

This implies that

$$\|(\dot{u}(t,x),\dot{v}(t,x))^T\|_2 = O(e^{-\lambda t}), \tag{30}$$

where $(\dot{u}(t,x),\dot{v}(t,x))^T$ denotes the derivative of $(u(t,x),v(t,x))^T$ for argument t. According to Cauchy convergence principle, we conclude that system (4) has an equilibrium solution

$$(u^*,v^*)^T = (u_1^*,u_2^*,\cdots,u_n^*,v_1^*,v_2^*,\cdots,v_m^*)^T,$$

where $\lim_{t\to\infty} u(t,x) = u^*, \lim_{t\to\infty} v(t,x) = v^*$. That is, system (1) has an equilibrium point.

Step 2: We prove that the equilibrium point of system (1) is globally exponentially stable.

Let $(u^*,v^*)^T$ be an equilibrium point of system (1), $(u(t,x),v(t,x))^T$ is an arbitrary solution of system (1), and set $\tilde{u}_i(t,x) = u_i(t,x) - u_i^*, \tilde{v}_j(t,x) = v_j(t,x) - v_j^*,\ i \in \mathscr{I}, j \in \mathscr{J}$. It is easy to see that system (1) can be transformed into the following system

$$\begin{cases} \frac{\partial \tilde{u}_i(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial \tilde{u}_i(t,x)}{\partial x_r}) - a_i\tilde{u}_i(t,x) \\ \qquad + \sum\limits_{j=1}^{m} a_{ij}\Big(g_j(v_j) - g_j(v_j^*)\Big) \\ \qquad + \bigwedge\limits_{j=1}^{m} \alpha_{ij}\int_{-\infty}^{t} N_{ij}(t-s)g_j(v)ds \\ \qquad - \bigwedge\limits_{j=1}^{m} \alpha_{ij}\int_{-\infty}^{t} N_{ij}(t-s)g_j(v_j^*)ds \\ \qquad + \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij}\int_{-\infty}^{t} N_{ij}(t-s)g_j(v_j)ds \\ \qquad - \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij}\int_{-\infty}^{t} N_{ij}(t-s)g_j(v_j^*)ds, \quad t \neq t_k, \\ \tilde{u}_i(t_k^+,x) = \tilde{P}_{ik}(\tilde{u}_i(t_k^-,x)), \quad k \in N \\ \frac{\partial \tilde{v}_j(t,x)}{\partial t} = \sum\limits_{r=1}^{l} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial \tilde{v}_j(t,x)}{\partial x_r}) - b_j\tilde{v}_j(t,x) \\ \qquad + \sum\limits_{i=1}^{n} b_{ji}\Big(f_i(u_i) - f_i(u_i^*)\Big) \\ \qquad + \bigwedge\limits_{i=1}^{n} \beta_{ji}\int_{-\infty}^{t} K_{ji}(t-s)f_i(u_i)ds \\ \qquad - \bigwedge\limits_{i=1}^{n} \beta_{ji}\int_{-\infty}^{t} K_{ji}(t-s)f_i(u_i^*)ds \\ \qquad + \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji}\int_{-\infty}^{t} K_{ji}(t-s)f_i(u_i)ds \\ \qquad - \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji}\int_{-\infty}^{t} K_{ji}(t-s)f_i(u_i^*)ds, \quad t \neq t_k \\ \tilde{v}_j(t_k^+,x) = \tilde{Q}_{jk}(\tilde{v}_j(t_k^-,x)), \quad k \in N, \end{cases} \tag{30}$$

where $\tilde{P}_{ik}(\tilde{u}_i(t,x)) = \bar{P}_{ik}(\tilde{u}_i(t,x) + u_i^*) - \bar{P}_{ik}(u_i^*)$, $\tilde{Q}_{jk}(\tilde{v}_j(t,x)) = \bar{Q}_{jk}(\tilde{v}_j(t,x) + v_j^*) - \bar{Q}_{jk}(v_j^*)$, and the Dirichlet boundary conditions and initial conditions of (3.18) are respectively

$$\begin{cases} \tilde{u}_i(t,x) = 0, & t \geq 0, \quad x \in \partial\Omega, \quad i \in \mathscr{I}, \\ \tilde{v}_j(t,x) = 0, & t \geq 0, \quad x \in \partial\Omega, \quad j \in \mathscr{J}, \end{cases} \tag{31}$$

and

$$\begin{cases} \tilde{\phi}_u(s,x) = u(s,x) - u^* = \phi_u(s,x) - u^*, & s \in (-\infty,0], \\ \tilde{\phi}_v(s,x) = v(s,x) - v^* = \phi_v(s,x) - v^*, & s \in (-\infty,0]. \end{cases} \tag{32}$$

Multiply both sides of first equation and third equation in (30) by $\tilde{u}_i(t,x)$ and $\tilde{v}_j(t,x)$, respectively, and then integrate them. By a minor modification of the proof of **Step 1**, we can easily derive that

$$\begin{cases} D^+\tilde{u}_i(t) \leq -r_i\tilde{u}_i(t) + \sum\limits_{j=1}^{m} p_{ij}\tilde{v}_j(t) \\ \qquad + \sum\limits_{j=1}^{m} q_{ij}\int_0^{+\infty} N_{ij}(s)\tilde{v}_j(t-s)ds, \quad i \in \mathscr{I}, \\ D^+\tilde{v}_j(t) \leq -\bar{r}_j\tilde{v}_j(t) + \sum\limits_{i=1}^{n} \bar{p}_{ji}\tilde{u}_i(t) \\ \qquad + \sum\limits_{i=1}^{n} \bar{q}_{ji}\int_0^{+\infty} K_{ji}(s)\tilde{u}_i(t-s)ds, \quad j \in \mathscr{J}, \end{cases} \tag{33}$$

where parameters $r_i, \bar{r}_j, p_{ij}, \bar{p}_{ji}, q_{ij}, \bar{q}_{ji}\,(i \in \mathscr{I}, j \in \mathscr{J})$ are the same as (22). Uniformity, from condition (C1), there exist $\lambda > 0$ and $\xi = (\xi_1,\xi_2,\cdots,\xi_n)^T > 0, \eta = (\eta_1,\eta_2,\cdots,\eta_m)^T > 0$ such that

$$\begin{cases} (\lambda - r_i)\xi_i + \sum\limits_{j=1}^{m}(p_{ij} + q_{ij}\int_0^{+\infty} N_{ij}(s)e^{\lambda s}ds)\eta_j < 0, \\ (\lambda - \bar{r}_j)\eta_j + \sum\limits_{i=1}^{n}(\bar{p}_{ji} + \bar{q}_{ji}\int_0^{+\infty} K_{ji}(s)e^{\lambda s}ds))\xi_i < 0 \end{cases} \tag{34}$$

for $i \in \mathscr{I}, j \in \mathscr{J}$.

Taking $\kappa == \frac{\|\phi_u - \phi_{u^*}\|_2 + \|\phi_v - \phi_{v^*}\|_2}{\min\limits_{1\leq i \leq n, 1 \leq j \leq m}\{\xi_i,\eta_j\}}$, it is easy to prove that

$$\begin{cases} \|\tilde{u}(t)\|_2 \leq \kappa\xi e^{-\lambda t}, & -\infty < t \leq 0 = t_0, \\ \|\tilde{v}(t)\|_2 \leq \kappa\eta e^{-\lambda t}, & -\infty < t \leq 0 = t_0. \end{cases}$$

From Lemma 3, we obtain

$$\begin{cases} \|\tilde{u}(t)\|_2 \leq \kappa\xi e^{-\lambda t}, & t_0 \leq t < t_1, \\ \|\tilde{v}(t)\|_2 \leq \kappa\eta e^{-\lambda t}, & t_0 \leq t < t_1. \end{cases} \tag{35}$$

Suppose that for $l \leq k$, the inequalities

$$\begin{cases} \|\tilde{u}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_{l-1}\xi e^{-\lambda t}, & t_{l-1} \leq t < t_l, \\ \|\tilde{v}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_{l-1}\eta e^{-\lambda t}, & t_{l-1} \leq t < t_l. \end{cases} \tag{36}$$

hold, where $\mu_0 = 1$. When $l = k+1$, we note that

$$\begin{aligned} \|\tilde{u}(t_k)\|_2 &= \|\tilde{u}(t_k^+,x)\|_2 = \|\tilde{P}_k(\tilde{u}(t_k^-,x))\|_2 \\ &\leq \Gamma_k\|\tilde{u}(t_k^-,x)\|_2 \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\Gamma_k\xi \lim_{t\to t_k^-} e^{-\lambda t} \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t_k}, \end{aligned} \tag{37}$$

and

$$\begin{aligned} \|\tilde{v}(t_k)\|_2 &= \|\tilde{v}(t_k^+,x)\|_2 = \|\tilde{Q}_k(\tilde{v}(t_k^-,x))\|_2 \\ &\leq \bar{\Gamma}_k\|\tilde{v}(t_k^-,x)\|_2 \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\bar{\Gamma}_k\eta \lim_{t\to t_k^-} e^{-\lambda t} \\ &\leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t_k}. \end{aligned} \tag{38}$$

From (37), (38) and $\mu_k \geq 1$, we have

$$\begin{cases} \|\tilde{u}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t}, & -\infty \leq t \leq t_k, \\ \|\tilde{v}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t}, & -\infty \leq t \leq t_k. \end{cases} \tag{39}$$

Combining (33),(34),(39) and Lemma 3, we obtain that

$$\begin{cases} \|\tilde{u}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, & t_k \leq t < t_{k+1}, \\ \|\tilde{v}(t)\|_2 \leq \kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, & t_k \leq t < t_{k+1}. \end{cases} \tag{40}$$

Applying the mathematical induction, we can obtain the following inequalities

$$\begin{cases} \|\tilde{u}(t)\|_2 \le \kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, & t\in[t_k,t_{k+1}),\ k\in N,\\ \|\tilde{v}(t)\|_2 \le \kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, & t\in[t_k,t_{k+1}),\ k\in N. \end{cases} \tag{41}$$

According to (C2), we have $\mu_k \le e^{\mu(t_k-t_{k-1})} < e^{\lambda(t_k-t_{k-1})}$, so we have

$$\begin{aligned} &\|\tilde{u}(t)\|_2\\ \le\ & \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\xi e^{-\lambda t}\\ =\ & \kappa\xi e^{\mu t_{k-1}}e^{-\lambda t}\le \kappa\xi e^{-(\lambda-\mu)t},\ \ t\in[t_{k-1},t_k),\ \ k\in N, \end{aligned}$$

and

$$\begin{aligned} &\|\tilde{v}(t)\|_2\\ \le\ & \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\eta e^{-\lambda t}\\ =\ & \kappa\eta e^{\mu t_{k-1}}e^{-\lambda t}\le \kappa\eta e^{-(\lambda-\mu)t},\ \ t\in[t_{k-1},t_k),\ \ k\in N. \end{aligned}$$

That is

$$\begin{cases} \|\tilde{u}(t)\|_2 \le \kappa\xi e^{-(\lambda-\mu)t}, & t\in(-\infty,t_k), \quad k\in N,\\ \|\tilde{v}(t)\|_2 \le \kappa\eta e^{-(\lambda-\mu)t}, & t\in(-\infty,t_k), \quad k\in N. \end{cases} \tag{42}$$

It follows that

$$\begin{aligned} &\sum_{i=1}^n\|u_i(t,x)-u_i^*\|_2+\sum_{j=1}^m\|v_j(t,x)-v_j^*\|_2\\ =\ & \sum_{i=1}^n\|\tilde{u}_i(t)\|_2+\sum_{j=1}^m\|\tilde{v}_j(t)\|_2\\ \le\ & \sum_{i=1}^n\kappa\xi_i e^{-(\lambda-\mu)t}+\sum_{j=1}^m\kappa\eta_j e^{-(\lambda-\mu)t}\\ =\ & \frac{\sum_{i=1}^n\xi_i+\sum_{j=1}^m\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}(\|\tilde{\phi}_u\|_2+\|\tilde{\phi}_v\|_2)e^{-(\lambda-\mu)t}\\ =\ & \frac{\sum_{i=1}^n\xi_i+\sum_{j=1}^m\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}\\ & \times\Big(\|\phi_u-u^*\|_2+\|\phi_v-v^*\|_2\Big)e^{-(\lambda-\mu)t}. \end{aligned}$$

Let $M=\frac{\sum_{i=1}^n\xi_i+\sum_{j=1}^m\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}$, then we have

$$\begin{aligned} &\sum_{i=1}^n\|u_i(t,x)-u_i^*\|_2+\sum_{j=1}^m\|v_j(t,x)-v_j^*\|_2\\ \le\ & M\Big(\|\phi_u-u^*\|_2+\|\phi_v-v^*\|_2\Big)e^{-(\lambda-\mu)t}. \end{aligned}$$

The proof is completed.

Remark 1 From the conditions of Theorem 1, one can know that the diffusion coefficient, the the Dirichlet boundary conditions, the delay kernel functions and system parameters have key effect on the stability of system (1). This theorem also shows that the diffusion phenomenon can conduce to stabilization of neural system.

Remark 2 In order to obtain more precise estimate of the exponential convergence rate of system (1) (or system (4)), it is necessary to solve the following optimization problem

$$(\mathrm{OP})\quad \begin{cases} \max\lambda,\\ s.t.\,(\mathrm{C1})\,holds. \end{cases}$$

When $D_{ir}=0$, $\bar{D}_{jr}=0$ or $D_{ir}\ll l_r$, $\bar{D}_{jr}\ll l_r\,(r=1,2,\cdots,l)$, that is, the diffusion effect can be ignored, we have the following corollary.

Corollary 1: Under assumptions (H1), (H2) and (H3), if the following conditions hold:

(C1′) there exist constant $\lambda>0$ and vectors $\xi=(\xi_1,\xi_2,\cdots,\xi_n)^T>0,\eta=(\eta_1,\eta_2,\cdots,\eta_m)^T>0$ such that

$$\begin{cases} (\lambda-a_i)\xi_i+\sum\limits_{j=1}^m\Big[|a_{ij}|+(|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)\\ \quad\times\int_0^{+\infty}e^{\lambda s}N_{ij}(s)ds\Big]G_j\eta_j<0,\quad i\in\mathscr{I}\\ (\lambda-b_j)\eta_j+\sum\limits_{i=1}^n\Big[|b_{ji}|+(|\beta_{ji}|+|\tilde{\beta}_{ji}|)\\ \quad\times\int_0^{+\infty}e^{\lambda s}K_{ji}(s)ds\Big]F_i\xi_i<0,\quad j\in\mathscr{J}; \end{cases}$$

(C2) $\mu=\sup\limits_{k\in N}\left\{\frac{\ln\mu_k}{t_k-t_{k-1}}\right\}<\lambda$, where $\mu_k=\max\limits_{1\le i\le n,1\le j\le m}\{1,\gamma_{ik},\bar{\gamma}_{jk}\}$, $k\in N$;

then system (1) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\lambda-\mu$.

Remark 3 Note that Lemma 2 transforms the fuzzy AND ($\bigwedge$) and the fuzzy OR ($\bigvee$) operation into the SUM operation ($\sum$). So above results can be applied to the following classical impulsive BAM neural networks with distributed delays and reaction-diffusion terms

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t}=\sum\limits_{r=1}^l\frac{\partial}{\partial x_r}(D_{ir}\frac{\partial u_i(t,x)}{\partial x_r})-a_iu_i(t,x)\\ \qquad+\sum\limits_{j=1}^m a_{ij}g_j(v_j(t,x))\\ \qquad+\sum\limits_{j=1}^m\alpha_{ij}\int_{-\infty}^t N_{ij}(t-s)g_j(v_j(s,x))ds\\ \qquad+I_i,\quad t\ge0,\quad t\ne t_k,\quad i\in\mathscr{I},\\ u_i(t^+,x)=u_i(t^-,x)+P_{ik}(u_i(t^-,x)),\\ \qquad t=t_k,\,k\in N,\,i\in\mathscr{I},\\ \frac{\partial v_j(t,x)}{\partial t}=\sum\limits_{r=1}^l\frac{\partial}{\partial x_r}(\tilde{D}_{jr}\frac{\partial v_j(t,x)}{\partial x_r})-b_jv_j(t,x)\\ \qquad+\sum\limits_{i=1}^n b_{ji}f_i(u_i(t,x))\\ \qquad+\sum\limits_{i=1}^n\beta_{ji}\int_{-\infty}^t K_{ji}(t-s)f_i(u_i(s,x))ds\\ \qquad+J_j,\quad t\ge0,\quad t\ne t_k,\quad j\in\mathscr{J},\\ v_j(t^+,x)=v_j(t^-,x)+Q_{ik}(v_j(t^-,x)),\\ \qquad t=t_k,\,k\in N,\,j\in\mathscr{J} \end{cases} \tag{43}$$

and its corresponding continuous system

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum_{r=1}^{l} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial u_i(t,x)}{\partial x_r}) - a_i u_i(t,x) \\ \quad + \sum_{j=1}^{m} a_{ij} g_j(v_j(t,x)) \\ \quad + \sum_{j=1}^{m} \alpha_{ij} \int_{\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \quad + I_i, \quad t \geq 0, \quad i \in \mathscr{I}, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum_{r=1}^{l} \frac{\partial}{\partial x_r}(\tilde{D}_{jr}\frac{\partial v_j(t,x)}{\partial x_r}) - b_j v_j(t,x) \\ \quad + \sum_{i=1}^{n} b_{ji} f_i(u_i(t,x)) \\ \quad + \sum_{i=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \quad + J_j, \quad t \geq 0, \quad j \in \mathscr{J}. \end{cases} \tag{44}$$

For system (43) and system (44), it is easy to obtain the following results.

Theorem 2: Under assumptions (H1) and (H2), if the following conditions hold:

($\bar{C}1$) there exist vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$, $\eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ and positive number $\lambda > 0$ such that

$$\begin{cases} \Big(\lambda - a_i - \sum_{r=1}^{l} \frac{D_{ir}}{l_r^2}\Big)\xi_i + \sum_{j=1}^{m} \Big[|a_{ij}| + |\alpha_{ij}| \\ \quad \times \int_0^{+\infty} e^{\lambda s} N_{ij}(s) ds\Big] G_j \eta_j < 0, \quad i \in \mathscr{I} \\ \Big(\lambda - b_j - \sum_{r=1}^{l} \frac{\bar{D}_{jr}}{l_r^2}\Big)\eta_j + \sum_{i=1}^{n} \Big[|b_{ji}| + |\beta_{ji}| \\ \quad \times \int_0^{+\infty} e^{\lambda s} K_{ji}(s) ds\Big] F_i \xi_i < 0, \quad j \in \mathscr{J}; \end{cases}$$

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \leq i \leq n, 1 \leq j \leq m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}$, $k \in N$;

then system (43) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\lambda - \mu$.

Corollary 2: Under assumption (H1) and (H2) , system (44) has exactly one globally exponentially stable equilibrium point if condition $\bar{\text{C}}1$ holds.

Remark 4 Although those models in [6], [8], [12], [13], [20] are different from system (44) because of the difference of diffusion terms and boundary conditions, but all criteria in [6], [8], [12], [13], [20] are independent of diffusion effect. Comparing with some corresponding criteria in [6], [8], [12], [13], [20], Corollary 2 is less conservative.

IV. An illustrative example

In order to illustrate the feasibility of our above-established criteria in the preceding sections, we provide a concrete example. Although the selection of the coefficients and functions in the example is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Consider the following impulsive BAM fuzzy neural networks with distributed delays and reaction-diffusion terms:

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum_{r=1}^{3} \frac{\partial}{\partial x_r}(D_{ir}\frac{\partial u_i(t,x)}{\partial x_r}) - a_i u_i(t,x) \\ \quad + \sum_{j=1}^{2} a_{ij} g_j(v_j(t,x)) + \sum_{j=1}^{2} \tilde{a}_{ij} w_j + I_i \\ \quad + \bigwedge_{j=1}^{2} \alpha_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \quad + \bigvee_{j=1}^{2} \tilde{\alpha}_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_j(v_j(s,x)) ds \\ \quad + \bigwedge_{j=1}^{2} T_{ij} w_j + \bigvee_{j=1}^{2} H_{ij} w_j, \quad t \geq 0, \quad t \neq t_k, \\ u_i(t,x) = u_i(t^-,x) + (-1 + e \sin t_k) u_i(t^-,x), \\ \qquad t = t_k,\ t_0 = 0,\ t_k - t_{k-1} = 10,\ k \in N, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum_{r=1}^{3} \frac{\partial}{\partial x_r}(\tilde{D}_{jr}\frac{\partial v_j(t,x)}{\partial x_r}) - b_j v_j(t,x) \\ \quad + \sum_{i=1}^{2} b_{ji} f_i(u_i(t,x)) + \sum_{i=1}^{2} \tilde{b}_{ji} \tilde{w}_i + J_j \\ \quad + \bigwedge_{i=1}^{2} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \quad + \bigvee_{i=1}^{2} \tilde{\beta}_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_i(u_i(s,x)) ds \\ \quad + \bigwedge_{i=1}^{2} \tilde{T}_{ji} \tilde{w}_i + \bigvee_{i=1}^{2} \tilde{H}_{ji} \tilde{w}_i, \quad t \geq 0, \quad t \neq t_k \\ v_j(t,x) = v_j(t^-,x) + (-1 + e \cos t_k) v_j(t^-,x), \\ \qquad t = t_k,\ t_0 = 0,\ t_k - t_{k-1} = 10,\ k \in N \end{cases} \tag{45}$$

for $i = 1, 2$, $j = 1, 2$, where $\Omega = \{(x_1, x_2, x_3)^T \,|\, |x_r| < 1, r = 1, 2, 3\}$, and

$D_{11} = 0.01$, $D_{12} = 0.03$, $D_{13} = 0.01$, $D_{21} = 0.01$, $D_{22} = 0.02$, $D_{23} = 0.02$,
$a_1 = 2.95$, $a_2 = 2.95$, $a_{11} = 1.5$, $a_{12} = -0.5$, $a_{21} = -0.5$, $a_{22} = 0.75$,
$\tilde{a}_{11} = 0.45$, $\tilde{a}_{12} = -0.45$, $\tilde{a}_{21} = -0.36$, $\tilde{a}_{22} = 0.36$, $\alpha_{11} = 0.25$, $\alpha_{12} = -0.25$,
$\alpha_{21} = -0.125$, $\alpha_{22} = 0.75$, $\tilde{\alpha}_{11} = 0.25$, $\tilde{\alpha}_{12} = 0.25$, $\tilde{\alpha}_{21} = 0.375$, $\tilde{\alpha}_{22} = 0.5$,
$T_{11} = 0.5$, $T_{12} = 0.5$, $T_{21} = 0.5$, $T_{22} = 0.5$, $H_{11} = 0.25$, $H_{12} = 0.25$,
$H_{21} = 0.25$, $H_{22} = 0.25$, $I_1 = -0.75$, $I_2 = -0.75$, $w_1 = 1$, $w_2 = 1$,
$\bar{D}_{11} = 0.01$, $\bar{D}_{12} = 0.03$, $\bar{D}_{13} = 0.01$, $\bar{D}_{21} = 0.01$, $\bar{D}_{22} = 0.02$, $\bar{D}_{13} = 0.02$,
$b_1 = 2.95$, $b_2 = 2.95$, $b_{11} = 0.75$, $b_{12} = -0.5$, $b_{21} = 1.25$, $b_{22} = 0.55$,
$\tilde{b}_{11} = -0.45$, $\tilde{b}_{12} = -0.35$, $\tilde{b}_{21} = -0.35$, $\tilde{b}_{22} = 0.25$, $\beta_{11} = 0.125$, $\beta_{12} = 0.25$,
$\beta_{21} = -0.45$, $\beta_{22} = 0.225$, $\tilde{\beta}_{11} = 0.125$, $\tilde{\beta}_{12} = -0.25$, $\tilde{\beta}_{21} = 0.3$, $\tilde{\beta}_{22} = 0.225$,
$\tilde{T}_{11} = 0.25$, $\tilde{T}_{12} = 0.25$, $\tilde{T}_{21} = 0.25$, $\tilde{T}_{22} = 0.25$, $\tilde{H}_{11} = 0.5$, $\tilde{H}_{12} = 0.5$,
$\tilde{H}_{21} = 0.5$, $\tilde{H}_{22} = 0.5$, $J_1 = 0.05$, $J_2 = -0.65$, $\tilde{w}_1 = 1$, $\tilde{w}_2 = 1$,
$N_{ij}(s) = e^{-s}$, $K_{ji}(s) = 2e^{-2s}$;
$f_1(u) = f_2(u) = \tanh(u)$, $g_1(v) = g_2(v) = \frac{|v+1| - |v-1|}{2}$.

It is easy to verify that assumptions (H1) and (H2) are satisfied, and it is easy to calculate that

$$F = G = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \quad \Gamma_k = \bar{\Gamma}_k = \begin{pmatrix} e & \\ & e \end{pmatrix}, \quad \mu_k = \max\{1, e\} = e, \quad \mu = \sup_{k\in N} \frac{\ln e}{t_k - t_{k-1}} = 0.1.$$

Solving the following optimization problem

$$\begin{cases} \max \lambda, \\ (\lambda - a_1 - \sum_{r=1}^{3} \frac{D_{1r}}{l_r^2})\xi_1 + \sum_{j=1}^{2} \Big[|a_{1j}| + (|\alpha_{1j}| + |\tilde{\alpha}_{1j}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} N_{ij}(s) ds\Big] G_j \eta_j < 0, \\ (\lambda - a_2 - \sum_{r=1}^{3} \frac{D_{2r}}{l_r^2})\xi_2 + \sum_{j=1}^{2} \Big[|a_{2j}| + (|\alpha_{2j}| + |\tilde{\alpha}_{2j}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} N_{ij}(s) ds\Big] G_j \eta_j < 0, \\ (\lambda - b_1 - \sum_{r=1}^{3} \frac{\bar{D}_{1r}}{l_r^2})\eta_1 + \sum_{i=1}^{2} \Big[|b_{1i}| + (|\beta_{1i}| + |\tilde{\beta}_{1i}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} K_{ji}(s) ds\Big] F_i \xi_i < 0, \\ (\lambda - b_2 - \sum_{r=1}^{3} \frac{\bar{D}_{2r}}{l_r^2})\eta_2 + \sum_{i=1}^{2} \Big[|b_{2i}| + (|\beta_{2i}| + |\tilde{\beta}_{2i}|) \\ \quad \times \int_0^{+\infty} e^{\lambda s} K_{ji}(s) ds\Big] F_i \xi_i < 0, \\ \lambda > 0, \quad \xi = (\xi_1, \xi_2)^T > 0, \quad \eta = (\eta_1, \eta_2)^T > 0, \end{cases}$$

we obtain that $\xi = (1709700, 1232282)^T > 0, \eta = (1130721, 1787826)^T > 0, \lambda \approx 0.164678 > 0.1 = \mu$. From Theorem 1, the equilibrium point of system (45) is globally exponentially stable, and its exponential convergence rate $\lambda - \mu \approx 0.064678$.

V. Conclusions

A class of impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms has been formulated and investigated. The global exponential stability criteria for impulsive BAM fuzzy cellular neural networks with distributed delays and reaction-diffusion terms have been derived. In particular, the more precise estimate of the exponential convergence rate is also provided, which depends on the system parameters, boundary conditions, delays and impulses. An illustrate example is given to show the effectiveness of obtained results. In addition, the sufficient conditions what we obtained are easily verified. This has practical benefits, since easily verifiable conditions for the global exponential stability are important in the design and applications of neural networks.

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