

Designing an Irregular Tensegrity as a Monumental Object

Buntara Sthenly Gan

Abstract—A novel and versatile numerical technique to solve a self-stress equilibrium state is adopted herein as a form-finding procedure for an irregular tensegrity structure. The numerical form-finding scheme of a tensegrity structure uses only the connectivity matrix and prototype tension coefficient vector as the initial guess solution. Any information on the symmetrical geometry or other predefined initial structural conditions is not necessary to get the solution in the form-finding process. An eight-node initial condition example is presented to demonstrate the efficiency and robustness of the proposed method in the form-finding of an irregular tensegrity structure. Based on the conception from the form-finding of an eight-node irregular tensegrity structure, a monumental object is designed by considering the real world situation such as self-weight, wind and earthquake loadings.

Keywords—Tensegrity; Form-finding; Design; Irregular; Self-stress; Force density method.

I. INTRODUCTION

SINCE the invention of tensegrity structures by Snelson, Fuller and Emmerich [1], the ingenious forms, simplicity conception, light weight and deployability of tensegrity structures were brought to a rapid development stage in civil structures [2-3], space structures [4-5], mechanical cells [6-8] and robotics [9]. As a result, analytical methods and numerical techniques based on mathematical principles and theories have been developed [10-11]. However, most of the form-finding methods are still limited to simple and regular tensegrity structures.

Form-finding of a tensegrity structure is a process to decide structural configurations which give self-stress equilibrium states of the tensegrity structure. The form-finding of tensegrity structures is one of the fundamental objectives in the design of any statically indeterminate structures including tensegrity structures. The work of Schek [12] in force density method was establishing an introduction to the form-finding of network tensile structures.

In the form-finding process, some initial conditions or assumptions of structural configurations are usually made on the mathematical and mechanical models such as a twisting angle, a strut to cable length ratio, or a force to length ratio. A vast amount of researches in form-finding of tensegrity

structures have resulted in reliable techniques; however there has been relatively few research on design procedures by using less assumption as the initial conditions.

In this paper, a novel iterative numerical form-finding procedure developed by Estrada et al. [13] is adopted as a tensegrity structure form-finding tool used in the genetic algorithm proposed. The formulation was based on the force density formulation with less design variables such as connectivity information and prototype tension coefficient. Since the iterative procedure will result in one particular structural configuration of tensegrity structure in self-stress equilibrium state.

II. TENSEGRITY STRUCTURE

A tensegrity structure consists of a set of continuous cables in tension and a set of discontinuous struts in compression. Tensegrity structures are usually associated with pin-jointed mechanism which is stabilized by the action of pre-stress. The tensegrity structure can be classified into self-stress and pre-stressed structures. Present paper interest is in the self-stress tensegrity structures, since they can free standing without any support while maintaining their self-stress equilibrium states. Tensegrity structures exist only in a self-stress equilibrium state which requires the calculation of member forces in a particular spatial arrangement. Without application of external forces at nodes, the associated mathematical models and numerical algorithms have to represent nontrivial solutions for the member forces of tensegrity structures which are in equilibrium state in space.

A. Equilibrium Equation of Tensegrity Structure

Fig. 1 is used to illustrate the equations of static equilibrium of a reference node *a* which is connected to nodes *b* and *c* by members *a-b* and *a-c*, respectively.

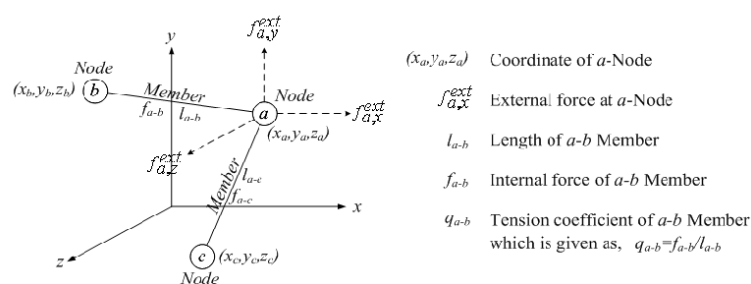


Fig. 1 Equilibrium at a reference node *a*

Buntara Sthenly Gan is with the Architecture Department, College of Engineering, Nihon University, Koriyama, 963-8642, Japan (phone: +81-24-956-8735; fax: +81-24-956-8735; e-mail: buntara@arch.ce.nihon-u.ac.jp)

Neglecting the self weight of each member, the equilibrium equations at the node a in Fig. 1 are given in the three dimensional axis directions by

$$\begin{aligned} (x_a - x_b)f_{a-b}/l_{a-b} + (x_a - x_c)f_{a-c}/l_{a-c} &= f_{a,x}^{ext} \\ (y_a - y_b)f_{a-b}/l_{a-b} + (y_a - y_c)f_{a-c}/l_{a-c} &= f_{a,y}^{ext} \\ (z_a - z_b)f_{a-b}/l_{a-b} + (z_a - z_c)f_{a-c}/l_{a-c} &= f_{a,z}^{ext} \end{aligned} \quad (1)$$

The so called tension coefficient [14], or force density coefficients [12] is often used to simplify the equilibrium equations which can be written as

$$\begin{aligned} (x_a - x_b)q_{a-b} + (x_a - x_c)q_{a-c} &= f_{a,x}^{ext} \\ (y_a - y_b)q_{a-b} + (y_a - y_c)q_{a-c} &= f_{a,y}^{ext} \\ (z_a - z_b)q_{a-b} + (z_a - z_c)q_{a-c} &= f_{a,z}^{ext} \end{aligned} \quad (2)$$

or as,

$$\begin{aligned} (q_{a-b} + q_{a-c})x_a - q_{a-b}x_b - q_{a-c}x_c &= f_{a,x}^{ext} \\ (q_{a-b} + q_{a-c})y_a - q_{a-b}y_b - q_{a-c}y_c &= f_{a,y}^{ext} \\ (q_{a-b} + q_{a-c})z_a - q_{a-b}z_b - q_{a-c}z_c &= f_{a,z}^{ext} \end{aligned} \quad (3)$$

where all the notations in Eqs. (1-3) are given in Fig. 1.

B. Connectivity Matrix of Tensegrity Structure

A tensegrity structure in a self-stressed means that there is no external load applied. Therefore, the equilibrium equations in (2) can be written as

$$A \mathbf{t} = \begin{pmatrix} C^T \text{diag}(\mathbf{C}\mathbf{x}) \\ C^T \text{diag}(\mathbf{C}\mathbf{y}) \\ C^T \text{diag}(\mathbf{C}\mathbf{z}) \end{pmatrix} \mathbf{t} = \mathbf{0}, \quad (4)$$

Equilibrium Matrix

where A is the equilibrium matrix, $\text{diag}(\ast)$ is a square matrix with the vector (\ast) filling diagonal of the matrix, $\mathbf{t} = \{q_1, q_2, \dots, q_{nb}\}^T$ is the tension coefficient vector and C is the connectivity matrix of $nb \times nn$ matrix which defined by

$$C_{a-b} = \begin{cases} +1 & \text{if } b \text{ is the starting node of member } a \\ -1 & \text{if } b \text{ is the terminal node of member } a \\ 0 & \text{otherwise} \end{cases}, \quad (5)$$

where nb is the number of members and nn is number of nodes. Eq. (4) relates the projected lengths to the tension coefficients along each axis direction.

The simplest triplex tensegrity structure shown in Fig. 2 is used to illustrate the creation of matrix C in which the structure

consists of twelve members ($nb=12$, nine cables and three struts) and six nodes. The connectivity matrix C for the tensegrity is shown in Fig. 2. The rows of the matrix C specify the connectivity information between two nodes connecting a member; hence the columns of the matrix C indicate the sequence of nodal number information.

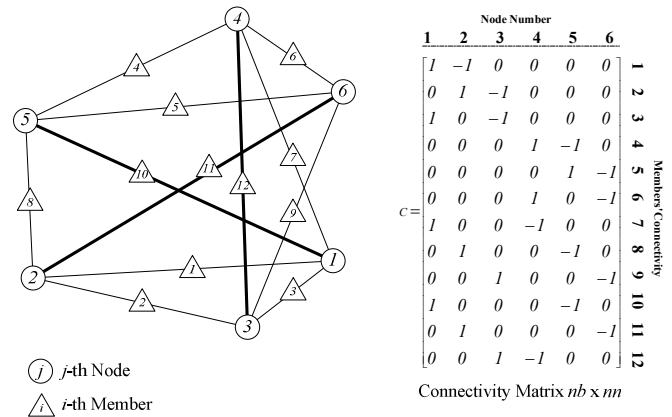


Fig. 2 Connectivity matrix of a triplex tensegrity example

The matrix representation of Eq. (3) relates a symmetric matrix D , known as the *force density matrix* (FDM) and the nodal coordinates as

$$D [\mathbf{x} \ \mathbf{y} \ \mathbf{z}] = \underbrace{(C^T \text{diag}(\mathbf{t})C)}_{FDM} [\mathbf{x} \ \mathbf{y} \ \mathbf{z}] = [\mathbf{0} \ \mathbf{0} \ \mathbf{0}], \quad (6)$$

The matrix components d_{ij} of matrix D can be expressed as

$$d_{ij} = \begin{cases} -q_{a-b} & \text{if } a \neq b, \\ \sum_{a \neq b}^{nb} q_{a-b} & \text{if } a = b, \\ 0 & \text{if nodes } a \text{ and } b \text{ are not connected.} \end{cases}, \quad (7)$$

where q_{a-b} is a tension coefficient between two nodes a and b .

C. Rank conditions of Tensegrity Structure

For a tensegrity structure in a self-stress equilibrium state, two necessary but not sufficient rank conditions have to be satisfied in a d -dimensional space [1]. The deficiency of the rank conditions is to ensure the existence of at least one self-stress equilibrium state, if

$$r = \text{rank}(A) < nb. \quad (8)$$

In [15], Calladine and Pellegrino gave a full description on the first order infinitesimal mechanisms where inextensional mechanisms, m , and of self-stress equilibrium states, s , which are necessary for a nontrivial solution of Eq. (4). This rank deficiency provides the number of independent self-stress equilibrium states as

$$s = nb - r \geq 1, \quad (9)$$

where the total number of infinitesimal mechanisms, m , is given by

$$m = d \times nn - r, \quad (10)$$

for a structure in d -dimensional space with number of node nn [16].

Tensegrity structures are not only statically, but often kinematically, indeterminate structures. Table I shows the classification of pin-jointed structures based on the values of s and m (Refer to Pellegrino [17-18] for a comprehensive description on the classification of pin-jointed structures).

TABLE I
 CLASSIFICATION OF PIN-JOINTED STRUCTURES

Category	s and m values	Type of Structure
I	$s=0;$ $m=0$	Statically determinate and kinematically determinate
II	$s=0;$ $m>0$	Statically determinate and kinematically indeterminate
III	$s>0;$ $m=0$	Statically indeterminate and kinematically determinate
IV	$s>0;$ $m>0$	Statically indeterminate and kinematically indeterminate

The second rank condition is related to the positive semi-definite matrix D in Eq. (6) as follows:

$$\text{rank}(D) = nn - d. \quad (11)$$

The nullity of the tensegrity structure is $(d+1)$ as the largest possible rank condition of matrix D in order to find a tensegrity structure [19-20].

III. NUMERICAL FORM-FINDING OF TENSEGRITY STRUCTURE

The numerically form-finding method of tensegrity presented herein is in a similar procedure proposed by Estrada et al. [13], but a genetically algorithm is used for form-finding of an irregular tensegrity structure instead. A full description on the iterative numerical procedures and formulations of form-finding of a tensegrity structure can be found in the reference. Brief explanations on the procedures which used as a form-finding tool of a tensegrity structure are repeated hereafter for clarification purposes.

A. Form-finding of Tensegrity Using Tension Coefficient Prototype

In contrast to the most existing form-finding procedures [1,24] which require initial assumptions on the length of members, geometry or the symmetry of structure, Estrada et al.

[13] proposed a procedure using a predefined connectivity matrix C and a prototype of tension coefficient vector q^0 for all members' information. To calculate the rank deficiency requirement of a tensegrity structure, the spatial dimension d of the problem is also necessary.

The prototype tension coefficient vector q^0 is assigned with coefficient of $+1$ or -1 to members that are chosen to be in tension or in compression, respectively as

$$q^0 = \left[\underbrace{+1 \ +1 \ +1 \ \dots}_{\text{tension}} \quad \underbrace{-1 \ -1 \ -1 \ \dots}_{\text{compression}} \right]^T \quad (12)$$

Subsequently, the vector q^0 replaces vector t to create the matrix D in Eq. (6) to satisfy the rank condition given in Eq. (11). The procedure guides both matrices D and A to be rank deficient, i.e. a proper rank, by selecting the appropriate eigenvector(s) in each decomposition which lead to the existence of at least one self-stress equilibrium state.

B. Approximation of Coordinates from Tension Coefficients

If the matrix D is positive semi-definite of maximal rank [21], i.e. satisfies the rank condition Eq. (11), a Schur decomposition to the D matrix can be expressed by

$$D = UVU^T, \quad (13)$$

where the first $(d+1)$ columns of the matrix $U = [u_1 \ u_2 \ u_3 \ \dots \ u_{nn}]$, contain the basis of the nodal coordinates; and the diagonal matrix V has $d+1=4$ zero eigenvalues for a three dimensional problem. The matrix U is a unitary matrix which has the basis for the nodal coordinates as columns of the null space vector which solves the homogeneous Eq. (6) (Refer to Meyer [22] for more information on the null spaces and decompositions). The approximated coordinates are then, given by

$$[x \ y \ z] = [u_1 \ \dots \ u_{d+1}]^T. \quad (14)$$

Here, a transformation matrix T is necessary to determine the configuration of the structure from nodal coordinates. An infinite number of geometrically different self-stress equilibrium configurations thus can be found for a single vector q . Since the tension coefficients do not change under affine transformations [23], the eigenvectors of the null space are used directly. To pick up the first three eigenvectors of the null space, the T matrix proposed by Estrada et al. [13] is given as follow:

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad (15)$$

where, only the first three eigenvectors are used for computing the approximation of nodal coordinates.

However, the tensegrity structures generated by a vector of tension coefficients q^0 in Eq. (12), unlikely satisfies Eq. (11) and therefore the structures are not in a self-stress equilibrium state. The static equilibrium can be achieved from the approximation of the column vectors which do not correspond to the zero eigenvalues in Eq. (13) as

$$D[u_x \ u_y \ u_z] \approx [0 \ 0 \ 0]. \quad (16)$$

A full explanation how to handle the non-equilibrium of the Eq. (16) can be found in [13]. At this point of the form-finding procedure, an equilibrium configuration that fulfills Eq. (16) can be approximated.

C. Approximation of Tension Coefficients from Coordinates

By using the approximated nodal coordinates computed from Eq. (14), the equilibrium matrix A in Eq. (4) can be decomposed by using the Singular Value Decomposition [18] as follows:

$$A = \begin{pmatrix} C^T \text{diag}(Cx) \\ C^T \text{diag}(Cy) \\ C^T \text{diag}(Cz) \end{pmatrix} = G Y W^T, \quad (17)$$

where the matrices G and W have the following the null spaces as

$$G = [g_1 \ g_2 \ \dots \ g_r \ | \ m_1 \ m_2 \ \dots \ m_{d \times nn-r}] \quad (18)$$

and

$$W = [w_1 \ w_2 \ \dots \ w_r \ | \ q_1 \ q_2 \ \dots \ q_{nb-r}], \quad (19)$$

where r is the rank of the diagonal matrix A , m is the vectors of infinitesimal mechanisms and the vectors q is the tension coefficients of self-stress at equilibrium states, each of which solves the homogeneous condition of Eq. (4).

However, if the structure is not in a self-stress equilibrium state, the null spaces of A as defined in Eq. (17) do not exist. That is the case when the matrix A is calculated with an approximation of the nodal coordinates. Alternatively, A can be modified to be rank deficient, and apply a matrix operation, that uses $[x \ y \ z]$ to compute an approximation of the tension coefficient vector q .

A self-stress equilibrium state that fulfills Eq. (17) can be approximated as,

$$A q = \begin{pmatrix} C^T \text{diag}(Cx) \\ C^T \text{diag}(Cy) \\ C^T \text{diag}(Cz) \end{pmatrix} q \approx 0, \quad (20)$$

A comprehensive explanation how to handle the

non-equilibrium of the Eq. (20) can be found in [13].

In summary, the form-finding procedure in the [13] iterates Eqs. (13) and (17) until the rank condition of Eq. (11) is satisfied. The tension coefficients and nodal coordinates are updated for the next equilibrium matrix, and so on until $s > 0$. Finally, the tension coefficient vector q that fulfill Eq. (20) and the nodal coordinates $[u_x \ u_y \ u_z]$ that fulfill Eq. (16) are the solutions. Fig. 3 shows the outline of the numerical form-finding procedure of a tensegrity structure.

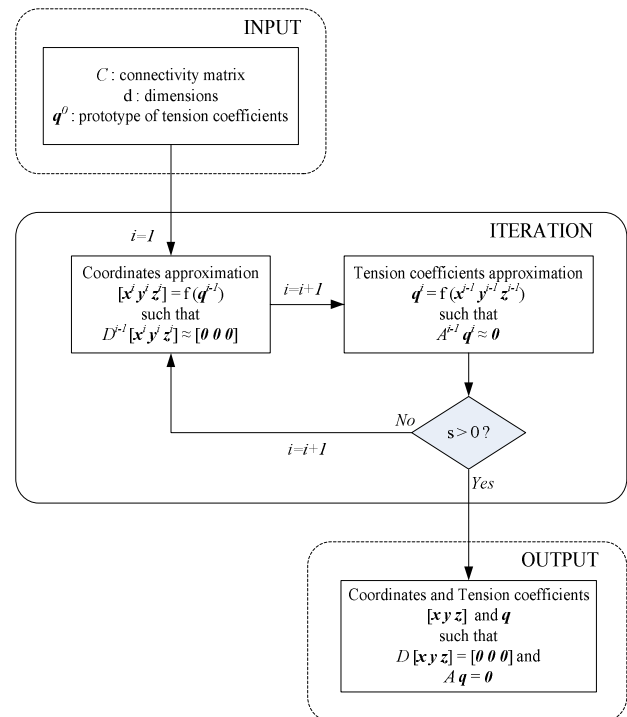


Fig. 3 The outline of the numerical form-finding procedure

IV. DESIGNING AN EIGHT-NODE IRREGULAR TENSEGRITY

The designing of a monumental object herein is started by using the numerical form-finding procedure from a random eight-node model preferred as a design parameter. After the form-finding of a tensegrity structure succeed with a unique configuration of an eight-node irregular tensegrity structure, the model is used as an initial design for a real monumental object by using the finite element analysis technique. Finally, for the safety and stability of the structure, the member strengths and stability checking are performed.

A. Form-Finding of the Eight-Node Irregular Tensegrity

Following the process of numerical form-finding described in previous sections, an eight-node irregular tensegrity was found by using the following connectivity matrix C and tension coefficient vector t as the initial try.

The connectivity matrix shows the one to one relationship between two nodes. As shown in Fig. 4, the initial tension coefficient vector consist of positive value of one, which indicates tension members, and negative different values,

which indicate the variation of compressive members. These negative varying values are intended to design the compressive members with different cross sectional properties.

$$C = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \quad q^0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -8 \\ -3 \\ -15 \\ -7 \end{bmatrix}$$

Fig. 4 The connectivity matrix C and initial tension coefficient vector t

As a result of form-finding process, an eight-node irregular tensegrity structure is found as shown in Fig. 5.

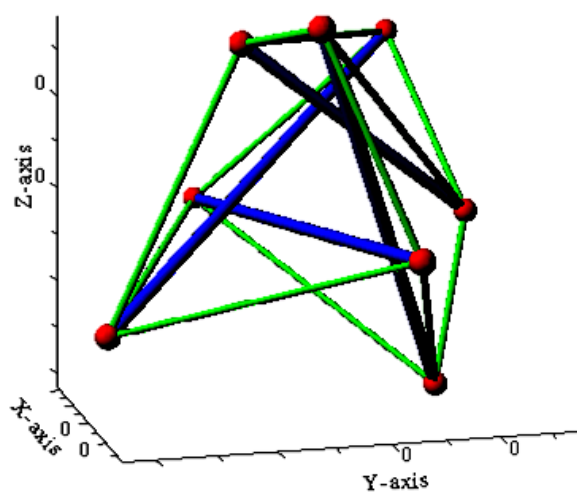


Fig. 5 The result of form-finding of an eight-node irregular tensegrity

The result of form-finding of an irregular tensegrity structure is shown in Tables II and III.

TABLE II
 NODAL COORDINATES

Node Number	x	y	z
1	-0.046997	-0.612765	-0.392801
2	0.100338	-0.267531	0.369638
3	0.462185	-0.155165	0.368403
4	0.502951	0.101945	-0.184040
5	-0.638767	-0.233056	-0.026372
6	-0.334700	0.331275	0.480091
7	-0.029177	0.470376	-0.063288
8	-0.015839	0.364918	-0.551630

TABLE III
 TENSION COEFFICIENTS

Member Connectivity	Tension Coefficient	Member Type
1-2	1.5331	Cable
2-3	3.2162	Cable
3-4	4.9010	Cable
1-4	1.6192	Cable
5-6	2.8351	Cable
6-7	1.8807	Cable
7-8	1.7724	Cable
5-8	1.3201	Cable
1-5	3.4248	Cable
2-6	2.7533	Cable
3-7	1.6502	Cable
4-8	5.0517	Cable
1-6	-3.1641	Strut
2-7	-2.0067	Strut
3-8	-3.7128	Strut
4-5	-3.2504	Strut

Since the initial tension coefficient vector used for the form-finding is not a real member forces, the resulting coordinates are correspondingly small. It can be shown that the results shown are scalable in length dimension and member forces. To designing a real tensegrity structure, the nodal coordinates resulted from form-finding procedure are rotated, translated and scaled up as given in Table IV such that the nodes 1, 2 and 3 which connect the three larger dimension of the pipes are projected to the ground.

TABLE IV
NODAL COORDINATES FOR DESIGN

Node Number	X (mm)	Y (mm)	Z (mm)
1	0.00	0.00	0.00
2	1437.34	-689.33	1688.40
3	2529.21	0.00	0.00
4	2015.31	1808.41	0.00
5	2261.04	-1106.38	1220.02
6	2062.49	902.53	2809.20
7	2517.07	1352.97	1185.58
8	66.22	1142.40	1839.52

B. Members Design of the Eight-Node Irregular Tensegrity

A 4-mm stainless wire cable made from material SUS-304 with 7x7 strands is used for designing the tensile members. Polished hollow stainless pipe made from material SUS-304 with two types of thickness are used for designing the compressive strut members. Table V shows the material properties used for designing the tensegrity.

TABLE V
SECTIONAL PROPERTIES OF MEMBERS

Member	Type	Dimension
2 – 7	Strut	$\phi = 60.5 \text{ mm}$, $t=2.0 \text{ mm}$
4 – 5	Strut	$\phi = 114.3 \text{ mm}$, $t=2.0 \text{ mm}$
1 – 6	Strut	$\phi = 165.2 \text{ mm}$, $t=3.0 \text{ mm}$
3 – 8	Strut	$\phi = 216.3 \text{ mm}$, $t=3.0 \text{ mm}$
Others	Cable	$\phi = 4.0 \text{ mm}$, $\phi \text{ strand} = 0.44 \text{ mm}$

C. Structural Analysis of the Eight-Node Irregular Tensegrity

The tensegrity structure is then, modeled in the finite element analysis scheme to evaluate the member strengths. The tensegrity structure will be put on the ground without any fixed supports thus, very small spring constants in three axis directions are provided in the finite element model in order to avoid any rigid body movement in the analysis. Fig. 6 shows the finite element model used for the eight-node irregular tensegrity with the chosen configuration where the three bigger dimensions of pipes are rested on the ground with the smallest dimension of the pipe is hanging in the air supported by the surrounding cables connected to the others pipes.

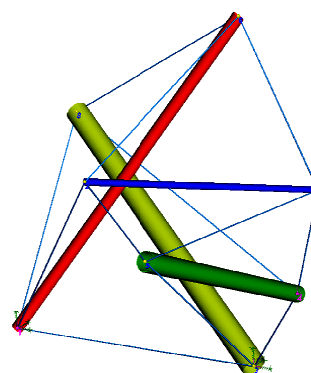


Fig. 6 The finite element model of the eight-node irregular tensegrity

D. Loadings and Member Evaluations

Beside the self weight of the structure, wind loading and static equivalent seismic loadings in 12 directions as illustrated in Fig. 7 are considered to ensure the safety of the structure.

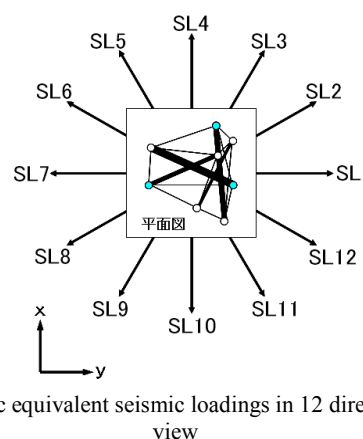


Fig. 7 The static equivalent seismic loadings in 12 directions of plane view

The safeties of the cables are evaluated by using the allowable breaking strength design of the stainless SUS-304 material. The minimum safety factor of 6.0 is achieved for the cable with the maximum tensile strength under various loading conditions. The struts are evaluated against member's elastic buckling strength design of the hollow stainless pipe SUS-304 material. The minimum safety factor beyond 50.0 is assured for the strut with the maximum compressive strength under the same various loading conditions.

V. SUMMARY

The structural behavior of an irregular tensegrity structure is different from the conventional real world structures. The tensegrity structure is designed under the assumption where there is no external loading applied everywhere at the structure. The equilibrium condition of the tensegrity structure is principally provided in a unique configuration where the compressive struts and the tension cables are in self-stress state.

In order to use the tensegrity structure as a monumental object, the results of form-finding configuration must be selected by finding the most stable positioning on the ground. The structure must also properly being designed when subjected to the self-weight, wind and earthquake loadings.

The proposed tensegrity structure is being planned to be constructed inside our campus in front of the library building. Figs. 8-9 show the illustrative image pictures from different views after the construction. This tensegrity which will be built as a monumental object hence it is expected to give an impression of modern look inside our campus.

ACKNOWLEDGEMENT

The author would like to thank the Nihon University, College of Engineering for supporting this work through the Research Grant of College of Engineering, Nihon University, JFY-23, Class 2-I. He is also grateful to Manabu Yamamoto the former graduate student for helping the calculations and design works of the present project.



Fig. 8 Illustration of installation of the Tensegrity as a monumental object



Fig. 9 Illustration of installation of the Tensegrity as a monumental object

REFERENCES

- [1] Motro R. Tensegrity: Structural Systems for the Future. London and Sterling: An imprint of Kogan Page Science; 2003.
- [2] Geiger DH, Stefaniuk A, Chen D. The design and construction of two cable domes for the Korean Olympics, shells, membranes and space frames. Osaka: Proceedings of IASS Symposium; 1986, p. 265-272.
- [3] Geiger DH. Design details of an elliptical cable dome and a large span cable dome (210m) under construction in the United States. Bangalore: Proceedings of the IASS ASCE International Symposium on Innovative Applications of Shells and Spatial Forms; 1988.
- [4] Furuya H. Concept of deployable tensegrity structures in space application. International Journal of Space Structures 1992;7:143-151.
- [5] Hanaor A. Double-layer tensegrity grids as deployable structures. International Journal of Space Structures 1993;8:135-145.
- [6] Ingber DE. Cellular tensegrity: defining new rules of biological design that govern the cytoskeleton. Journal of Cell Science 1993;104:613-627.
- [7] Ingber DE. Tensegrity: the architectural basis of cellular mechanotransduction. Annual Review of Physiology 1997;59:575-599.
- [8] Ingber DE. Tensegrity I. Cell structure and hierarchical systems biology. Journal of Cell Science 2003;116:1157-1173.
- [9] Paul C, Lipson H, Valero-Cuevas F. Design and control of tensegrity robots for locomotion. IEEE Transaction on Robotics 2006;22:944-957.
- [10] Juan SH, Tur JMM. Tensegrity frameworks: static analysis review. Mechanism and Machine Theory 2008;43:859-881.
- [11] Tibert AG, Pellegrino S. Review of form-finding methods for tensegrity structures. International Journal of Space Structures 2003;18(4):209-223.
- [12] Schek HJ. The force density method for form finding and computation of general networks. Computer Methods in Applied Mechanics and Engineering 1974;3:115-134.
- [13] Estrada GG, Bungartz HJ, Morhdieck C. Numerical form-finding of tensegrity structures. International Journal of Solids and Structures 2006;43:6855-6868.
- [14] Southwell RV. Primary stress-determination in space frames. Engineering 1920;CIX:165-8.
- [15] Calladine CR, Pellegrino S. First-order infinitesimal mechanisms. International Journal of Solids and Structures 1991;27(4):505-515.
- [16] Calladine CR. Buckminster Fuller's 'tensegrity' structures and Clerk Maxwell's rules for the construction of stiff frames. International Journal of Solids and Structures 1978;14(2):161-172.
- [17] Pellegrino S. Analysis of prestressed mechanisms. International Journal of Solids and Structures 1990;26:1329-1350.
- [18] Pellegrino S. Structural Computations with the singular value decomposition of the equilibrium matrix. International Journal of Solids and Structures 1993;30(21):3025-3035.
- [19] Graver J, Servatius B, Servatius H. Combinatorial rigidity. Graduate Studies in Mathematics. American Mathematical Society 1993;2.
- [20] Connelly R. Rigidity. In P.M. Gruber and J.M. Wills, editors, Handbook of convex geometry, pages 223-271. Elsevier Science Publishers, 1993.
- [21] Connelly R. Tensegrity structures: why are they stable? Rigidity Theory and Applications. Kluwer Academic/Plenum Publishers; 1999.
- [22] Meyer CD. Matrix Analysis and Applied Linear Algebra. SIAM; 2000.
- [23] Tarnai T. Duality between plane trusses and grillages. International Journal of Solids and Structures 1989;25(12):1395-1409.