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Fatigue Properties and Strength Degradation of Carbon Fibber Reinforced Composites

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***Abstract*—**A two-parameter fatigue model explicitly accounting for the cyclic as well as the mean stress was used to fit static and fatigue data available in literature concerning carbon fiber reinforced composite laminates subjected tension-tension fatigue. The model confirms the strength–life equal rank assumption and predicts reasonably the probability of failure under cyclic loading. The model parameters were found by best fitting procedures and required a minimum of experimental tests.

***Keywords*—**Fatigue life, strength, composites, Weibull distribution.

I. Introduction

The fatigue behavior of composites is very peculiar, resulting in the accumulation of diffuse damage having different origin and location, rather than the propagation of a single crack as in metals. Fiber fracture, matrix cracking, matrix crazing, fiber buckling, fiber-matrix interface failure and delamination are interacting sources of damage accumulation showing different growth rates [1]-[8]. Further the fatigue lifetime of composite materials is influenced primarily from the matrix and fibers type, the geometry of reinforcement (unidirectional, mat, fabric, braiding), the laminate stacking sequence, the environmental conditions (mainly temperature and moisture), the loading history (stress ratio, R, cyclic frequency.) and boundary conditions. A subtle phenomenon influencing fatigue of polymer based-composites is the viscoelastic nature of the matrix that accumulates residual stresses depending on the manufacturing process characteristics. Thermal cycling plays an important role during service lifetime even if this aspect has been rarely attacked systematically. Furthermore it was shown that the structural relaxation of the in situ resin (i.e. the resin constrained into the fibers lattice) may influence its nominal glass transition [9]-[15]. Samples conditioned in the vicinity of Tg showed higher characteristic strength, which resulted in a correspondingly higher fatigue life compared to the as manufactured materials. All the above phenomena would be preferably included in a fatigue damage modeling. However, this task is really difficult and expensive. The first reasons are the several scales where damage mechanisms are present. Secondly, it is impossible reproducing perfectly identical specimens for fatigue as well as static characterization. In turn, the different micro-structural specimens features claims for a statistical approach that requires time-consuming testing procedures.

On the other hand many models have been established for laminates with a particular stacking sequence, under given loading conditions. Thus, all the above considerations explain why the extrapolation to real structures is almost impossible, owing to the fact that the stacking sequence may be a variable and the "in service" loading history is much more complex than that used in a laboratory environment. The complexity of factors influencing fatigue and the usually large scatter in mechanical properties explain why one of the most intricate tasks in designing with composite laminates is the definition of reliable allowable strength. Among the others the residual stresses arising in the polymer matrix that act as non-mechanical loads and influence the strength of the composites [11]-[18]. The difficulty to predict the long-term behavior of carbon fiber reinforced composites comes from the thermodynamic intricacies resulting from exposure to adverse conditions, e.g. aggressive environments, near glass transition, Tg, service temperature or cyclic loadings [19]-[28]. For instance, in the vicinity of the glass transition temperature the structural relaxation alters the viscoelastic behavior of the matrix and affects its long-term behavior [29]-[37].

This complicates the modeling of composites response to fatigue, which is also strongly dependent on external factors such as maximum applied stress, stress ratio, frequency and stress history.

Commonly, the cyclic fatigue data are termed S-N data and are presented as the maximum cyclic stress, σ_{max}, or the normalized maximum stress, σ_{max}/σ_0, (σ_0 being the static material strength) or stress amplitude, $\sigma_{max} - \sigma_{min}$, as a function of number of cycles to failure, N. The fatigue data are obtained at a fixed value of the maximum stress and the stress ratio, $R = \sigma_{min}/\sigma_{max}$, the latter parameter representing the severity of loading conditions. However, it is a matter of fact that, given σ_{max}, higher stress ratios, R, imply higher cycles to failure, thus the phenomenological analytic models appeared in literature assume R as the main variable [5]. The loading frequency is strictly related to temperature problems arising from hysteretic heating. In fact, for most thermoplastic and thermoset systems the temperature rise on the surface of the specimen should be monitored in exploratory tests and should not exceed a few degrees Celsius, which is usually unavoidable. No standard frequency is universally recognized. However, values between one and ten Hertz are typically selected to ensure against hysteretic heating. Further, in [3], [5], [7], [13] the loading frequency was adjusted as function of maximum stress amplitude to give a quasi-constant loading rate.

The phenomenological fatigue life models extract information from the S-N curves and possibly propose a fatigue failure criterion. Thus, they do not take into account damage accumulation, but predict the number of cycles at which fatigue failure occurs under fixed loading conditions.

This paper presents a re-elaboration of static and fatigue data available in literature [38], [39] concerning carbon fiber reinforced laminates. The data were used to obtain the parameters of a recent model [3] accounting for both mean

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stress and stress amplitude.

II. MODELLING

In [3] a two parameter model was proposed for the prediction of the fatigue lifetime of Random Glass Fiber Reinforced Plastics (RGFRP). The starting point of the model is the hypothesis that the material strength undergoes a continuous decrease with fatigue cycle evolution, according to a power law.

Expressing the cyclic and the mean stress as function of the stress ratio R = $\sigma_{min}/\sigma_{max}$, and using the boundary condition n = 1 i.e. $\sigma_{n=}\ \sigma_0$, where σ_0 is the strength of the virgin material, the following relationship was obtained:

$$\sigma_0 - \sigma_n = \sigma_{\max}\left(\frac{\varphi}{2\beta}\right)\left(n^{\beta}-1\right)$$

$$\varphi = \alpha(1-R)-(1-\alpha)(1+R) \tag{1}$$

where σ_n is the residual material strength after *n* cycles, α and *b* are positive constants and $\Delta\sigma = (\sigma_{max} - \sigma_{min})/2$, and $\sigma_{mean} = (\sigma_{max} + \sigma_{min})/2$ are the cyclic and the mean stresses, respectively. α (0<α<1) is the parameter partitioning the fatigue sensitivity to cyclic and mean stress.

According to (1), the evolution of strength degradation with fatigue cycling can be calculated, provided the constants α and β, only dependent on the material and loading conditions, are known. Assuming that the failure will happen when the residual strength equals the maximum applied stress during fatigue, the critical number of cycles for failure, N, can be calculated putting $\sigma_n = \sigma_{max}$ in (1). Solving for N, we obtain:

$$N = \left[\left(\frac{\sigma_0}{\sigma_{\max}}-1\right)\frac{2\beta}{\varphi}+1\right]^{1/\beta}$$

$$\varphi = \alpha(1-R)-(1-\alpha)(1+R) \tag{2}$$

Equation (2) can be rearranged to obtain:

$$\left(N^{\beta}-1\right)\frac{\varphi}{2\beta} = \left(\frac{\sigma_0}{\sigma_{\max}}-1\right)$$

$$\varphi = \alpha(1-R)-(1-\alpha)(1+R) \tag{3}$$

or, equivalently:

$$\frac{\sigma_{\max}}{\sigma_0} = \left[\left(N^{\beta}-1\right)\frac{\varphi}{2\beta}+1\right]^{-1}$$

$$\varphi = \alpha(1-R)-(1-\alpha)(1+R) \tag{4}$$

from which the parameters α and β are obtained by best fitting procedure from fatigue data usually expressed in terms of σ_{max}/σ_0 vs. number of cycles to failure, N, at given stress ratio, R.

Moreover, when (4) is solved for σ_0 one obtains:

$$\sigma_0 = \sigma_{0N} = = \sigma_{\max}\left[\left(N^{\beta}-1\right)\frac{\varphi}{2\beta}+1\right] \tag{5}$$

$$\varphi = \alpha(1-R)-(1-\alpha)(1+R)$$

From a physical viewpoint (5) indicates that the ultimate strength of the virgin material can be calculated from the fatigue life, *N*. The symbol σ_{0N} has been used in (5) just to distinguish the calculated strength from that, σ_0 directly measured in a static characterization test.

However the reliability of our approach has still to be assessed for carbon fiber laminates. To do that, in this paper, the scheme already adopted in references [3], [9] will be followed. The experimental data are taken from reference [38], where both monotonic and fatigue tests were carried out on T300/934 graphite/epoxy laminates. In Reference [38], tension–tension fatigue test were performed with R=1/36 and R=0, using different levels for σ_{max}. The different stress ratios were adopted for laminates having different stacking sequences. In this paper we use the static and the fatigue data obtained at R=1/36.

The static strength data are presented in Fig. 1 (open circle) where the characteristic strength, σ_0=608.6 MPa, and the shape parameter, γ=21.6, of the two-parameter Weibull distribution are evaluated by best fitting procedures.

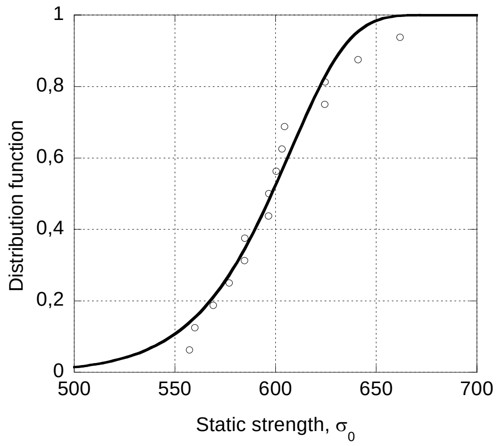

Fig. 1 Statistical distribution of the measured monotonic strength, σ_0, for T300/934 graphite/epoxy laminates. Experimental data taken from reference [38]

The tension–tension fatigue data obtained with R=1/36, are reported in Fig. 2 in a classical σ_{max}/σ_0 vs. the number of cycles to failure, N, together with the best fit curve based on (4). The fitting process allowed obtaining the model parameters, α=0.536, β=0.11.

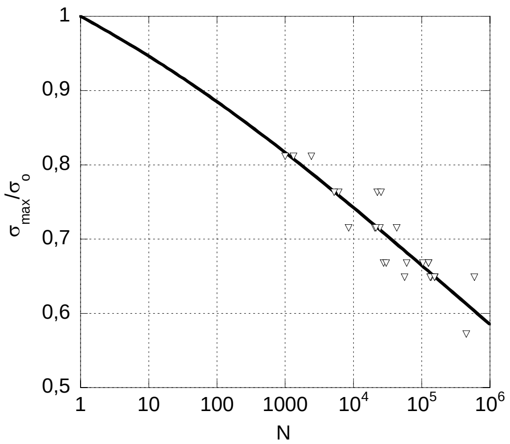

Fig. 2 Re-elaboration of tension-tension fatigue data according to (4) Experimental data taken from reference [38]

A useful way to test the model reliability consists of its potential ability to predict the monotonic strength from fatigue data as described by (5).

It assumed that the scatter in the monotonic material strength, σ_0, is well represented by a two-parameter Weibull distribution. Therefore the probability of finding a σ_0 value≤x is given by:

$$F_{\sigma_0}(x) = P(\sigma_{\max} \le x) = 1 - \exp\left[-\left(x/\gamma\right)^{\delta}\right] \tag{6}$$

where γ is the scale parameter or the characteristic strength, and δ is the shape parameter. Accordingly, in Fig. 3 the static strength and the theoretical static strength, σ_{0N}, coming out from our analytical model expressed by (6) and resulting from fatigue data, are reported.

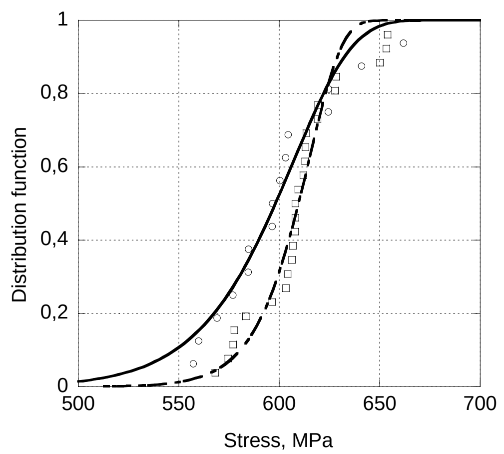

Fig. 3 Statistical distribution of static strength σ_o (circle symbols) and the calculated, according to (6), monotonic strength, σ_{on} (square symbols) , for T300/934 graphite/epoxy laminates. Broken and continuous lines represent the best fitting with a two parameters Weibull distribution for σ_o and σ_{on}, respectively. Experimental data taken from reference [38]

A reasonable agreement between the two statistical distributions can be recognized. The observable discrepancies between the Weibull parameters, (namely, the characteristic strength and the shape parameter) are attributed to the different sampling of experimental static strength data, σ_0, and the calculated monotonic strength, σ_{0N}, spurted from fatigue data according to (5).

It appears that the scatter in fatigue life is substantially due to the variability in monotonic strength according to the strength-life equal rank assumption [10], [11].

If this is true, then the modeling approach could be further implemented statistically. In fact, according to (5) and (6), it can be easily recognized that the relationship allowing for the calculation of the probability F(N*) to find an N lower that N* can be written as follows:

$$F_N(N^*) = 1 - \exp\left\{ \frac{\sigma_{\max}^{\delta}\left[\left(N^{*\beta} - 1\right)\varphi + 1\right]^{\delta}}{\gamma^{\delta}} \right\} \tag{7}$$

$$\varphi = \frac{\alpha(1-R) - (1-\alpha)(1+R)}{2\beta}$$

Where the parametersα,β, γ and δ have been already defined.

This approach was verified by one of us [40] for laminates with different stacking sequence and stress ratio (namely, R=0). Some small discrepancies between the experimental data and the model prediction were observed [40].

The present approach yielded similar response compared to the data elaborated in references [9], [38], with negligible discrepancies. To this end, (5) still remains an empirical expression. However it has the potential to describe both the fatigue life and the scatter in tension-tension fatigue of composite laminates.

III. Conclusion

A fatigue model explicitly accounting for the cyclic as well as the mean stress was tested on the basis of static and fatigue data for a graphite/epoxy laminate. The model confirms the strength–life equal rank assumption and predicts reasonably the probability of failure under cyclic loading. The model has the potential of being statistically implemented assuming a distribution of static strength according to a two-parameter Weibull distribution. However the phenomenological nature of the model limits its applicability when the parameters governing the fatigue life are modified. Of course, the laminate stacking sequence dictate also the evolution of fatigue damage and the strength degradation kinetics, given the loading history, so that generalizing the model to different stacking sequences appears rather difficult if not impossible. We restrict our future work trying to extend the phenomenological approach for uniaxial constant amplitude loading to more general loading condition, such as block type and spectrum loading and to take into account the effect of cyclic frequency and multiaxial loads.

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