Comparative Analysis of Vibration between Laminated Composite Plates with and without Holes under Compressive Loads

Bahi-Eddine Lahouel and Mohamed Guenfoud

***Abstract*—**In this study, a vibration analysis was carried out of symmetric angle-ply laminated composite plates with and without square hole when subjected to compressive loads, numerically. A buckling analysis is also performed to determine the buckling load of laminated plates. For each fibre orientation, the compression load is taken equal to 50% of the corresponding buckling load. In the analysis, finite element method (FEM) was applied to perform parametric studies, the effects of degree of orthotropy and stacking sequence upon the fundamental frequencies and buckling loads are discussed. The results show that the presence of a constant compressive load tends to reduce uniformly the natural frequencies for materials which have a low degree of orthotropy. However, this reduction becomes non-uniform for materials with a higher degree of orthotropy.

***Keywords*—**Vibration, Buckling, Cutout, Laminated composite, FEM.

I. Introduction

LAMINATED composite structures are being increasingly used in aerospace, automotive, marine, and civil areas. This is primarily due to their large values of specific strength and stiffness and the advantage that their properties can be tailored to meet practical requirements. The vibration and buckling load are one of the most important design considerations for laminated composite plates [1]-[3]. The parametric dependence of natural frequencies and buckling loads on the layup configuration and fiber orientation, etc. can be found in some handbooks and even texts, [4]-[6]. The presence of cutouts changes the dynamic characteristics of the plate. Therefore, it becomes necessary to study the effect of cutouts on the vibration response and stability of plates to avoid mechanical failure due to resonance and fatigue due to sustained vibrations.

In the literature, there are a range of published studies on the vibration and buckling of composite plates. Paramasivam [7] proposed a method to determine the effect of square openings on the fundamental frequency of square isotropic plates for different boundary conditions using the finite difference method. Results were obtained for simply supported and clamped boundary conditions. Aksu and Ali [8] developed a theory to study the dynamic characteristics of isotropic and orthotropic rectangular plates with one or two rectangular cutouts. They employed a method based on the use of variational principles in conjunction with finite difference technique. Rajamani and Prabhakaran [9], [10] studied the effect of a centrally located square cutout on the natural frequencies of square, simply supported and clamped symmetrically laminated composite plates for free and forced vibration cases. Lee and Lim [11] analyzed free vibration response of simply supported isotropic and orthotropic square plates with a square cutout subjected to an inplane force using the Rayleigh method. They concluded that the in-plane tensile force increases the natural frequency where as increased compressive force decreases the natural frequency until the state of buckling is reached. Sabir and Davies [12] used finite element method to determine natural frequencies of flat square plates containing an eccentrically located square hole. Plates were subjected to in-plane uniaxial or biaxial compression or uniformly distributed shear along the four outer edges, which were either simply supported or clamped.

For predicting the buckling load of a structure in the finite element program, the linear (or eigenvalue) buckling analysis is an existing technique for estimation. Ghannadpour *et al.* [13] studied the influences of a cutout on the buckling performance of rectangular plates made of polymer matrix composites (PMC). Kong *et al.* [14] analyzed buckling and postbuckling behaviors both numerically and experimentally for composite plates with a hole. In the finite element analysis, the updated Lagrangian formulation and the eight-node degenerated shell element were used. The effect of hole sizes and stacking sequences was examined on the compression behavior of the plate. Experiments showed fine agreement with the finite element results in the buckling load and the postbuckling strength.

The objective of the present investigation is to understand the effect of the presence of a rectangular cutout on the vibration response of a symmetric angle-ply laminated plates when subjected to compressive loads. The study also includes the effect of material orthotropy and stacking sequence on natural frequencies of the laminate.

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II. Methods and Materials

In this study, the effects of square hole on the vibration response and buckling load of laminated composite plates have been investigated numerically. Linear (or eigenvalue) buckling analysis based on stress softening theory determines the bifurcation points of a perfect structure, where two or more load deflection curves intersect. In eigen buckling analysis, eigenvalues and eigenvectors need to be solved. Eigenvalue equation has the form of

$$[K]\{\phi_i\} = \lambda_i [S]\{\phi_i\} \tag{1}$$

where [K] and [S] are the global stiffness and stress stiffness matrices, respectively, ϕ_i and λ_i are the eigenvector and eigenvalue, respectively.

The mode-frequency analysis is formulated as an eigenvalue problem. It has the form of

$$\left([K]-\omega_i^2[M]\right)\{\phi_i\} = 0 \tag{2}$$

where [M] and ω_i are the structure mass matrix and circular natural frequencies, respectively. For pre-stressed modal analyses, the [K] matrix includes the stress stiffness matrix [S]. Equations (1) and (2) are a set of homogeneous linear equations. For non-trivial solution, the determinant is equal to zero and the eigenvalues correspond to natural frequencies and buckling loads of the laminated plate.

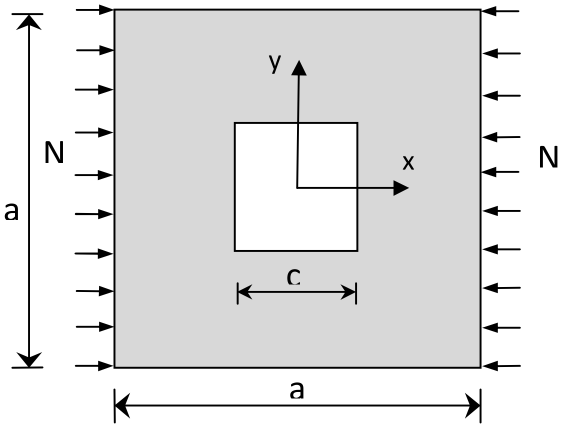

Fig. 1 Geometry and loadings of the model

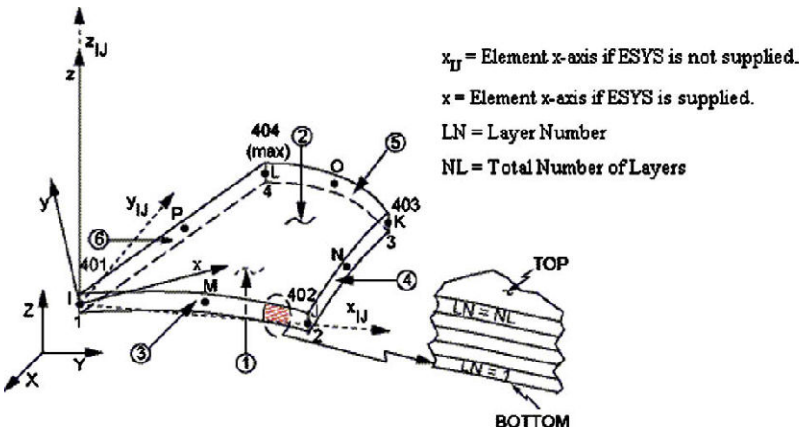

Fig. 2 SHELL91 element geometry [15]

For the purpose of the present study, a symmetric angle-ply laminate $[\pm\theta]_{2S}$ and simply supported on all the four edges is considered. The laminate has side-to-thickness ratio of 100 (a/h) and the properties of each lamina are E_1/E_2 = open, $E_2 = E_3$, $G_{12} = G_{13} = 0.6\ E_2$, $G_{23} = 0.5\ E_2$, $\upsilon_{12} = \upsilon_{13} = \upsilon_{23} = 0.25$. The density of the material (ρ) is assumed to be unity. Computations were carried out for various degree of orthotropy of individual layers (E_1 / E_2 = 3, 10, 40). The geometry and loadings of the model are shown in Fig. 1, the composite plate was considered as a square with dimensions of 100 mm × 100 mm, the cutout shape was assumed a square hole centered in the square plate with c / a = 0.333 in this work. Meanwhile, the laminated plates were also analyzed without a hole when c/a = 0 to compare the influences having a hole and without a hole conditions on natural frequencies and buckling loads. During the analysis, ANSYS which is known general purpose finite element software was preferred as numerical tool. SHELL91 [15] element type illustrated in Fig. 2 was used to produce mesh structure. Since, SHELL91 can be used for layered applications of a structural shell model. Additionally, the element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

Two different meshes, the graded and the uniform (10×10) mesh, are employed to study the plates with and without holes, respectively. The graded mesh is shown in Fig. 3. As seen from this figure, the model was divided with quadrilateral elements for a good mesh generation. Additionally, a refine mesh process was performed for surrounding of hole, since the close areas of the hole were very critical for FEM solutions. The refine mesh provided a sensitive solution. Therefore, the obtained results could be calculated as good with refine mesh than normal mesh.

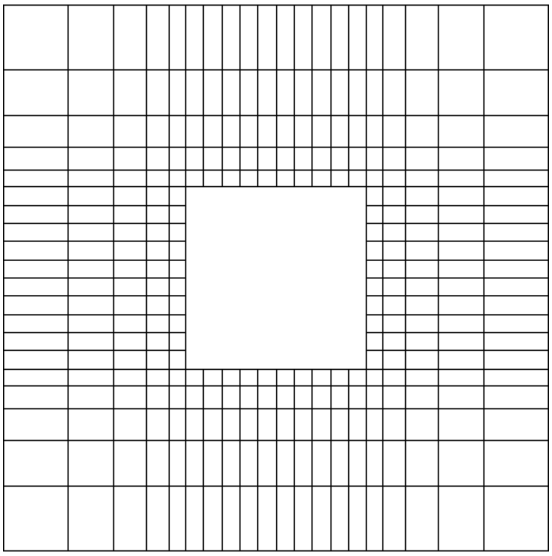

Fig. 3 Mesh with geometrically graded elements near cutout

III. Results and Discussion

Preliminary calculations were carried out to determine the required element mesh to produce a converged result. Acceptable results were obtained for the both cases, vibration and buckling analysis, with the graded and regular meshes used. The results of this study are presented in the following non-dimensionalized forms:

– natural frequency: $\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$;
– buckling load: $N_{cr} = N a^2 / E_2 h^3$.

A. Buckling Analysis

In this section the effects of stacking sequence and degree of material orthotropy on buckling loads are shown in Fig. 4 for symmetric angle-ply laminated plates with and without a cutout. As seen from this figure, firstly, in all cases the buckling loads are increased by increasing of degree of orthotropy and the fibre orientation angle for the maximum value of the critical buckling load is 45°. Second important point in Fig. 4, the buckling load is decreased with very small percentages for plates with cutout when the degree of material orthotropy takes the values 3 or 10; however, in case of materials with higher degree of orthotropy, the decrease of the critical load is larger for laminates with the angle of fibre orientation varying from 0° to 45°. Furthermore, we could observe that the critical buckling load of the plate with a cutout approaches that of a plate without a cutout for orientation angles 60–90°.

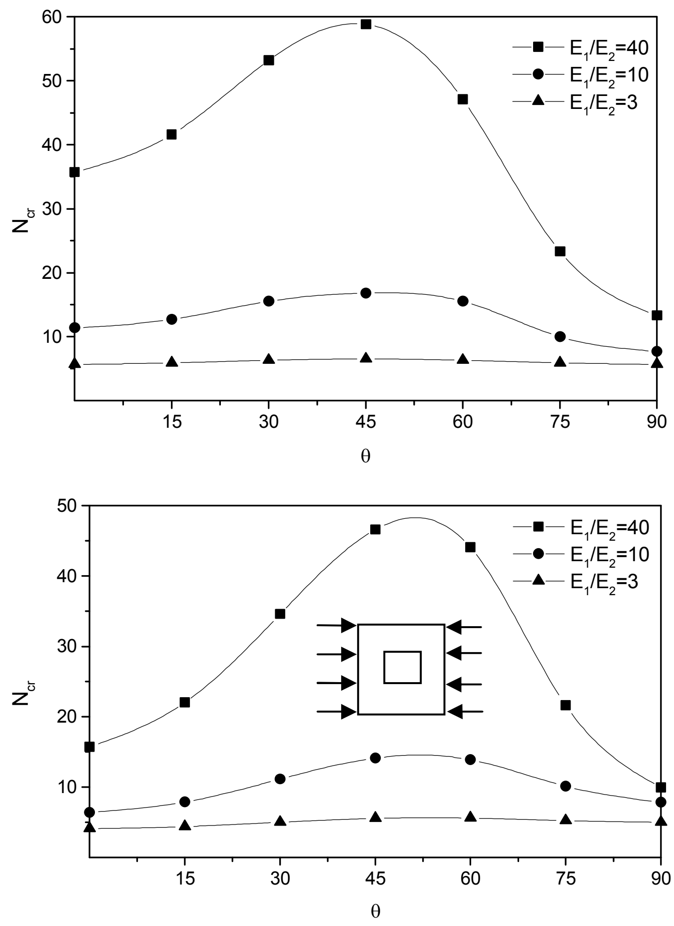

Fig. 4 Variation of non-dimensional buckling load with fibre orientation

B. Vibration Analysis

The effects of stacking sequence and degree of material orthotropy on the natural frequency of a laminated composite plate with and without a cutout when loaded by a uniform uniaxial in-plane load are shown in Figs 5, 6 and 7. Results are given for in-plane uniaxial compression loads equal to 50% of the corresponding buckling loads. In all cases, we can observe that the natural frequency of laminated plates with cutout remain close to those without cutout.

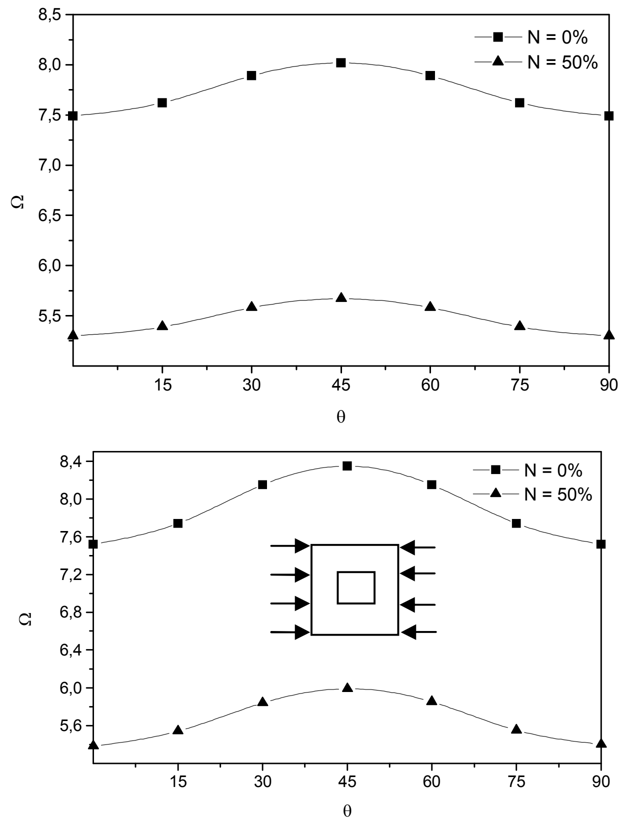

Fig. 5 Variation of non-dimensional fundamental frequency with fibre orientation ($E_1 / E_2 = 3$)

It is clearly that the natural frequency will increase gradually for plates with larger holes ($c / a > 0.333$). It is also observed when the plates are unloaded (N = 0), that the natural frequencies have a symmetric distribution about the maximum value corresponding to θ = 45°. When the plates are subjected to uniaxial compression the variation of the natural frequency is shown in Fig. 5 for materials with $E_1 / E_2 = 3$. The natural frequency is reduced uniformly, due to application of the uniaxial compression, by almost 28% and 29% for laminates with and without a cutout, respectively.

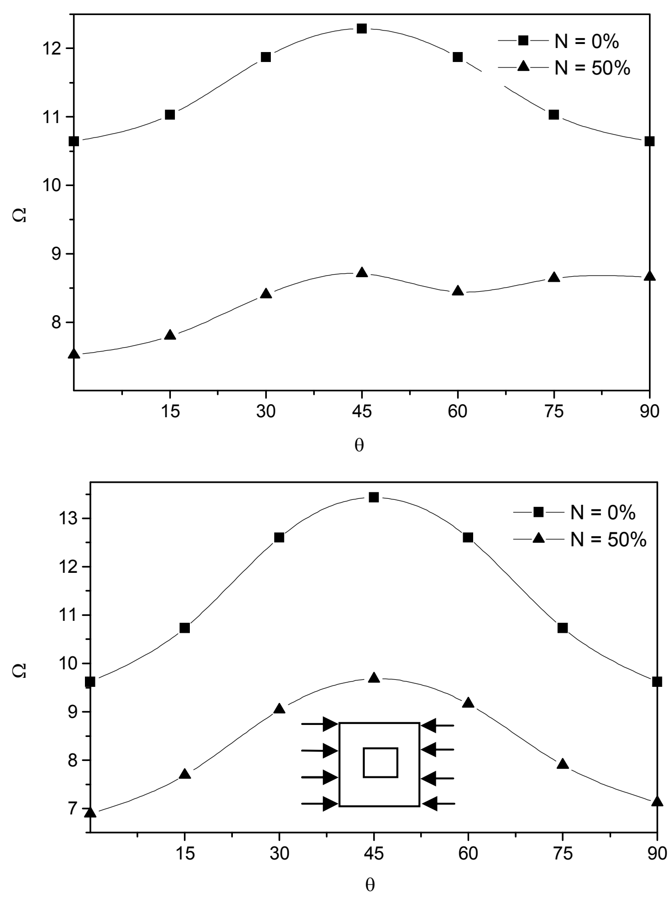

Fig. 6 Variation of non-dimensional fundamental frequency with fibre orientation ($E_1 / E_2 = 10$)

According to Fig. 6, in case of materials having $E_1 / E_2 = 10$, the differences between the natural frequencies are non-uniform for orientation angles 60–90°. The reducing is continued from 28% to 26% and from 29% to 18% for laminates with and without a cutout, respectively. Furthermore, the reduction of the natural frequency becomes more pronounced for materials with higher degree of orthotropy ($E_1 / E_2 = 40$), Fig. 7 shows that the reducing is from 28% to 15% and from 29% to 11% for laminates with and without a cutout, respectively.

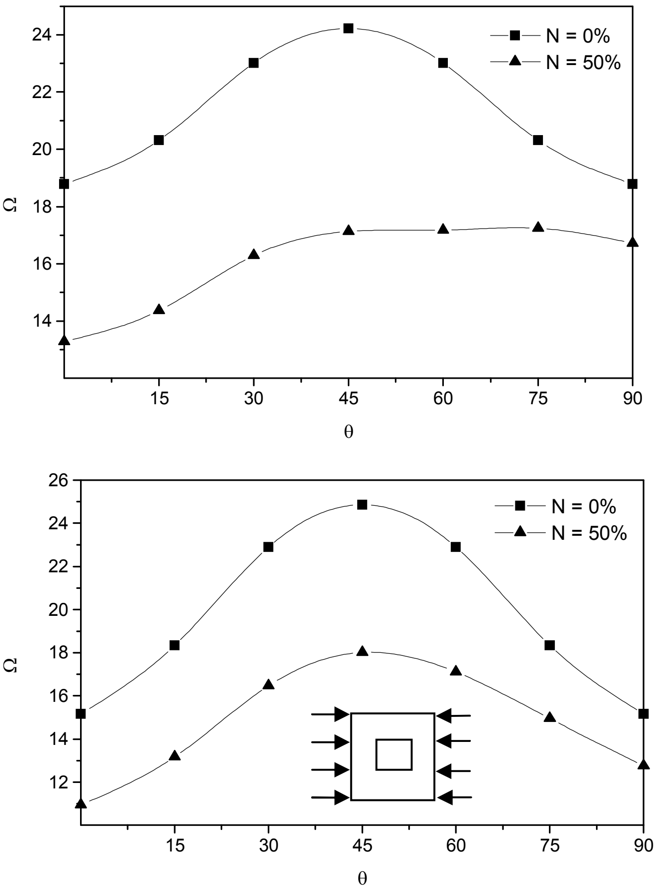

Fig. 7 Variation of non-dimensional fundamental frequency with fibre orientation (E_1 / E_2 = 40)

IV. Conclusion

In this study, the vibration response of symmetric angle-ply laminated composite plates with and without square hole when subjected to compressive loads is investigated. The uniaxial compression is taken equal to 50% of the corresponding buckling loads. Additionally, the effect of stacking sequences and degree of material orthotropy on natural frequencies and buckling loads are calculated. The modeling process and solutions were done with FEM using ANSYS finite element software. From the present study, the following main conclusions can be made. $\theta = 45°$ gives the highest value of natural frequency and buckling load in all studied cases. The material anisotropy has strong effect on buckling loads. In case of materials with higher degree of orthotropy, the critical buckling load of the plate with a cutout approaches that of a plate without a cutout for orientation angles 60–90°. When the plates are subjected to uniaxial compression load, the natural frequencies are reduced uniformly for materials which have a low degree of orthotropy. However, this reduction becomes non-uniform for materials with a higher degree of orthotropy and occurs for orientation angles ranging from 60° to 90°.

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