World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:5, No:11, 2011

Performance analysis of a flexible manufacturing line operated under surplus-based production control

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Abstract—In this paper we present our results on the performance analysis of a multi-product manufacturing line. We study the influence of external perturbations, intermediate buffer content and the number of manufacturing stages on the production tracking error of each machine in the multi-product line operated under a surplus-based production control policy. Starting by the analysis of a single machine with multiple production stages (one for each product type), we provide bounds on the production error of each stage. Then, we extend our analysis to a line of multi-stage machines, where similarly, bounds on each production tracking error for each product type, as well as buffer content are obtained. Details on performance of the closed-loop flow line model are illustrated in numerical simulations.

Keywords—flexible manufacturing systems, tracking systems, discrete time systems, production control, boundary conditions.

I. Introduction

A Manufacturing network consisting of workstations interconnected in a tandem manner, where at each station one machine serves several buffers (i.e., flexible machine), can be frequently encountered as a part of an industrial production process. For example, in case of semiconductor manufacturing it is typical to observe that at some stages the machines are working with multiple product types. In order to produce a wafer several layers of semiconductor material have to be put together, which implies that the product (wafer) has to undergo several times (some wafers more that others) through the same process before it is finally ready (see, e.g., [1]). In this case manufacturing machines work with intermediate products (wafers) of different processing stages. Another example of flexible manufacturing lines can be observed in the automotive industry (see, e.g., [2]).

Analysis on control and performance of networks, which present flexible behavior in the production process, has always attracted much attention of manufacturers, as well as of researchers. Thus control problems of flexible manufacturing lines are widely studied and a lot of valuable approaches including queuing theory, Petri nets, dynamic programming, linear programming, hybrid systems were proposed and some of them are implemented (for surveys see, e.g., [3]–[5]).

In this paper we focus on the performance analysis of a flexible production line controlled by a surplus-based[1] decentralized production control (see e.g., [6]). Specifically, given the presence of unknown but bounded production speed perturbations, as well as demand rate fluctuations, we investigate how close the cumulative production output of the network follows its cumulative production demand under this control policy.

In order to achieve our goal we use classical tools from control theory. The production flow process is described by means of difference equations and in order to analyse its performance, a Lyapunov theory approach is exploited (see, e.g., [7], [8] and references there in).

Each machine in the network is responsible for several production stages. At each stage the machine coordinates its individual production with those of the rest of the system. While working at one stage the machine does not switch to another one unless the primary control objective at this stage is fulfilled or product starvation occurs. The primary objective of each production stage may be viewed as manufacturing a sufficient quantity of parts to satisfy the demand of its immediate downstream production stage (belonging to the downstream machine) and some desired amount as back-up material storage in its downstream buffer. The production strategy itself is intuitive and it can be associated with a wide range of existing techniques such as Basestock policy (see, e.g., [6]), Hedging Point policy (see, e.g., [3]), and Clearing policy (see, e.g., [9]).
To the best of our knowledge, concerning the previous results on performance analysis of surplus-based approaches (see, e.g., [3]–[5], [10]–[14], the novelty of our results can be summarized as follows. The proposed production model is considered in discrete time. The production speed of each machine is defined as deterministic with bounded perturbations. The future production demands are assumed to be unknown and with bounded fluctuations. As a result, for one flexible manufacturing machine of N production stages, strict, so-called ”worst” case bounds on the production tracking error for each product type are obtained. Extending this strategy to a network of P machines with N production stages each, we present the obtained results regarding the bounds on the production tracking errors and buffer contents for each machine and its buffers. Furthermore, we show that, though the analysis given in this paper is focused on multi-product manufacturing lines, the obtained results can be easily extended to re-entrant configurations with one product type demand.

The paper is organized as follows. First, in Section 2 the flow model of one manufacturing machine with surplus-based pull control is presented. The detailed analysis of production error trajectories is developed in this section. Then the flow model of a flexible manufacturing line with surplus-based pull control is analyzed in Section 3. Here necessary conditions are derived to guarantee the uniform ultimate boundedness of the

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[1]In the surplus-based control, decisions are made based on the production tracking error, which is the difference between the cumulative demand and the cumulative output of the system.

production error trajectories of each machine. Performance and robustness issues of the closed-loop flow models are illustrated in numerical simulations in Section 4. Finally, Section 5 contains conclusions and our future developments.

II. Analysis of One Flexible Machine

Figure 1 shows a schematics of one machine M with N production stages, which directly correspond to the number of product types that it can serve. The machine is interconnected with N buffers $B_1 \ldots B_N$, each containing its infinite product supply of corresponding product type.

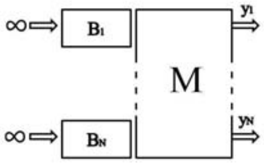

Fig. 1. Schematics of one flexible manufacturing machine M.

A. Flow Model

In discrete time the cumulative number of produced products in time k for a simple manufacturing machine can be described as the sum of its production rates at each time step till time k. Thus the flow model of each production stage of one flexible machine (see Figure 1) in discrete time is defined as

$$y_j(k+1) = y_j(k) + \beta_j(k)u_j(k),\ \forall k \in \mathbb{N}, j = 1, \ldots, N, \quad (1)$$

where $y_j(k) \in \mathbb{R}$ is the cumulative output of the machine for product type j in time k, $u_j(k) \in \mathbb{R}$ is the control input of the machine in processing stage of product type j and $\beta_j(k) = \mu_j + f_j(k)$ where μ_j is a positive constant that represents the processing speed of the machine for servicing the product type j and $f_j(k) \in \mathbb{R}$ is an unknown external disturbance affecting the performance of the machine at stage j. Under the assumption that there is always sufficient quantity of the raw material to feed the machine, the control aim is to track the non-decreasing cumulative production demand of each product type j on its output. We define the cumulative production demand by using $y_{dj}(k) \in \mathbb{R}$ given by

$$y_{dj}(k) = y_{dj0} + v_{dj}k + \varphi_j(k),\ \forall j = 1, \ldots, N, \quad (2)$$

where y_{dj0} is a positive constant that represents the initial production demand of product j, v_{dj} is a positive constant that defines the average desired demand rate of product j, and $\varphi_j(k) \in \mathbb{R}$ is the bounded fluctuation that is imposed on the linear demand $v_{dj}k$.

In order to give a solution to this tracking problem we consider the controller based on the production tracking error of each product type. The machine can only work at one stage at a time. The controller randomly selects the stage at which the machine must work, from those where production is needed. The machine works at this stage till its product demand is satisfied. Then the controller again selects a stage for the machine to work at. In case the product demand of all product types are satisfied, the controller idles the machine.

The above mentioned can be formulated by following control algorithm (see next paragraph for summary):

$\{q(k) = B_j\}$
if $\varepsilon_j(k) > 0$ **then**
$u_j(k) = 1,$
$u_s(k) = 0,\ \forall s \neq j,\ s, j = 1, \ldots, N,$
$q(k+1) = B_j,$
if $\varepsilon_j(k) \leq 0$ **and** $\exists s \neq j : \varepsilon_s(k) > 0$ **then**
$u_j(k) = 0,$
$u_s(k) = 1,$
$q(k+1) = B_s,$
if $\varepsilon_s(k) \leq 0,\ \forall s$ **then**
$u_j(k) = 0,$
$u_s(k) = 0,\ \forall s \neq j,\ s, j = 1, \ldots, N,$
$q(k+1) = 0,$ (3)

where $q(k)$ is the internal variable that specifies the buffer that machine M is processing, $\varepsilon_j(k) \in \mathbb{R}$ is the production tracking error at stage j. Note that all B_j buffers are considered to always have sufficient raw material.
Summarizing (3), the machine can only work on one buffer (product type) at a time. The control input $u_j(k)$ of each production stage j can only take the value of 0 (stop) or 1 (produce). The $u_j(k)$ receives the value of 1 only if production stage j needs to produce ($\varepsilon_j(k) > 0$). The machine will remain at its current state ($q(k) = B_j$) while all the conditions of the state are satisfied. The value of 0 is given to the control input of stage j if at least one of the conditions of the current state $q(k) = B_j$ is unsatisfied. The change in the value of the control signal of a stage j also implies a change in the machine's state $q(k)$. The machine has $N+1$ states. This is due to that N is the total number of processing stages (product types) that M can be working in, which directly relate to the states of the machine, plus the idle state ($q(k) = 0$).

The production tracking error at each stage of M is given by:

$$\varepsilon_j(k) = y_{dj}(k) - y_j(k),\ \forall k \in \mathbb{N}. \quad (4)$$

For further analysis, let us rewrite flow model (1) in a closed-loop with (3) in terms of production tracking errors as

$$\Delta\varepsilon_j(k) = v_{dj} + \Delta\varphi_j(k) - \beta_j(k)u_j(k), \quad (5)$$

where for all $j = 1, \ldots, N$, $\Delta\varepsilon_j(k) = \varepsilon_j(k+1) - \varepsilon_j(k)$ and $\Delta\varphi_j(k) = \varphi_j(k+1) - \varphi_j(k)$. Here we assume that system (5) satisfies the following assumptions.

Assumption 1 *(Boundedness of perturbations) There are constants c_1, c_2, c_3 and c_4 such that*

$$c_1 < \Delta\varphi_j(k) < c_2,\ \forall k,\ j = 1, \ldots, N, \quad (6)$$
$$c_3 < f_j(k) < c_4,\ \forall k,\ j = 1, \ldots, N. \quad (7)$$

From *Assumption 1*, it follows that $W_j(k) = \Delta\varphi_j(k) - f_j(k)$ satisfies

$$\alpha_1 < W_j(k) < \alpha_2,\ \forall k,\ j = 1, \ldots, N, \tag{8}$$

with $\alpha_1 = c_1 - c_4$ and $\alpha_2 = c_2 - c_3$.

Assumption 2 *(Capacity condition) Constants c_1, c_2, c_3 and c_4 satisfy the following inequalities*

$$c_1 \ > \ -v_{dj},\ \forall j = 1, \ldots, N, \tag{9}$$

$$\alpha_2 \ < \ \mu_j - v_{dj},\ \forall j = 1, \ldots, N, \tag{10}$$

and the following condition (also know as capacity condition) holds

$$0 < \sum_{j=1}^{N} \frac{v_{dj} + \Delta\varphi_j(k)}{\mu_j + f_j(k)} < 1. \tag{11}$$

By (9), (10), and (11) we state that, in the presence of market fluctuations bounded by (c_1, c_2), the demand rate for each product type can only be positive, the production speed at each manufacturing stage of the machine is always faster than the demand rate of its product and in general the processing speed of the machine is faster than its demand rate, respectively.

It is important to notice that machine M at each process step j has a processing speed of $\mu_j + f_j(k)$ lots per time unit, which can differ from the other processing steps.

B. *Results on Performance*

In this section we present the results respecting the production error trajectories behavior of flow model (5).

Theorem 1 *Assume that the discrete time system defined by (5) satisfies Assumptions 1 and 2. Then all solutions of (5) are ultimately bounded by*

$$\limsup_{k\to\infty} \sum_{j=1}^{N} \frac{\varepsilon_j(k) - v_{dj} - \alpha_2}{\mu_j + c_3} \leq 0, \tag{12}$$

$$\liminf_{k\to\infty} \varepsilon_j(k) \geq v_{dj} + \alpha_1 - \mu_j. \tag{13}$$

Note that by replacing $v_{dj} + \Delta\varphi_j(k)$ for $v_d + \Delta\varphi(k)$ this result can be also extended to a re-entrant production machine serving one product type.

Proof: see Appendix A. ■

The obtained bounds can be appreciated graphically through a phase portrait of the production error trajectories shown in Figure 2, which was made for a single machine producing 2 product types. The product demand rate $v_{dj} = 0.99$ [lots/time unit] and the production rate at each stage μ_j=2[lot/ time unit]. Here the experiment starts with initial production tracking errors $\varepsilon_1(0) = 2$ [lots] and $\varepsilon_2(0) = 2$ [lots]. It can be observed that first the controller activates stage 1 of M. The machine works with this stage till $\varepsilon_1(k) \leq 0$ and switches to stage 2. Eventually the trajectories of the tracking errors enter the zone depicted by the rectangular triangle, where they remain for the rest of the experiment. The legs of this triangle are given by (13) and the hypotenuse by (12).

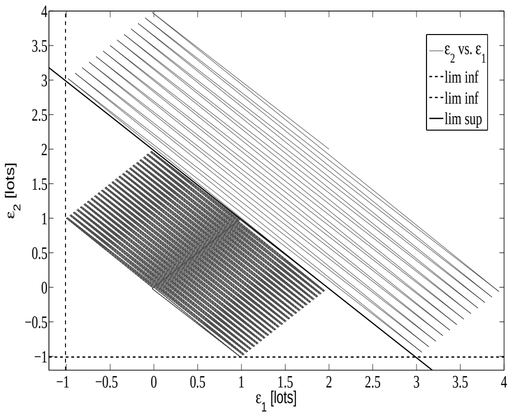

Fig. 2. Tracking Errors $\varepsilon_2(k)$ vs. $\varepsilon_1(k)$, with $v_{dj} = 0.99$ [lots/time unit] and μ_j=2[lot/ time unit].

III. Analysis of a Flexible Flow Line

Figure 3 shows a schematics of a flexible manufacturing line consisting of P machines $M_1, \ldots, M_P$ with N production stages each. Each machine M_i receives its intermediate products from N upstream buffers $B_{i,1}, \ldots, B_{i,N}$. The products flow through the network in unidirectional manner.

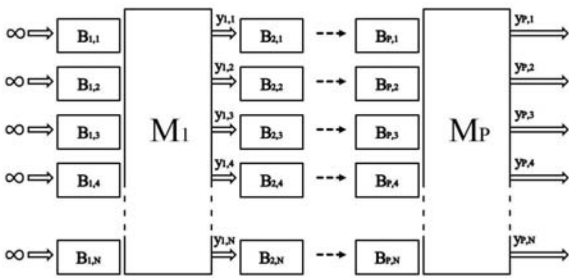

Fig. 3. Schematic of a flexible production line

A. *Flow Model*

The flow model of each production stage of a flexible line (Figure 3) in discrete time is defined as

$$y_{i,j}(k+1) \ = \ y_{i,j}(k) + \beta_{i,j}(k)u_{i,j}(k),\ \forall k, i, j, \tag{14}$$

where $i = 1, .., P$ is the machine number, $j = 1, ..., N$ is the processing stage (product type) number of machine i, $y_{i,j}(k) \in \mathbb{R}$ is the cumulative output of machine i in processing stage j in time k, $u_{i,j}(k) \in \mathbb{R}$ is the control input of machine i in processing stage j and $\beta_{i,j}(k) = \mu_{i,j} + f_{i,j}(k)$ where $\mu_{i,j}$ is a positive constant that represents the processing speed of the machine i at its stage j and $f_{i,j}(k) \in \mathbb{R}$ is an unknown external disturbance affecting the performance of the ith machine at its stage j.

Under the assumption that there is always sufficient quantity of the raw material to feed the input buffers $B_{1,j}$, the control aim is to track the non-decreasing cumulative production demands given by (2) on each output of the multi-product manufacturing line.

In order to give a solution to this tracking problem we consider the following control algorithm:

$$\begin{array}{l}
\{q_i(k) = B_{i,j}\} \\
\textbf{if } \varepsilon_{i,j}(k) > 0 \textbf{ and } w_{i,j}(k) \geq \beta_{i,j}(k) \textbf{ then} \\
u_{i,j}(k) = 1, \\
u_{i,s}(k) = 0, \forall s \neq j,\ s, j = 1 \ldots, N, \\
q_i(k+1) = B_{i,j}, \\
\textbf{if } (\varepsilon_{i,j}(k) \leq 0 \textbf{ or } w_{i,j}(k) < \beta_{i,j}(k)) \textbf{ and} \\
\exists s \neq j : \varepsilon_{i,s}(k) > 0 \textbf{ and } w_{i,s}(k) \geq \beta_{i,s}(k) \textbf{ then} \\
u_{i,j}(k) = 0, \\
u_{i,s}(k) = 1, \\
q_i(k+1) = B_{i,s}, \\
\textbf{if } (\varepsilon_{i,s}(k) \leq 0 \textbf{ or } w_{i,s}(k) < \beta_{i,s}(k)),\ \forall s \textbf{ then} \\
u_{i,j}(k) = 0, \\
u_{i,s}(k) = 0, \forall s \neq j,\ s, j = 1 \ldots, N, \\
q_i(k+1) = 0,
\end{array} \tag{15}$$

where $q_i(k)$ is the internal variable representing the current buffer that M_i is processing, $w_{i,j}(k)$ is the buffer content of $B_{i,j}$. For the current time step $\beta_{i,j}(k)$ is the minimal raw material content in buffer $B_{i,j}$, such that machine M_i is able to process if required at this stage. Note that $B_{1,j}$ is considered to always contain sufficient raw material. Thus the buffer content condition $w_{1,j}(k) \geq \beta_{1,j}(k)$ is assumed to be always satisfied. The tracking error for each product type j at each stage of M_i is given by:

$$\varepsilon_{i,j}(k) = \varepsilon_{i+1,j}(k) + w_{di+1,j} - w_{i+1,j}(k), \tag{16}$$
$$\varepsilon_{P,j}(k) = y_{dj}(k) - y_{P,j}(k), \tag{17}$$

where $i = 1, \ldots, P-1$, and $j = 1, \ldots, N$. Here $w_{i+1,j}(k) = y_{i,j}(k) - y_{i+1,j}(k)$ is the buffer content of buffer $B_{i+1,j}$ and $w_{d_{i+1,j}}$ is the constant that represents the desired buffer level (extra stock) of buffer $B_{i+1,j}$.

For further analysis, let us rewrite flow model (14) in a closed-loop with (15) in terms of tracking errors as

$$\Delta\varepsilon_{i,j}(k) = v_{dj} + \Delta\varphi_j(k) - \beta_{i,j}(k)u_{i,j}(k), \quad \forall j = 1, \ldots, N,\ i = 1, \ldots, P \tag{18}$$

where $\Delta\varepsilon_{i,j}(k) = \varepsilon_{i,j}(k+1) - \varepsilon_{i,j}(k)$.

Notice that machine M_i operates at each production step j under a processing speed of $\mu_i + f_i(k)$ lots per time unit, which is the same for each production stage of the machine, but can differ from the other machines in the network.

For system (18) the following assumptions are satisfied.

Assumption 3 *(Boundedness of perturbations) There are constants c_1, c_2, c_3 and c_4 such that*

$$c_1 < \Delta\varphi_j(k) < c_2,\ \forall k, j = 1, \ldots, N \tag{19}$$
$$c_3 < f_i(k) < c_4\ \forall k,\ i = 1, \ldots, P. \tag{20}$$

From *Assumption 3*, it follows that $W_{i,j}(k) = \Delta\varphi_j(k) - f_i(k)$ satisfies

$$\alpha_1 < W_{i,j}(k) < \alpha_2,\ \forall k, \tag{21}$$

with $\alpha_1 = c_1 - c_4$ and $\alpha_2 = c_2 - c_3$.

Assumption 4 *(Capacity condition) Constants c_1, c_2, c_3 and c_4 satisfy the following inequalities*

$$c_1 > -v_{dj},\ \forall j = 1, \ldots, N, \tag{22}$$
$$\alpha_2 < \mu_i - v_{dj},\ \forall i = 1, \ldots, P, \tag{23}$$

and the following condition (Capacity Condition) holds for each M_i in the network

$$0 < \frac{1}{\mu_i + f_i(k)} \sum_{j=1}^{N} (v_{dj} + \Delta\varphi_j(k)) < 1, \forall i. \tag{24}$$

One of the important physical limitations in the network is the buffer content restriction. In our model, in order for the positive control action ($u_{i,j}(k) = 1$) of the selected production stage ($B_{i,j}$) of M_i to take place, the buffer of this stage must satisfy the following condition on its content

$$w_{i,j}(k) \geq \beta_{i,j}(k),\ \forall i = 2, \ldots, P,\ j = 1, \ldots, N. \tag{25}$$

Thus, from (16) and (25), the following tracking error condition holds

$$\varepsilon_{i+1,j}(k) \geq \beta_{i+1,j}(k) - w_{d_{i+1,j}} + \varepsilon_{i,j}(k),$$

where $i = 1, \ldots, P-1,\ j = 1, \ldots, N$, and $w_{d_{i,j}}$ satisfies the following assumption:

Assumption 5 *(Desired buffer content condition) The constants $w_{d_{i,j}}$ comply with the following inequality*

$$w_{d_{i,j}} \geq \mu_{i,j} + N\mu_{i-1,j} + (N+1)c_4 + (N-1)(c_2 - c_1), \tag{26}$$

From (26) it follows that $w_{d_{i,j}} > \beta_{i,j}(k), \forall k$.

B. Results on Performance

In this section we present the results respecting the production error trajectories behavior of flow model (18).

Theorem 2 *Assume that the discrete time system defined by (18) satisfies Assumptions 3, 4, and 5. Then all solutions of (18) are ultimately bounded by*

$$\limsup_{k\to\infty} \sum_{j=1}^{N} (\varepsilon_{i,j}(k) - v_{dj} - \alpha_2) \leq 0, \tag{27}$$
$$\liminf_{k\to\infty} \varepsilon_j(k) \geq v_{dj} + \alpha_1 - \mu_j. \tag{28}$$

Note that by replacing $v_{d_j} + \Delta\varphi_j(k)$ by $v_d + \Delta\varphi(k)$ this result can be also extended to a re-entrant production line serving one product type.

Proof: Due to extensive technical details the proof of Theorem 2 is omitted in this paper and will be presented in its full version. ■

From (27), and (28) it can be deduced that for the buffer content $w_{i,j}(k)$ of each buffer $B_{i,j}$ defined by (16), it holds that

$$\limsup_{k\to\infty} w_{i,j}(k) \leq (N-1)\mu_i + N(\alpha_2 - \alpha_1) + \mu_{i-1} + w_{d_{i,j}}, \forall i = 2,\ldots,P. \quad (29)$$

Now, in order to support the present development let us present simulation results.

IV. Simulation Results

Consider the following example of a flexible production line consisting of 2 manufacturing machines with 2 production stages each (see Figure 3). The line is operating under surplus-based regulators (15). The processing speed of each machine is set to $\mu_i + f_i(k) = (10, 5)$ (lots per time unit), the desired buffer content of each buffer is selected considering (26) as $w_{d_2} = (w_{d_{2,1}}, w_{d_{2,2}}) = (26, 26)$ (lots), and the mean demand rate for each product type $v_{dj} = 2$ (lots per time unit) with fluctuation rate of $\Delta\varphi_j(k) = 0.4\sin(90k)$. The tracking error of each machine in the line is depicted in Figure 4. Here the initial conditions ($y_{1,1}(0)$, $y_{1,2}(0)$, $y_{2,1}(0)$, $y_{2,2}(0)$) are set to the zero value and $y_{d0} = 100$ (lots). After the first 245 time steps for product type 1 and 241 time steps for product type 2 , as it is shown in Figures 4 and 5, the system reaches its steady state . Tracking errors are maintained inside $[-8.4, 13.2]$ lots for M_1, and $[-3.4, 8.2]$ lots for M_2 (see the dashed lines of Figure 4), which satisfy the bounds given by (27) and (28). Figure 5 shows the buffer content of each $B_{i,j}$ in the network. After some transient behavior the inventory level of each buffer is maintained inside the obtained bound (29).

Another experimental result can be appreciated in Figure 6. This two graphics show the relation between the upper bound on the production tracking error $\varepsilon_{2,1}(k)$ and $\varepsilon_{2,2}(k)$ and the desired buffer content of the network from the previous example. Here it can be observed that the amount of extra storage for intermediate products has only limited influence on the tracking precision of the network and the threshold value of this influence is given by (26). In conclusion, the presented simulation results reflect the desired flow model behavior, i.e., all the values assigned to the parameters utilized in this section are consistent with the assumptions of Section 3 and the outcome of the simulation example satisfies the theoretical results.

V. Conclusion

The performances of a multi-product manufacturing network operated under surplus-based pull control has been studied. Developed results show uniform boundedness for trajectories of each production tracking error for one flexible machine considering that each production stage has a variable processing speed. Also bounds on the production tracking error of each stage of a multi-product manufacturing line were presented. For a line it was considered that each production machine has a variable processing speed. Simulation examples were presented and discussed in order to illustrate and support analytical results. One of the important outcomes of these examples is the relation between the amount of extra intermediate product storage and the production tracking error. It was shown that extra storage capacity has a limited influence on the production tracking error. The threshold value on the desired capacity for each buffer content was provided in *Assumption 5* of the flow model analysis.

Furthermore, studies on manufacturing networks under surplus-based pull control with the presence of production delays and setup times, as well as its comparison with other surplus-based pull strategies will be pursued in our future research.

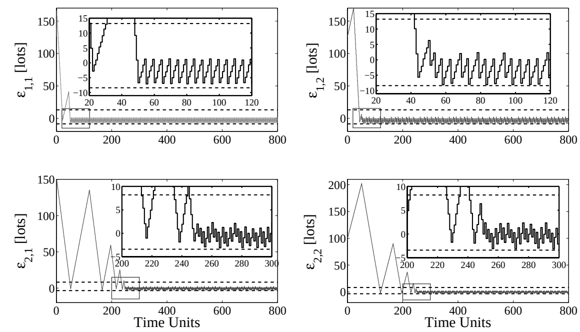

Fig. 4. Production errors and the obtained bounds (dotted lines), with $v_{dj} = 2$, $\Delta\varphi_j(k) = 0.4\sin(90k)$,$\forall j = 1, 2$, $w_{d_2} = (26, 26)$, $\mu_i + f_i(k) = (10, 5)$, and $y_{d0} = 100$.

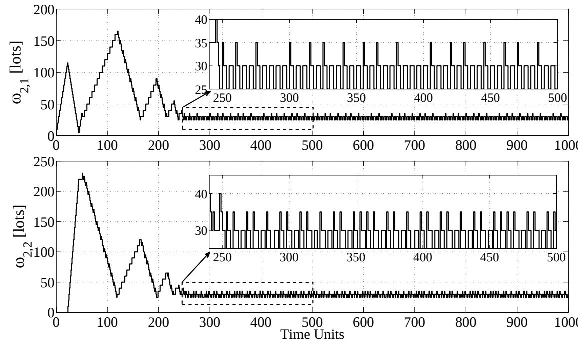

Fig. 5. Buffer contents, with $v_{dj} = 2$, $\Delta\varphi_j(k) = 0.4\sin(90k)$, $\forall j = 1, 2$, $w_{d_2} = (26, 26)$, $\mu_i + f_i(k) = (10, 5)$ and $y_{d0} = 100$.

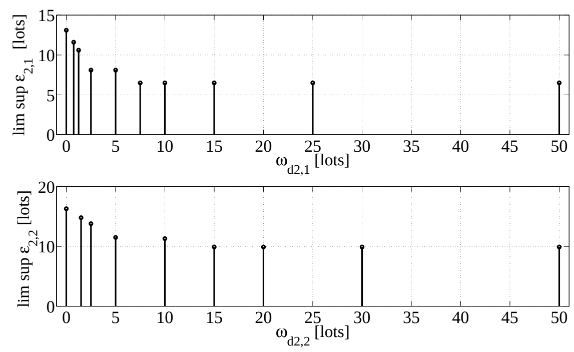

Fig. 6. Upper bound on output production errors vs. desired buffer contents, with $v_{dj} = 3.3$ [lots/time unit], $\mu_{1,j} + f_{1,j}(k) = (10, 5)$ [lots/time unit], $\mu_{2,j} + f_{2,j}(k) = (5, 10)$ [lots/time unit].

ACKNOWLEDGMENT

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. INFSO-ICT-223844.

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APPENDIX A
PROOF OF THEOREM 1

Let us prove that Theorem 1 holds for one machine with $j = 1, \dots, N$ defined by (5). With this goal, let us introduce the following Lyapunov function

$$V_k^{B_N} = \max \left\{ \begin{array}{c} -\varepsilon_1(k) - \mu_1 + v_{d1} + \alpha_1, \\ \vdots \\ -\varepsilon_N(k) - \mu_N + v_{dN} + \alpha_1, \\ \underbrace{\sum_{j=1}^{N} \frac{\varepsilon_j(k) - v_{dj} - \alpha_2}{\mu_j + c_3}}_{X_k}, \\ 0 \end{array} \right\}. \quad (30)$$

Here for the sake of brevity $V_k^{B_N} = V^{B_N}(\varepsilon_1(k), ..., \varepsilon_N(k))$, with $V^{B_N} = 0$, for all $\varepsilon_j(k) \in [v_{dj} + \alpha_1 - \mu_j, v_{dj} + \alpha_2 + (\mu_j + c_3) \sum_{s=1}^{N} \frac{\mu_s - \alpha_1 + \alpha_2}{\mu_s + c_3}]$, where $s \neq j$.

Thus, $\Delta V_k^{B_2}$ along the solutions of $\varepsilon_j(k)$ is given by

$$\Delta V_k^{B_N} = \underbrace{\max \left\{ \begin{array}{c} -\varepsilon_1(k) - \Delta\varphi_1(k) + \alpha_1 - \mu_1 + \beta_1(k) u_1(k), \\ \vdots \\ -\varepsilon_N(k) - \Delta\varphi_N(k) + \alpha_1 - \mu_N + \beta_N(k) u_N(k), \\ X_{k+1}, \\ 0 \end{array} \right\}}_{V_{k+1}^{B_N}} + \underbrace{\min \left\{ \begin{array}{c} \varepsilon_1(k) + \mu_1 - v_d - \alpha_1, \\ \vdots \\ \varepsilon_N(k) + \mu_N - v_{dN} - \alpha_1, \\ -X_k, \\ 0 \end{array} \right\}}_{-V_k^{B_N}}, \quad (31)$$

where $X_{k+1} = \sum_{j=1}^{N} \frac{\varepsilon_j(k) + W_j(k) - \alpha_2 - \beta_j(k) u_j(k)}{\mu_j + c_3}$.
In order to perform a more detailed analysis on $\Delta V_k^{B_N}$, let us divide this proof into 2 cases.

Case 1 ($q(k) = 0$)

Suppose that $q(k) = 0$, which from (3) implies that $u_{j,k} = 0$, for all $j = 1, ..., N$.

Then we can rewrite $\Delta V_k^{B_N}$ from (31) as

$$\Delta V_k^{B_2} = \underbrace{\max \left\{ \begin{array}{c} -\varepsilon_1(k) - \Delta\varphi_1(k) + \alpha_1 - \mu_1, \\ \vdots \\ -\varepsilon_N(k) - \Delta\varphi_N(k) + \alpha_1 - \mu_N, \\ X_{k+1}, \\ 0 \end{array} \right\}}_{V_{k+1}^{B_N}} + \underbrace{\min \left\{ \begin{array}{c} \varepsilon_1(k) + \mu_1 - v_{d1} - \alpha_1, \\ \vdots \\ \varepsilon_N(k) + \mu_N - v_{dN} - \alpha_1, \\ -X_k, \\ 0 \end{array} \right\}}_{-V_k^{B_N}}. \quad (32)$$

From (3), $\varepsilon_j(k)$ satisfies

$$\varepsilon_j(k) \leq 0, \quad (33)$$

for all k and j. Then we can reduce $\Delta V_k^{B_N}$ from (32) to

$$\Delta V_k^{B_N} = \underbrace{\max\left\{\begin{array}{c} -\varepsilon_1(k) - \Delta\varphi_1(k) + \alpha_1 - \mu_1, \\ \vdots \\ -\varepsilon_N(k) - \Delta\varphi_N(k) + \alpha_1 - \mu_N, \\ 0 \end{array}\right\}}_{V_{k+1}^{B_N}}$$

$$+ \underbrace{\min\left\{\begin{array}{c} \varepsilon_1(k) + \mu_1 - v_{d1} - \alpha_1, \\ \vdots \\ \varepsilon_N(k) + \mu_N - v_{dN} - \alpha_1, \\ 0 \end{array}\right\}}_{-V_k^{B_N}}. \quad (34)$$

Here, let us assume that for $V_{k+1}^{B_N}$ the maximum is reached in the j element of the function, i.e. $V_{k+1}^{B_N} = -\varepsilon_j(k) - \Delta\varphi_j(k) + \alpha_1 - \mu_j$. Then from the definition of $\min$ it holds that

$$\begin{aligned} \Delta V_k^{B_N} &\leq -\varepsilon_j(k) - \Delta\varphi_j(k) + \alpha_1 - \mu_j \\ &\quad +\varepsilon_j(k) + \mu_j - v_{dj} - \alpha_1, \\ \Delta V_k^{B_N} &\leq -v_{dj} - \Delta\varphi_j(k) \overset{(6,9)}{<} 0. \end{aligned} \quad (35)$$

For $V_{k+1}^{B_N}$ with maximum reached by its last element it holds that

$$\Delta V_k^{B_N} = -V_k^{B_N}. \quad (36)$$

Thus, for in this case for $V_k^{B_N} > 0$ given by (30) its increment $\Delta V_k^{BN} < 0$. This concludes the analysis of Case 1.

Case 2 ($q(k) = B_j$) Suppose that $\varepsilon_j(k)$ satisfies

$$\varepsilon_j(k) > 0 \quad (37)$$

for all k. Thus, the machine is working with buffer B_j ($q(k) = B_j$), which is considered to always have a sufficient raw material. Without loss of generality let us assume for now that $\varepsilon_s(k) \in \mathbb{R}$ for all $s \neq j$. Then we can rewrite $\Delta V_k^{B_N}$ from (32) as

$$\Delta V_k^{B_N} = \underbrace{\max\left\{\begin{array}{c} -\varepsilon_1(k) - W_1(k) + \alpha_1, \\ \vdots \\ -\varepsilon_N(k) - \Delta\varphi_N(k) + \alpha_1 - \mu_N, \\ X_{k+1}, \\ 0 \end{array}\right\}}_{V_{k+1}^{B_N}}$$

$$+ \underbrace{\min\left\{\begin{array}{c} \varepsilon_1(k) + \mu_1 - v_{dN} - \alpha_1, \\ \vdots \\ \varepsilon_N(k) + \mu_N - v_{dN} - \alpha_1, \\ -X_k, \\ 0 \end{array}\right\}}_{-V_k^{B_N}}. \quad (38)$$

Subcase 1: Let us first analyse (38) assuming that $\varepsilon_s(k)$ satisfies

$$\varepsilon_s(k) > 0 \quad (39)$$

for all k, $s \neq j$, and $s = 1, ..., N$. Then due to condition (37) and (39), the increment (38) satisfies

$$\Delta V_k^{B_N} \leq \underbrace{\max\left\{\begin{array}{c} \frac{\varepsilon_j(k)+W_j(k)-\alpha_2-\mu_j}{\mu_j+c_3} + \frac{\varepsilon_s(k)+\Delta\varphi_s(k)-\alpha_2}{\mu_s+c_3}, \\ 0 \end{array}\right\}}_{-V_{k+1}^{B_N*}}$$

$$+ \underbrace{\min\left\{\begin{array}{c} \frac{-\varepsilon_j(k)+v_{dj}+\alpha_2}{\mu_j+c_3} + \frac{-\varepsilon_s(k)+v_{ds}+\alpha_2}{\mu_s+c_3}, \\ 0 \end{array}\right\}}_{-V_k^{B_N*}}. \quad (40)$$

Consider now that for $V_{k+1}^{B_N*}$ the maximum is reached in its first element, i.e. $V_{k+1}^{B_N*} = \frac{\varepsilon_j(k)+W_j(k)-\alpha_2-\mu_j}{\mu_j+c_3} + \frac{\varepsilon_s(k)+\Delta\varphi_s(k)-\alpha_2}{\mu_s+c_3}$. Then from the definition of $\min$ it holds that

$$\Delta V_k^{B_N} \leq -\frac{\mu_j + f_j(k)}{\mu_j + c_3} + \frac{v_{dj} + \Delta\varphi(k)}{\mu_j + c_3} + \frac{v_{ds} + \Delta\varphi_s(k)}{\mu_s + c_3} \overset{(7,11)}{<} 0.$$

In case that for $V_{k+1}^{B_N*}$ given by (40) the maximum is reached in its second element, then from the definition of $\min$

$$\Delta V_k^{B_N} = -V_k^{B_N}. \quad (41)$$

Thus, for this subcase for $V_k^{B_N} > 0$ given by (30) its increment $\Delta V_k^{B_N} < 0$.

Subcase 2: Now let us analyse (38) assuming that $\varepsilon_s(k)$ satisfies

$$\varepsilon_s(k) \leq 0 \quad (42)$$

for all k, $s \neq j$, $s = 1, ..., N$. In analogy with the procedure followed in previous subcase it is obtained that

$$\left\{\begin{array}{l} \Delta V_k^{B_N} \leq -v_{ds} - \Delta\varphi_s(k) \overset{(6,9)}{<} 0 \\ \text{if } V_{k+1}^{B_N} = -\varepsilon_s(k) - \Delta\varphi(s) + \alpha_1 - \mu_s, \\ \Delta V_k^{B_N} \leq -\frac{\mu_j+f_j(k)}{\mu_j+c_3} + \frac{v_{dj}+\Delta\varphi_j(k)}{\mu_j+c_3} + \frac{v_{ds}+\Delta\varphi_s(k)}{\mu_s+c_3} \overset{(7,11)}{<} 0 \\ \text{if } V_{k+1}^{B_N} = \frac{\varepsilon_j(k)+W_j(k)-\alpha_2-\mu_j}{\mu_j+c_3} + \frac{\varepsilon_s(k)+\Delta\varphi_s(k)-\alpha_2}{\mu_s+c_3}, \\ \Delta V_k^{B_N} = -V_k^{B_N} \\ \text{if } V_{k+1}^{B_N} = 0. \end{array}\right.$$

Thus, for this subcase for $V_k^{B_N} > 0$ given by (30) its increment $\Delta V_k^{B_N} < 0$. This concludes the analysis of Case 2. Summarizing for 2 cases, we have shown that for $V_k^{B_N} > 0$ given by (30) its increment $\Delta V_k^{B_N} < 0$ for all $\varepsilon_j(k) \notin [v_{dj} + \alpha_1 - \mu_j, v_{dj} + \alpha_2 + (\mu_j + c_3)\sum_{s=1}^{N} \frac{\mu_s - \alpha_1 + \alpha_2}{\mu_s + c_3}]$, and $V^{B_N} = 0$ $\forall \varepsilon_j(k) \in [v_{dj} + \alpha_1 - \mu_j, v_{dj} + \alpha_2 + (\mu_j + c_3)\sum_{s=1}^{N} \frac{\mu_s - \alpha_1 + \alpha_2}{\mu_s + c_3}]$, where $s \neq j$. Thus, $\limsup_{k\to\infty} V_k^{B_N} = 0$, which completes our proof.