World Academy of Science, Engineering and Technology
International Journal of Civil and Environmental Engineering
Vol:7, No:4, 2013

Structural Cost of Optimized Reinforced Concrete Isolated Footing

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***Abstract*—**This paper presents an analytical model to estimate the cost of an optimized design of reinforced concrete isolated footing base on structural safety. Flexural and optimized formulas for square and rectangular footingare derived base on ACI building code of design, material cost and optimization. The optimization constraints consist of upper and lower limits of depth and area of steel. Footing depth and area of reinforcing steel are to be minimized to yield the optimal footing dimensions. Optimized footing materials cost of concrete, reinforcing steel and formwork of the designed sections are computed. Total cost factor TCF and other cost factors are developed to generalize and simplify the calculations of footing material cost. Numerical examples are presented to illustrate the model capability of estimating the material cost of the footing for a desired axial load.

***Keywords*—**Footing, Depth, Concrete, Steel, Formwork, Optimization, Material cost, Cost Factors.

I. Introduction

SAFETY and reliability were used in the flexural design of reinforced concrete footings using ultimate-strength design method used under the provisions of aci building code of design [1, 2, 3, 4]. Footings are very important structure members and the most common shape of reinforced concrete footing is square footing. Footing sizes are mostly governed by the axial load p, allowable soil pressure qa, unit weight of concrete γc, soil unit weight γs, and the depth of the footing base below the final grade. The optimized dimensions of reinforced concrete footing could be achieved by minimizing the optimization function of slab depth and reinforcing steel area, fig. 1, [5, 6, 7].

This paper presents an analytical model to estimate the cost of an optimized design of reinforced concrete footings with yield strength of nonprestressed reinforcing steel bars$\boldsymbol{f_y}$ and compression strength of concrete $\boldsymbol{f`_c}$ base on shear and flexural capacities of the footing. The optimization of footings is formulated to achieve the best footing dimension that will give the most economical section to resist the external axial loads **P** that is made of summation of dead loads **DL** and live loads **LL**. The optimization is subjected to the design constraints of the building code of design ACI such as maximum and minimum reinforcing steel area, footing depth, developmental length in tension and compression, [8, 9, 10].

The total cost of the footing materials is equal to the summation of the cost of concrete, steel and formwork. Total cost factor TCF, cost factor of concrete CFC, Cost Factor of steel CFS, and cost factor of timber CFT are developed to generalize and simplify the estimation of footing material cost. The required footing areaF_A is computed based on the axial load P and the effective soil pressure Qe:

$$F_A = \frac{P}{Q_e} = \frac{DL + LL}{Q_e} \quad (1)$$

$$Qe = Qa - Wc - Ws \quad (1a)$$

$$Wc = \gamma c * h \quad (1b)$$

$$Ws = \gamma s * D_f \quad (1c)$$

where
F_A = Footing Area
Qa = Allowable Soil Pressure
Qe =Effective Soil Pressure
Wc = Concrete Weight
Wc = Soil Weight
h = Total Footing

A. Shear and Flexural Design Formulas

Both one-way shear and two-way shear are considered for estimating the footing effective depth, Fig. 2.

The ACI design code 2008 formulas for one way shear depth:

$$d_{one\,way} = \frac{Vu}{\varphi_s\ v_c\ \ b_w} (\text{ACI Eq. } 11 - 3)$$

where
d = Effective depth
Vu =Factored shear force
φ_s= Shear reduction factor
v_c = Shear stress carried by the concrete
b_w =Footing width

The ACI design code 2008 formulas for two way shear depth, is the largest value obtained from the following equations:

$$d_{2wI} = \frac{6\,Vu}{\varphi_s\ \left(1 + \frac{8}{\beta_C}\right)\sqrt{f`c}\,b_o} (\text{ACI Eq. } 11 - 31)$$

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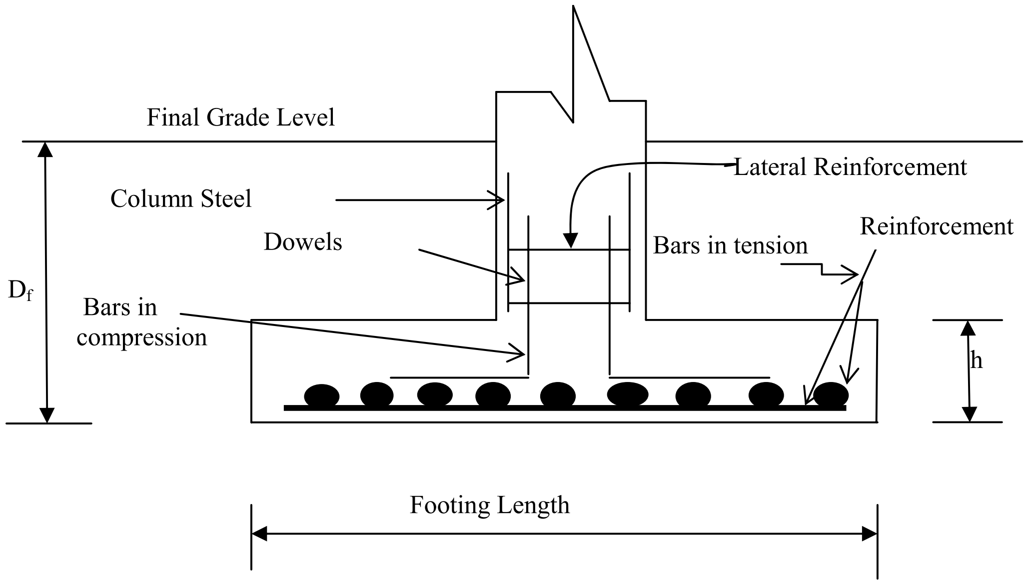

Fig. 1 Isolated Footing Dimensions and Reinforcement Detailing

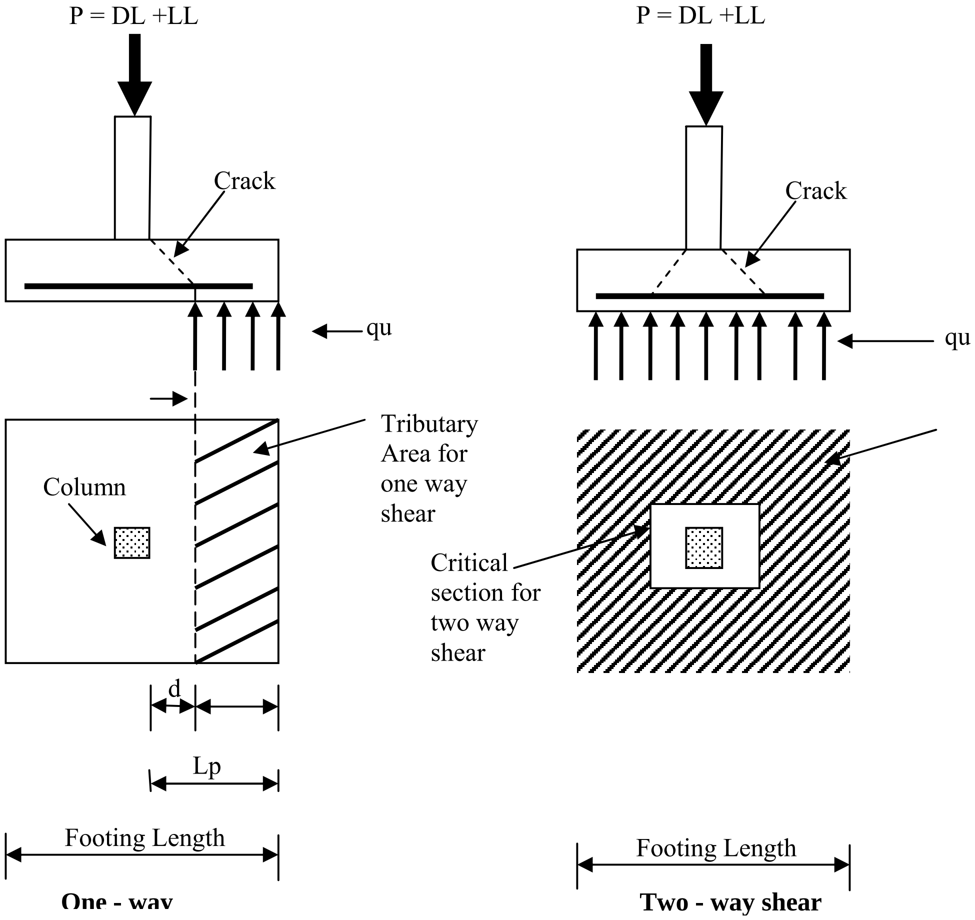

Fig. 2 Shear in footing

The ACI design code 2008 formulas for one way shear depth:

$$d_{one\,way} = \frac{Vu}{\varphi_s\ v_c\ b_w} \text{(ACI Eq. 11 − 3)}$$

where
d = Effective depth
Vu =Factored shear force
φ_s= Shear reduction factor
v_c = Shear stress carried by the concrete
b_w =Footing width

The ACI design code 2008 formulas for two way shear depth, is the largest value obtained from the following equations:

$$d_{2wI} = \frac{6\,Vu}{\varphi_s\ \left(1+\frac{8}{\beta_C}\right)\sqrt{f`c}\,b_o} \text{(ACI Eq. 11 − 31)}$$

$$d_{2wII} = \frac{12\,Vu}{\varphi_s\ \left(2+\frac{\propto_S}{b_o}\right)\sqrt{f`c}\,b_o} \text{(ACI Eq. 11 − 32)}$$

$$d_{2wIII} = \frac{3\,Vu}{\varphi_s\ \sqrt{f`c}\,b_o} \text{(ACI Eq. 11 − 33)}$$

where
β_C =Ratio of long side of the column to the short of the column
$f`c$ = Specified compression strength of concrete
b_o = Perimeter around the punching area
$\propto_S$= Ratio equals to 40, 30 and 20 for interior column, edge column and corner column respectively

The bending moments Mu in both axes are considered at the face of the column Fig. 3.

$$Mu = \frac{L_p^2}{2} q_u b_w \tag{2}$$

$$qu = \frac{Pu}{F_A} = \frac{DL*DLF+LL*LLF}{F_A} \tag{2a}$$

where
Mu = Fully factored bending moment
L_p = Projected length
q_u = Bearing pressure for strength design
DLF = Dead Load Factor equal 1.2
LLF = Live Load Factor equal 1.6

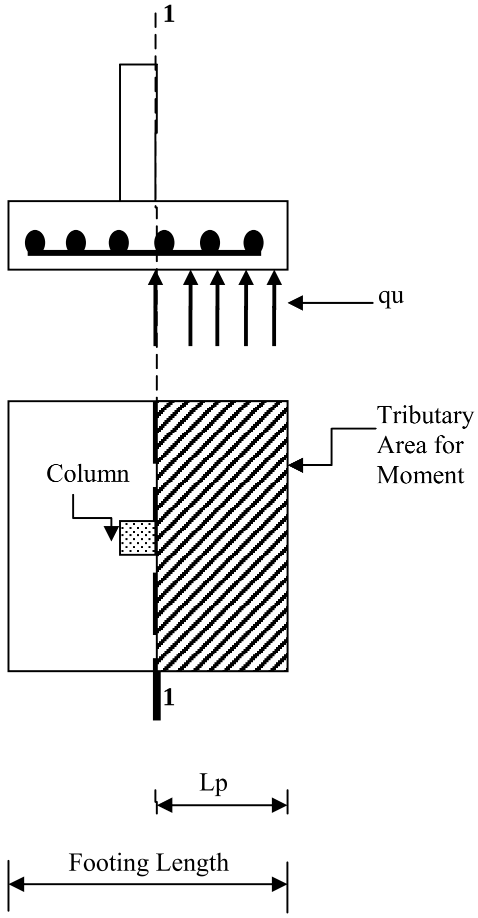

Fig. 3 Moment in footing about section 1-1

The reinforcement area As of the footing

$$As = \frac{Mu}{\varphi_b\ fy\ \left(d-\frac{a}{2}\right)} \tag{3}$$

where
φ_b= Bending reduction factor
fy = Specified yield strength of nonprestressed reinforcing
As = Area of tension steel
d = Effective depth
a =Depth of the compression block

B. Footing Optimization

The optimization of isolated footing is formulated to achieve the best footing dimensions that will give the most economical footing size and steel reinforcement to resist the ultimate bending moment Mu. The optimization is subjected to the constraints of the building code of design ACI for depth, reinforcement and footing size. The optimization function of the footing
Minimize

$$F(As,d) = \varphi_b\ As\ fy\ \left(d-\frac{a}{2}\right) - M_u \tag{4}$$

Must satisfy the following constraints:

$$d_S^L \leq d \leq d_S^U \tag{4a}$$

$$As_S^{Mini} \leq As \leq As_S^{Max} \tag{4b}$$

$$As^{Max} = 0.75 * \beta 1 * \frac{f`c}{fy}\left(\frac{600}{600+fy}\right)bd \tag{4c}$$

$$As^{Mini} = \left(\frac{1.4}{fy}\right)bd \tag{4d}$$

$$\beta 1 = 0.85 \;\; for \;\; f`c \leq 30\, MPa \tag{4e}$$

$$\beta 1 = 0.85 - 0.008(f`c - 30) \geq 0.65 \;\; for \;\; f` > 30\, MPa \tag{4f}$$

where d_B^L and d_B^L are footing depth lower and upper bounds, and As_B^{Mini} and As_B^{Max} are slab steel reinforcement area lower and upper bounds. The reinforcing bars must have the required length to provide sufficient strength. In other words, the bars must extend developmental length Ld from the face of the column.

$$Ld < LdT_{AVAILABLE} \tag{4g}$$

where
$Ld =$ Required bar developmental length
(ACI Section 12.2)

$LdT_{AVAILABLE}$ =Available length in tension
(ACI Section 12.3)

For the dowel bars under compression

$$As_{dowels} \geq 0.005\, A_{Column} \tag{4h}$$

$$Ld_{Comp} < LdC_{AVAILABLE} \tag{4i}$$

$$h > Ld_{Comp} + Cover + 2d_b \tag{4j}$$

where
$As_{dowels} =$ Steel area of the dowels
$A_{Column} =$ Column area
$Ld_{Comp} =$ Required bar developmental length in compression

$LdC_{AVAILABLE} =$ Available length in compression
$h =$ Total footing depth
$Cover$ =Concrete cover thickness
$d_b =$ Bar diameter

C. Footing Formwork Materials

The form work material is limited to footing four sides of 20 mm thickness, Fig. 4. The formwork area AF of the isolated footing

$$AF_{SQUARE} = 4(0.2(B * h)) \tag{5}$$

$$AF_{RECTANGULAR} = 2 * 0.2 * h(B + L) \tag{5a}$$

D. Footing Cost Analysis

The total cost of the beam materials is equal to the summation of the cost of the concrete, steel and the formwork per square meter:

$$\frac{Total\ Cost}{m^2} = \frac{Ag(m^2)}{m} * Cc + \frac{As(m^2)}{m} * \frac{\gamma_s\left(\frac{Ton}{m^3}\right)}{m} * Cs + \frac{AF(m^2)}{m} * Cf \tag{6}$$

where
Cc = Cost of 1 m^3 of ready mix reinforced concrete in dollars
Cs = Cost of 1 Ton of steel in dollars
Cf = Cost of 1 m^3timber in dollars
γ_s = Steel density = $7.843\,\frac{Ton}{m^3}$

Total Cost Factor TCF and other cost factors are developed to generalize and simplify the calculations of footing material cost.

$$CFC = \frac{(Concrete\ Cost)}{m^2} = \frac{Ag(m^2)}{m} * Cc \tag{7}$$

$$CFS = \frac{Steel\ Cost}{m^2} = \frac{As(m^2)}{m} * \gamma_s\left(\frac{Ton}{m^3}\right) * Cs \tag{8}$$

$$CFT = \frac{Timber\ Cost}{m^2} = \frac{AF(m^2)}{m} * Cf \tag{9}$$

and

$$TCF = CFC + CFS + CFT = \frac{Total\ Cost}{m^2} \tag{10}$$

where
CFC = Cost Factor of Concrete
CFS = Cost Factor of Steel
CFT = Cost Factor of Timber
TCF = Total Cost Factor

Therefore the total cost of the footing is equal to the product of the total cost factor TCF and the footing area.

Fig. 4 Footing formwork material – Four sides

TABLEI
FOOTING DIMENSIONS

Axial Design Loads kN		Soil Pressure Qa	Footing Dimensions					
			Analytical Model			Software STAAD Foundation		
Service Load kN	Ultimate Load kN	kN/m^2	Depth mm	Steel Area mm^2	Length m	Depth mm	Steel Area mm^2	Length m
500	700	200	340	1502	1.7	350	1863	1.63
900	1260		455	2913	2.3	440	3310	2.19
1400	1960		550	4400	2.9	540	4972	2.74
2100	2940		640	6686	3.55	650	7218	3.4
2500	3500		700	8125	3.9	700	8687	3.7
3200	4480		780	1034	4.4	800	10604	4.2

TABLE II
FOOTING TOTAL COST

Axial Design Loads kN		Footing Area	Total Cost $	
Service Load DL + LL kN	Ultimate Load DL(1.2) + LL(1.6) kN	m^2	Qatar	USA
800	1120	4.84	396	409
1500	2100	9	958	990
2000	2800	12.25	1457	1505
2600	3640	16	2100	2177
3300	4620	20.25	3037	3135
4000	5600	25	4044	4174

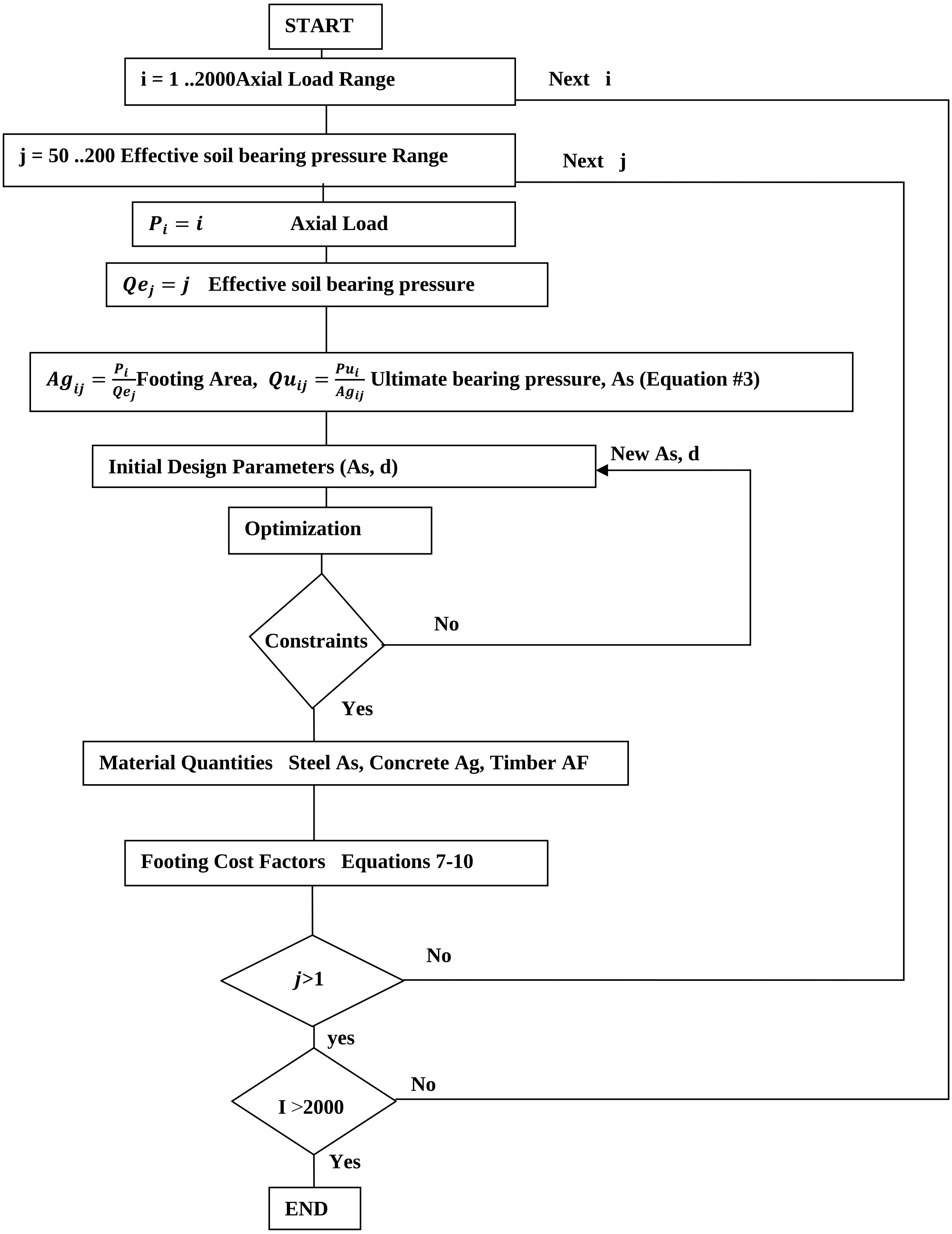

Fig. 5 The Process of Computing Cost Factors

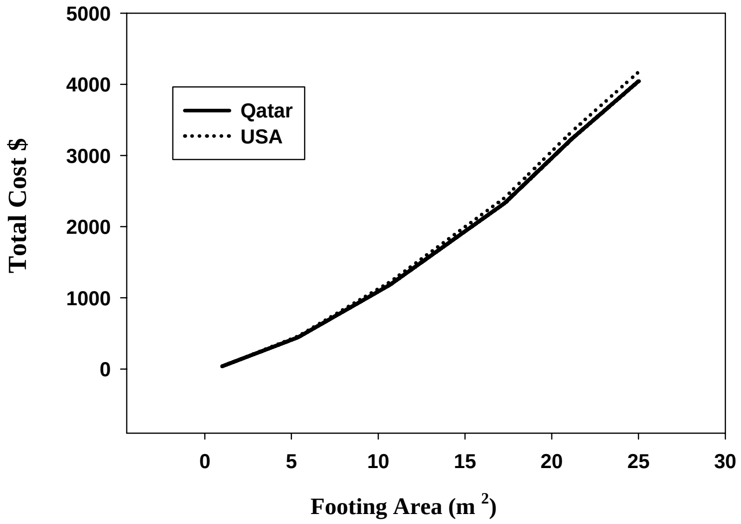

Fig. 6 Total Material Cost of Isolated Footing $

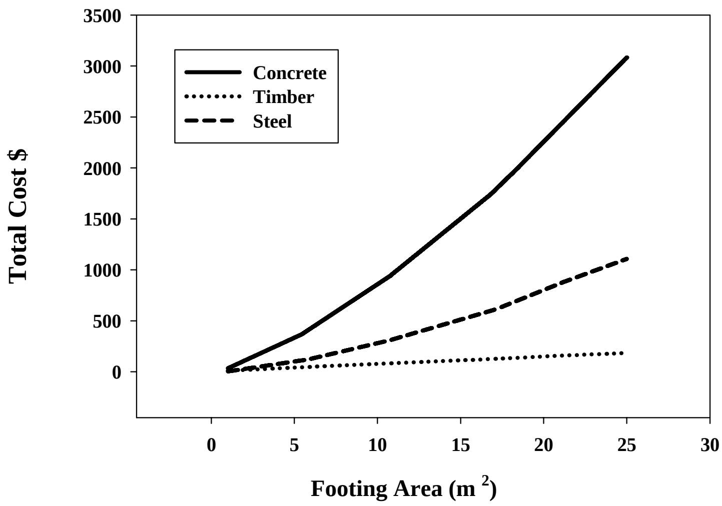

Fig. 7 Material Cost of Isolated Footing

II. Result and Discussion

The footing were analyzed and designed optimally to ACI code of design in order to minimize the total cost of the footing that includes cost of concrete, cost of steel, and formwork cost. The footings were subjected to different dead load DL, live load LL, and effective soil pressure Qe. In order to optimize the footing size, thickness, and steel area, a list of constraints (equations 4-4J) that contain shear, bending and developmental length have to be met. Areas of concrete, reinforcing steel and timber area AF (equations 5-5A) are computed based on optimum footing dimensions. The formwork area AF of the slab cross section is made of four vertical sides of 20mm thickness and height of footing total depth. The total cost of footing material is calculated using equation6 base on Qatar and USA prices respectively of \$100,\$131 for 1 m3 of ready mix concrete, \$1070,\$1100 for 1 ton of reinforcing steel bars, and \$531.\$565 for 1 m3 of timber. Total Cost Factor TCF, Cost Factor of concrete CFC, Cost Factor of steel CFS, and Cost Factor of Timber CFT, are developed in equations 7 - 10 to generalize and simplify the calculation of footing material cost. To determine the cost factors that are to be used for estimating the footing material cost, an iterative cost safety procedure of estimating the footing material cost base on optimal criteria is applied to external axial load range of 10kN to 2000kNas the maximum axial load, Fig. 5. The design parameter used in iterative cost procedure are 400 MPa, 30 MPa, 200KPa, 25kN/m3 , 15kN/m3 and 1meter for fy, f'c, Qa, γc, γs and Df respectively. In order to verify the design results made by the analytical method a comparison of footing designed by the analytical method with the software STAAD Foundation is presented in Table I [10].

Table II shows the total cost of different footing areas due to different axial loads and allowable soil bearing pressure Qa. As an example, an axial load of 1500kN and allowable soil pressure of 200kN/m^2 will require a footing cost $1457 and $1505in Qatar and USA prices respectively, Fig. 6.

III. CONCLUSION

Footing optimum design analytical model is developed to estimate the cost of footing materials base on various design constraints. Total cost factor TCF, cost factor of concrete CFC, Cost Factor of steel CFS, and cost factor of timber CFT are developed and presented as formulas to approximate material cost estimation of optimized reinforced concrete footing base on ACI code of design. Cost factors were used to produce footing cost charts that relate the required footing area F_Adue to axial load, soil bearing pressure, and other design parameters to the slabmaterial cost. The model is flexible and could be used for other codes of design by modifying equations of shear, flexural and optimization to check cost estimates for isolated footings.

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