

Realization of Fractional-Order Capacitors with Field-Effect Transistors

Steve Hung-Lung Tu, and Yu-Hsuan Cheng

Abstract—A novel and efficient approach to realize fractional-order capacitors is investigated in this paper. Meanwhile, a new approach which is more efficient for semiconductor implementation of fractional-order capacitors is proposed. The feasibility of the approach has been verified with the preliminary measured results.

Keywords—Fractional-order, field-effect transistors, RC transmission lines.

I. INTRODUCTION

IN recent years, many applications of fractional-order capacitors have been proposed [1]-[5]. On the electrical side, the behaviors of traditional capacitors and inductors are governed by the first-order differential equations. Hence, any system constructed with such n^{th} -order elements is described by an n^{th} -order system of differential equations. With Laplace transform, it also generates n^{th} -order terms to represent the equation. The applications of fractional-order operations such as fractional derivatives and integrals (FDIs) have motivated the theory of chaos and their corresponding research on viscoelasticity / damping [6]-[10], and chaos / fractals [11]-[14] as well as the state-of-the-art areas such as biology [15], electronics [16], control [17]-[19], and signal processing [20]-[21].

Four decades ago, the feasibility of realizing a fractional-order capacitor has been investigated [22]-[23]. A finite element approximation of the special case $Z = 1/C(j\omega)^{1/2}$, was reported in [24]. This finite element approximation relies on the possibility of emulating a fractional-order capacitor via semi-infinite self-similar RC trees. The technique was later developed further [25]-[27] in order to approximate any $Z = 1/C(j\omega)^{\pm\alpha}$, in which α is arbitrary and the device with such an impedance was termed “fractance device”.

Although these fractional-order element realization approaches were developed, all of them are based on an approximation method, which implies that a certain amount of discrete components are required to implement the fractional-order impedance device. Until more recently, the implementation of two-terminal fractional device was reported in [28]-[29]. In which the implemented probe was based on a

metal-insulator liquid interface and was employed in [29] to realize a fractional-order differentiator circuit. The approach may develop the real fractional element whereas it is difficult to implement with integrated circuits. In this paper, we therefore renew the development of discovery of the fractional-order capacitor and take advantage of the nonlinear characteristics of FETs to simulate the RC transition lines as the fractional-order capacitors [30].

To the best of our knowledge, this is the first time that FETs are announced to be employed as the fractional-order elements. Moreover, in this paper we will investigate the large-signal frequency response and also we will propose a new approach to implement fractional-order capacitors.

II. RC TRANSMISSION LINE WITH FETs

The conducting channel of a FET operating at triode region can be modeled as a nonlinear RC transmission line as shown in Fig.1[30], in which the resistance and capacitance per unit of line length, $R(v)$ and $C(v)$ are functions of the voltage v . The instantaneous values of voltage and current are given by

$$i_{x,t} = -\frac{1}{R(v_{x,t})} \frac{\partial v_{x,t}}{\partial x} \quad (1)$$

and

$$\frac{\partial i_{x,t}}{\partial x} = -\frac{\partial}{\partial t} \int_0^{v_{x,t}} C(v_{x,t}) dv_{x,t} \quad (2)$$

For an insulated gate FET, the resistance $R(v)$ and capacitance $C(v)$ can be derived as follows,

$$R(v) = \frac{L}{\mu C_t (v - V_0)} \quad (3)$$

$$C(v) = \frac{C_t}{L} \quad (4)$$

Note that L is the length of the channel, μ is the mobility of carriers in the channel, C_t is the total gate-channel capacitance, and V_0 is the threshold voltage (pinch-off).

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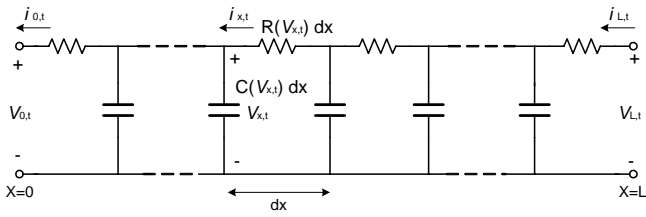


Fig. 1 Lumped equivalent circuit of a nonlinear RC line

Large-signal transient response analysis presented in [30] demonstrated the same steady-state behavior as that of the RC line itself, which the series distributed resistance was modeled with n lumped resistors and the shunt distributed capacitance was also modeled with $n-1$ lumped capacitors. As a result, by the employment of a large number of resistors and capacitors for the lumped equivalent circuit and letting the node $x=0$ grounded, we can obtain the impedance $Z(s)$ at node $x=L$ as shown in Fig. 2(a). Also by rearranging the circuit, a recursive circuit structure can be obtained as shown in Fig. 2(b).

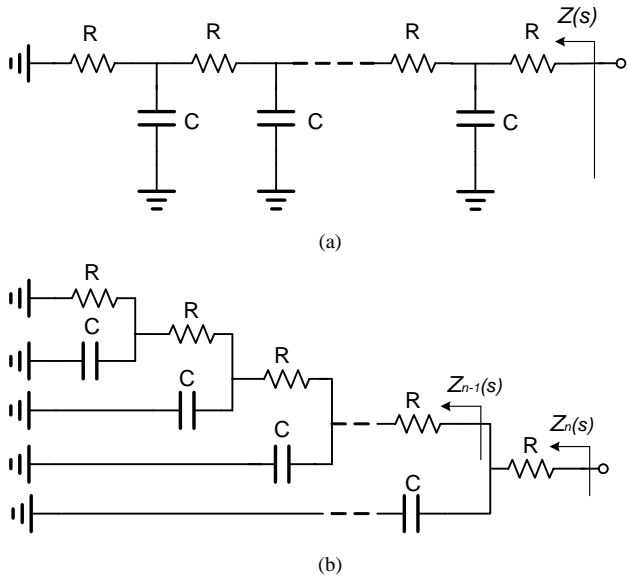


Fig. 2 Lumped equivalent circuit of a FET with one terminal grounded

Notice that for a large number of R and C , the impedance $Z_n(s)$ is approximately equal to $Z_{n-1}(s)$. Therefore, for $Z_n(s) = Z_{n-1}(s) = Z(s)$, we can obtain

$$\left(Z(s) \parallel \frac{1}{sC} \right) + R = Z(s) \quad (5)$$

By letting $\frac{4}{sRC} \gg 1$ yields

$$\begin{aligned} Z(s) &= \frac{1}{2} \left(R + 2\sqrt{\frac{R}{C}} \cdot \frac{1}{s^{1/2}} \right) \\ &= \frac{1}{2} \left[R + 2\sqrt{\frac{R}{C}} \omega^{-1/2} \exp(-j\pi/4) \right] \end{aligned} \quad (6)$$

where $s = j\omega$. Obviously, by dividing the nonlinear RC transmission line into an infinite number of sections, we can obtain a fractional-order capacitance as indicated in Equation (6).

A preliminary Hspice simulation has been performed with a single FET M1 operating at triode region. The configuration shown in Fig. 3 is employed to investigate the fractional-order grounded capacitor. We also let the drain as port 1 and the grounded-source as port 2 with the biased voltage at the gate.

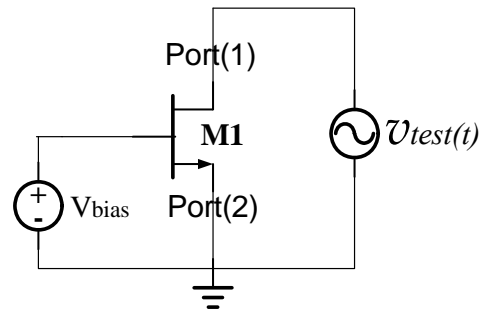


Fig. 3 Configuration of a FET operating at triode region acts as a fractional-order grounded capacitor

Fig. 4(a) and (b) show the simulated results with the biased voltages $V_{GS}=1V$ and $2V$, respectively. Obviously, from the S_{11} plots the resistance and capacitive reactance of the impedance $Z(s)$ decrease with an increase of frequency, which can be explained with Equation (6).

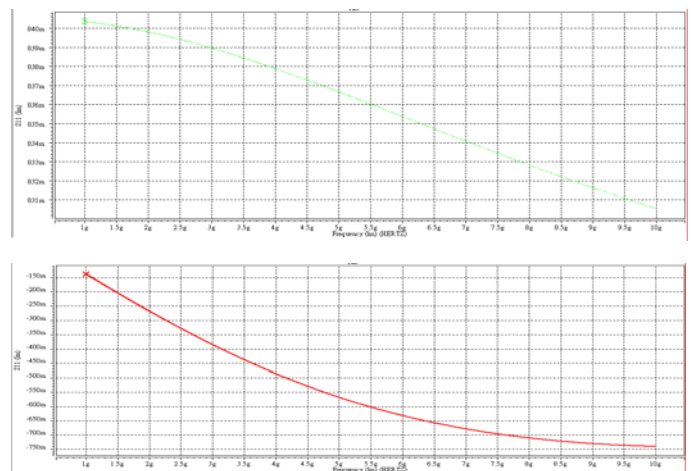
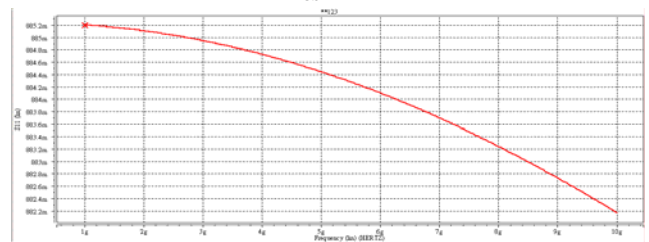


Fig. 4 (a) S_{11} plots of magnitude (upper plot) and phase (lower plot) for $V_{GS}=1V$



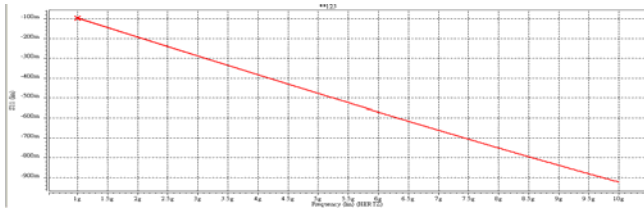


Fig. 4 (b) S_{11} plots of magnitude (upper plot) and phase (lower plot) for $V_{GS}= 2V$

III. IMPLEMENTATION OF FRACTIONAL-ORDER CAPACITORS WITH FIELD-EFFECT TRANSISTORS

To gain more insight, from Equation (6) the resistance and capacitive reactance of the impedance $Z(s)$ decrease with an increase of frequency. Fig. 5 shows the measured S_{11} for the device of enhancement-mode pHEMT FET, which the frequency range of the plot in a $50-\Omega$ system is from 1.9GHz to 4.5GHz for the biased voltage of $V_{GS}=0.8V$ with the same configuration shown in Fig. 3. For the gate-to-source biased voltage, the resistance and capacitive reactance of the impedance $Z(s)$ decrease with an increase of frequency. Due to packaging inductive effect, the reactance becomes positive for the operating frequency over 4.5GHz.

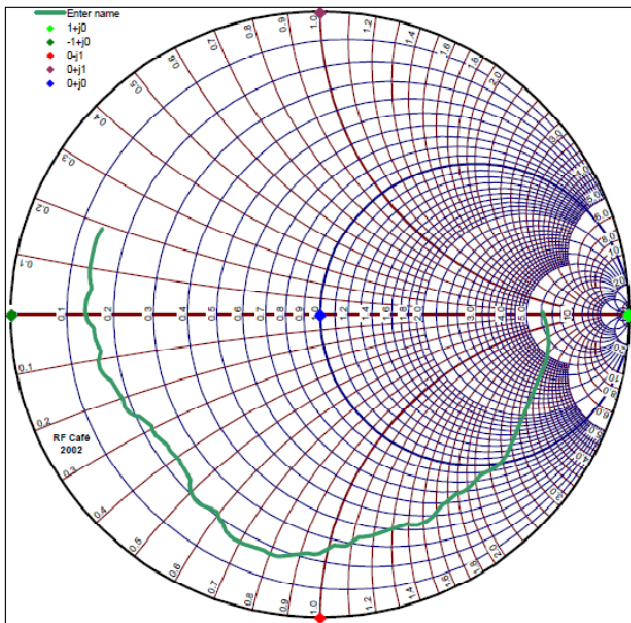


Fig. 5 S_{11} plot with signal frequencies from 1.9GHz to 4.5GHz for gate-to-source voltage $V_{GS}=0.8V$

Moreover, the resistance R and capacitance C can be investigated with the parasitic resistance of the conductive channel and the gate-to-channel capacitance for different V_{GS} . For a higher V_{GS} , due to the higher biased voltage, the channel resistance will be lower. Fig.6 (a) and (b) show the resistance and reactance of the measured impedances for three different biased voltages, respectively. Due to the additional packaging parasitic reactance, the imaginary-part is a bit higher than real-part of the measured results whereas the resistance $Re(Z(s))$

and capacitance $Im(Z(s))$ of the fractional-order impedance decrease with an increase of frequency. Moreover, for a higher biased voltage V_{GS} the channel resistance will be lower and in turn, from Equation (6) the impedance will be lower.

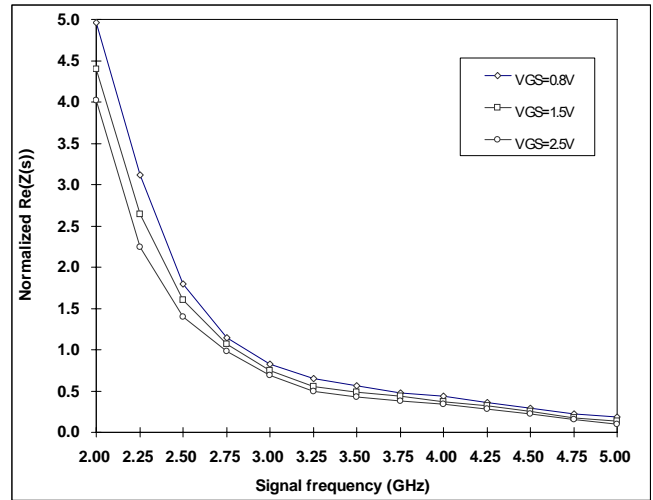


Fig. 6 (a) Normalized real-part of fractional-order impedance versus frequency for different gate-to-source biased voltages

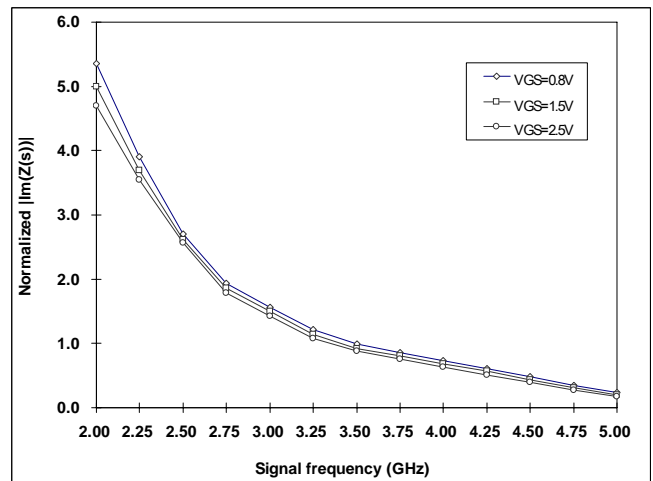


Fig. 6 (b) Normalized absolute value of imaginary-part of fractional-order impedance versus frequency for different gate-to-source biased voltages

IV. CONCLUSION

This paper has explored the feasibility of monolithic realization of fractional-order capacitors. We obtain a good agreement between the mathematics derivation and measured results. Obviously, the advantage of this approach is the possibility to realize the fractional-order devices and some high-performance systems with integrated circuits.

REFERENCES

- [1] R. Yadav, M. Gupta, "Design of fractional order differentiators and integrators using indirect discretization scheme," pp. 1-6, *India International Conference on Power Electronics*, Jan. 2011.
- [2] D. Mondal, K. Biswas, "Performance study of fractional order integrator using single-component fractional order element," *IET Circuits, Devices & Systems*, Vol. 5, No. 4, pp. 334-342, July 2011.

- [3] Z. Gao, X. Liao, "Improved Oustaloup approximation of fractional-order operators using adaptive chaotic particle swarm optimization," *Journal of Systems Engineering and Electronics*, Vol. 23, No. 1, pp. 145-153, Feb. 2012.
- [4] D. A. Robinson, "Lateral directional fractional order (PI)ⁿ control of a small fixed-wing unmanned aerial vehicles: controller designs and flight tests," *IET Control Theory & Applications*, Vol. 5, No. 18, pp. 2156-2167, Dec. 2011.
- [5] W. Ahmad, R. El-khazali, A. S. Elwakil, "Fractional-order Wien-bridge oscillator," *IET Electronics Letters*, Vol. 37, pp. 1110-1112, 2001
- [6] P. Xie, Z. Xu, Q. Zou, "Compensation for the dynamics effect on nanoscale broadband viscoelasticity measurements using adaptive filtering approach," *IEEE Transactions on Instrumentation and Measurement*, Vol. 60, No. 4, pp. 1155-1162, April 2011.
- [7] N. Markris, "Dynamic analysis of viscoelastic-fluid dampers," *Journal of Engineering Mechanics*, Vol. 121, No. 10, pp. 1114-1121, 1995.
- [8] B. S. Liebst, "Asymptotic approximations for systems incorporating fractional derivative damping," *ASME Journal of Dynamic Systems, Measurement, and Control* Vol. 118, pp. 572-579, 1996.
- [9] A. Fenander, "Modal synthesis when modeling damping by use of fractional derivatives," *AIAA Journal*, Vol. 34, No. 5, pp. 1051-1058, 1996.
- [10] X. Lei, F. Feng, J. Sun, "Optimal damping of EMI filter input impedance," *IEEE Transactions on Industry Applications*, Vol. 47, No. 3, pp. 1432-1440, May 2011.
- [11] J. P. Clerc, "A. C. response of fractal networks," *Le Journal de Physique-Lettres* Vol. 45, No. 19, pp. 913-924, 1984.
- [12] S. H. Liu, "Fractal model for the AC response of a rough interface," *Physical Review Letters* Vol. 55, No. 5, pp. 529-532, 1985.
- [13] K. Theodore, "Self-AÆne fractal model for a metal-electrolyte interface," *Physical Review B* Vol. 35, No. 10, pp. 5379-5381, 1987.
- [14] A. L. Mehaute, *Fractal Geometries: Theory and Applications*. Penton Press, 1991.
- [15] T. J. Anastasio, "The fractional-order dynamics of brainstem vestibulo-oculomotor neurons," *Biological Cybernetics*, Vol. 72, pp. 69-79, 1994.
- [16] A. Oustaloup, "Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulation," *IEEE Transactions on Circuits and Systems*, Vol. 28, No. 10, pp. 1007-1009, 1981.
- [17] I. Podlubny, *Fractional Differential Equations*. Academic Press, San Diego, 1999.
- [18] J. A. T. Machado, "Analysis and design of fractional-order digital control systems," *SAMS-Journal Systems Analysis, Modeling, Simulation*, Vol. 27, pp. 107-122, 1997.
- [19] J. A. T. Machado, "Position/force fractional control of mechanical manipulators," *IEEE International Workshop on Advanced Motion Control*, 1998.
- [20] J. R. M. Hosking, "Fractional differencing," *Biometrika*, Vol. 68, No. 1, pp. 165-176, 1981.
- [21] M. Haldun, "Digital computation of the fractional Fourier transform," *IEEE Transactions on Signal Processing*, Vol. 44, No. 9, pp. 2141-2150, 1996.
- [22] S. Roy, "On the realization of a constant-argument immittance or fractional operator," *IEEE Transactions on Circuits and Systems*, Vol. 14, pp. 264-274, 1967.
- [23] G. Carlson and C. Halijak, "Approximation of fractional capacitors (1/s)^{1/n} by a regular Newton process," *IEEE Transactions on Circuits and Systems*, Vol. CAS-11, No. 2, pp. 210-213, Mar. 1964.
- [24] K. Steiglitz, "An RC impedance approximation to s^{-1/2}," *IEEE Trans. Circuits Syst.*, Vol. 11, pp. 160-161, 1964.
- [25] M. Nakagawa, K. Sorimachi, "Basic characteristics of a fractance device," *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, Vol. 12, pp. 1814-1819, 1992.
- [26] K. Saito and M. Sugi, "Simulation of power-law relaxations by analog circuits: fractal distribution of relaxation times and non-integer exponents," *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, E76, Vol. 2, pp. 205-209, 1993.
- [27] M. Sugi, Y. Hirano, Y. F. Miura and K. Saito, "Simulation of fractal immittance by analog circuits: An approach to the optimized circuits," *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, E82, Vol. 8, pp. 1627-1634, 1999.
- [28] K. Biswas, S. Sen, and P. Dutta, "Modelling of a capacitive probe in apolarizable medium," *Sens. Actuat. Phys.*, Vol. 120, pp. 115-122, 2005.
- [29] K. Biswas, S. Sen, and P. Dutta, "Realization of a constant phase element and its performance study in a differentiator circuits," *IEEE Circuits Syst. II, Exp. Briefs*, Vol. 53, No. 8, pp. 802-806, Aug. 2006.
- [30] V. P. Popov and T. A. Bickart, "RC transmission line with nonlinear resistance: large-signal response computation," *IEEE Circuits Syst. Exp. Briefs*, Vol. 21, No. 5, pp. 666-671, Sep. 1974.