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Performance Analysis of Space-Time Trellis Coded OFDM System

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Abstract— **This paper presents the performance analysis of space-time trellis codes in orthogonal frequency division multiplexing systems (STTC-OFDMs) over quasi-static frequency selective fading channels. In particular, the effect of channel delay distributions on the code performance is discussed. For a STTC-OFDM over multiple-tap channels, two extreme conditions that produce the largest minimum determinant are highlighted. The analysis also proves that the corresponding coding gain increases with the maximum tap delay. The performance of STTC-OFDM, under various channel conditions, is evaluated by simulation. It is shown that the simulation results agree with the performance analysis.**

Keywords: Space-time trellis code, OFDM, delay profile.

I. Introduction

Space-time trellis coding (STTC) technique has been proposed to achieve both the diversity and coding gains in multi-input multi-output (MIMO) fading channels [1]. The orthogonal frequency division multiplexing (OFDM) technique is currently widely used to combat intersymbol interference (ISI) by transforming a frequency selective fading channel into a set of parallel correlated flat fading channels. Recently, various STTCs in OFDM systems, referred to as STTC-OFDMs, in frequency selective fading channels have been investigated [2][3][4][5].

The diversity gain of STTC-OFDM systems is investigated in [3] and [4]. It was pointed out in [4] that the performance of space-time coded OFDM systems depends on the channel delay profile. To reduce this dependence and simplify the code design, *ideal* interleaving was usually used to scramble the coded symbols.

In contrast to the analysis in [4] with ideal interleaving, the worst case, where no interleaving is employed in the transmitter, was considered in [5], as the optimization for the worst case can provide a robust system design [6]. For STTC-OFDMs in quasi-static frequency selective fading channels, the maximum possible diversity gain is the product of the number of transmit antennas n_T, the number of receive antennas n_R and the number of the channel taps L [3]. Since the low memory order STTCs in OFDM systems cannot achieve the maximum possible diversity gain [3], the performance of STTC-OFDM is analyzed in terms of the coding gain [5]. However this analysis only applies to the STTCs with the minimum error event length p_{min} of 2. This results in a restriction since some STTCs, especially high memory order codes, have the minimum error event length p_{min} greater than 2 [6].

In this paper, we will address this issue. We extend the analysis in [5] to the general case, in which STTCs have the minimum error event length p_{min} no less than 2. Then the effect of the channel delay distribution on the coding gain is discussed. The code performance of STTC-OFDM over quasi-static frequency selective fading channels is evaluated by simulations. It is shown that the simulation results agree with the performance analysis.

This paper is organized as follows. Section II introduces the system model. Section III presents the pairwise error probability (PWEP) of the STTC-OFDMs in quasi-static frequency selective fading channels. In Section IV, the code performance of STTC-OFDM based on the diversity and coding gains has been investigated. In particular, the effect of various channel delay distributions on the code performance is discussed in terms of the coding gain. Section V presents the simulation results. Finally, conclusions are drawn in Section VI.

II. System Model

We consider an OFDM system with n_T transmit antennas, n_R receive antennas and K subcarriers in a quasi-static frequency selective fading channel. Each OFDM frame consists of $n_T K$ M-PSK STTC symbols, where the encoded symbol $x_i(k)$, $i \in \{1, 2, ..., n_T\}$, $k \in \{1, 2, ..., K\}$, is transmitted on the k-th subcarrier from the i-th transmit antenna. After matched filtering, sampling and fast Fourier transform (FFT), the received signal at the j-th receive antenna and on the k-th subcarrier is given by

$$r_j(k) = \sum_{i=1}^{n_T} H_{ij}(k) x_i(k) + n_j(k), \quad (1)$$

where $H_{ij}(k)$, $j \in \{1, 2, ..., n_R\}$, denotes the channel frequency response from the transmit antenna i to receive antenna j and subcarrier k, $n_j(k)$ is the noise component at receive antenna j through subcarrier k, which is an independent complex Gaussian random variable with zero-mean and variance $N_0/2$ per dimension.

The quasi-static fading channel in this paper is assumed to be static during one OFDM frame but varies from one frame to another. The fading channels between different transmit and receive antennas are assumed to be uncorrelated. Assuming the fading channel has L non-zero taps, the time-domain channel impulse response can be modeled by an L tap-delay line [7].

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The channel impulse response from the i-th transmit antenna to the j-th receive antenna is expressed as [3]

$$h_{ij}(\tau,t)=\sum_{l=0}^{L-1}\tilde{h}_{ij}(l,t)\delta\left(\tau-\frac{n_l}{K\Delta f}\right), \qquad (2)$$

where $\delta(\cdot)$ denotes the Dirac delta function, $\tilde{h}_{ij}(l,t)$ denotes the complex amplitude of the l-th non-zero tap with the delay of t_l. The $\tilde{h}_{ij}(l,t)s$ are modeled by the wide-sense stationary (WSS) and narrowband complex Gaussian processes, which are independent for different paths with $E[|\tilde{h}_{ij}(l,t)|^2]=\sigma_l^2$. We normalize the channel power such that we have $\sum_l \sigma_l^2 = 1$.

In (2), n_l is the normalized time delay for the l-th tap and it is given by $n_l = t_l K\triangle f = t_l/T_s$, where $l \in \{0,1,...,L-1\}$, $\triangle f$ is the subcarrier separation, and T_s is the sampling interval of the OFDM systems. We call the delays $[t_0,t_1,...,t_l,...,t_{L-1}]$ of the L non-zero taps the channel delay distribution. Let t_{l_2} and t_{l_1} denote the delays of the l_2-th and l_1-th taps, respectively, where $l_1,l_2\in\{0,1,...,L-1\}$, $l_2>l_1$ and $t_{l_2}>t_{l_1}$. The interval $t_{l_2}-t_{l_1}$ is assumed to be not less than the sampling interval T_s.

Since the performance analysis is done within one OFDM frame, the time index t in (2) is omitted hereafter. A cyclic prefix (CP) with the length of T_{cp}, where $T_{cp}>t_{L-1}$, is appended to each OFDM frame to avoid the ISI. With proper cyclic extension and tolerable leakage, the channel frequency response between the i-th transmit antenna and the j-th receive antenna is given by [3]

$$\begin{aligned} H_{ij}(k) &= \sum_{l=0}^{L-1}\tilde{h}_{ij}(l)e^{-j2\pi kn_l/K} && (3)\\ &= \mathbf{h}_{ij}^{*}\mathbf{w}(k), && (4)\end{aligned}$$

where $\mathbf{h}_{ij} = \left[\tilde{h}_{ij}(0),\tilde{h}_{ij}(1),\cdots,\tilde{h}_{ij}(L-1)\right]^*$ is the channel vector, $\mathbf{w}(k) = [e^{-j2\pi kn_0/K}, e^{-j2\pi kn_1/K}, ..., e^{-j2\pi kn_{L-1}/K}]^T$ is the FFT coefficient vector, $*$ and T denote the Hermitian and transpose operation, respectively. Note that the delay of the first channel tap is $t_0=0$. Thus, the FFT coefficients can be rewritten as

$$\mathbf{w}(k)=[1,e^{-j2\pi kt_1\triangle f},e^{-j2\pi kt_2\triangle f},...,e^{-j2\pi kt_{L-1}\triangle f}]^T. \qquad (5)$$

III. Pairwise Error Probability (PWEP) of STTC-OFDM

Assuming that the perfect channel state information (CSI) is known to the receiver, a codeword $\mathbf{x} = (\mathbf{x}(1),...,\mathbf{x}(k),...,\mathbf{x}(K))$, where $\mathbf{x}(k) = (x_1(k),x_2(k),...,x_{n_T}(k))$, $k\in\{1,2,...,K\}$, is transmitted and erroneously decoded as $\widehat{\mathbf{x}}=(\widehat{\mathbf{x}}(1),\ ...,\ \widehat{\mathbf{x}}(k),\ ...,\ \widehat{\mathbf{x}}(K))$, where $\widehat{\mathbf{x}}(k) = (\widehat{x}_1(k),\ \widehat{x}_2(k),\ ...,\ \widehat{x}_{n_T}(k))$. The pairwise error probability (PWEP) of deciding erroneously using the maximum likelihood decoder (MLD), conditioned on $H_{ij} = [H_{ij}(1)\ ,\ H_{ij}(2)\ ,\ ...\ ,\ H_{ij}(K)]$, $i\in\{1,\ 2,\ ...,\ n_T\}$, $j\in\{1,\ 2,\ ...,\ n_R\}$, is upper bounded by [6]

$$P_r(\mathbf{x}\to\widehat{\mathbf{x}}\mid H_{ij})\le\exp\left(-\frac{E_s}{4N_0}\left(\sum_{j=1}^{n_R}\mathbf{h}_j^{*}\mathbf{D}(\mathbf{x},\widehat{\mathbf{x}})\mathbf{h}_j\right)\right), \qquad (6)$$

where

$$\begin{aligned}\mathbf{h}_j^{*} &= [\mathbf{h}_{1j}^{*},\mathbf{h}_{2j}^{*},...,\mathbf{h}_{n_Tj}^{*}]_{1\times Ln_T}^{*},\\ \mathbf{D}(\mathbf{x},\widehat{\mathbf{x}}) &= \sum_{k=1}^{K}\mathbf{W}(k)\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)\mathbf{W}^{*}(k), \qquad (7)\\ \boldsymbol{\Delta}(k) &= \begin{bmatrix} x_1(k)-\widehat{x}_1(k),\\ x_2(k)-\widehat{x}_2(k),\\ \vdots\\ x_{n_T}(k)-\widehat{x}_{n_T}(k),\end{bmatrix}_{n_T\times 1},\\ \mathbf{W}(k) &= \begin{bmatrix}\mathbf{w}(k) & 0 & \cdots & 0\\ 0 & \mathbf{w}(k) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{w}(k)\end{bmatrix}_{Ln_T\times n_T},\end{aligned}$$

and E_s is the energy per symbol at each transmit antenna.

Averaging the conditioned PWEP in (6) with respect to the Rayleigh fading coefficients, the upper bound of the averaged PWEP is given by [3]

$$\begin{aligned}P_r(\mathbf{x}\ \to\ \widehat{\mathbf{x}}) &\le \left(\prod_{n=1}^{\xi}\left(1+\lambda_n\frac{E_s}{4N_0}\right)\right)^{-n_R} \qquad (8)\\ &= \left(\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\widehat{\mathbf{x}})\right)\right)^{-n_R}\alpha^{-\xi n_R},\end{aligned}$$

where ξ is the rank of matrix $\mathbf{D}(\mathbf{x},\ \widehat{\mathbf{x}})$, λ_n, $n\in\{1,2,...,\xi\}$, are the non-zero eigenvalues of the matrix $\mathbf{D}(\mathbf{x},\ \widehat{\mathbf{x}})$, $\alpha=E_s/4N_0$, $\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}}) = \frac{1}{\alpha}\mathbf{I}+\mathbf{D}(\mathbf{x},\widehat{\mathbf{x}})$, $\mathbf{I}$ is the identity matrix, and $\det(\mathbf{A})$ denotes the determinant of the matrix $\mathbf{A}$. As discussed in [1], we call the minimum value of $\left(\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\widehat{\mathbf{x}})\right)\right)^{1/\xi}$ the coding gain and the minimum value of ξn_R the diversity gain of the system.

In (7), $\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)$ is a rank-one matrix [6]. If the symbols of the codewords $\mathbf{x}$ and $\widehat{\mathbf{x}}$ corresponding to the k-th subcarrier in the given OFDM frame are the same, e.g. $\mathbf{x(k)}=\widehat{\mathbf{x}}(\mathbf{k})$, $\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)$ is an all zero matrix. Otherwise, we obtain $\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)\neq 0$.

Let p denote the length of the pairwise error event path, which is the number of the time instances in the code trellis such that $\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)\neq 0$. The minimum value of p over all possible codeword pairs is denoted by $p_{\min}$. Note that $\xi = rank(\mathbf{D}(\mathbf{x},\ \widehat{\mathbf{x}}))$. We thus obtain $min_{x,\widehat{x}}\ \xi\le min(p_{\min},\ n_TL)$ [6]. To achieve the maximum possible diversity gain n_Rn_TL, it requires that $p_{min}\ge n_TL$. This relation states that increasing the number of channel taps L results in a larger diversity gain. Otherwise, if $p_{min}\le n_TL$, the maximum possible diversity gain n_Rn_TL cannot be obtained [3]. As a consequence, to minimize the error probability, the coding gain, or equivalently, the minimum determinant $\left(\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})\right)\right)^{1/\xi}$ needs to be maximized over all codeword pairs.

Considering (7), it is obvious that both the matrix $\mathbf{D}(\mathbf{x},\ \widehat{\mathbf{x}})$ and its individual matrices $\mathbf{W}(k)\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^{*}(k)\mathbf{W}^{*}(k)$, where

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$k \in \{1,\ 2,\ ...,\ K\}$, are non-negative symmetric Hermitian. According to Minkowski inequality [8], the determinant of the matrix $\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})$ has the following property:

$$\begin{aligned}\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\widehat{\mathbf{x}})\right) &= \det\left(\frac{1}{\alpha}\mathbf{I}+\mathbf{D}(\mathbf{x},\widehat{\mathbf{x}})\right)\\ &= \det\left(\frac{1}{\alpha}\mathbf{I}+\sum_{k=1}^{K}\mathbf{W}(k)\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^*(k)\mathbf{W}^*(k)\right)\\ &\geq \det\left(\frac{1}{\alpha}\mathbf{I}+\sum_{k=1}^{K-1}\mathbf{W}(k)\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^*(k)\mathbf{W}^*(k)\right)\\ \geq ... &\geq \det\left(\frac{1}{\alpha}\mathbf{I}+\sum_{k=1}^{p_{\min}}\mathbf{W}(k)\boldsymbol{\Delta}(k)\boldsymbol{\Delta}^*(k)\mathbf{W}^*(k)\right). \end{aligned} \tag{9}$$

Let $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})\right)|_{p_{\min}}$ denote the minimum determinant of the matrix $\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})$ with the minimum length of the pairwise error event paths, $p_{\min}$. Therefore, to maximize the coding gain, the value of $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})\right)|_{p_{\min}}$ needs to be maximized.

IV. The Effect of Channel Delay Distribution on the Code Performance

In order to investigate the effect of the channel delay distributions on the code performance, we assume that the powers of the multi-paths are fixed while their relative delays are variable.

Consider the determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\ \widehat{\mathbf{x}})\right)|_{p_{\min}}$ with $n_T \geq 2$ and $L \geq 2$ in (10) [8], where $t_{l_2} - t_{l_1}$ is the time interval between the l_1-th and l_2-th taps, $t_{l_2} > t_{l_1}$, $l_2 > l_1$, $i_1, i_2 \in \{1,...,\ n_T\}$, $k_1, k_2 \in \{1,...,\ p_{\min}\}$, and $k_2 > k_1$. In (10), the coefficients β, η and γ_{k_1,k_2} are positive real values, which are determined by the STTC only. Note that the determinant consists of two parts, PART I and PART II, where PART I is a positive constant for a given STTC and only PART II is related to the channel delays. In order to evaluate the effect of channel delay distribution on the performance of the given STTC-OFDM, we focus on PART II.

In PART II, defining $\varpi = \pi\,(k_2\text{-}k_1)\,\mathbf{\Delta}f$, we have

$$\sin^2(\pi\,(k_2\text{-}k_1)\,\mathbf{\Delta}f\ \ (t_{l_2}\text{-}t_{l_1})) = \sin^2\left(\varpi\,(t_{l_2}\text{-}t_{l_1})\right). \tag{11}$$

Note that the maximum delay of the channel is t_{L-1}, where $t_{L-1} < T_{cp}$. Considering that $t_{l_2} - t_{l_1} \leq t_{L-1}$ and $k_2 - k_1 \leq p_{\min} - 1$, we have

$$\varpi\,(t_{l_2} - t_{l_1}) \leq \pi(p_{\min} - 1)\mathbf{\Delta}f\ t_{L-1}, \tag{12}$$

with $p_{\min} = \lfloor v/2 \rfloor + 1$ for STTCs with the memory order v [6, p. 122], where $\lfloor v/2 \rfloor$ denotes the maximum integer not greater than $v/2$. The maximum delay for indoor communications environment, such as Wireless LAN, is less than 500 ns [9][10]. For most wireless OFDM systems and all the STTCs designed in the literature [1][6][11], we have † [1]

[1]For Wireless LAN and Hiperlan OFDM systems [12][13], the value of $\varpi\left(t_{l_2}\text{-}t_{l_1}\right)$ is in the range of $(0, 0.45\pi]$. For wireless OFDM systems in the research literature, such as [2][4][6], the value of $\varpi\left(t_{l_2}\text{-}t_{l_1}\right)$ is in the range of $(0, 0.46\pi]$.

$$\varpi\,(t_{l_2}\text{-}t_{l_1}) \in (0, \pi/2). \tag{13}$$

It is obvious that the value of $sin^2(\varpi(t_{l_2} - t_{l_1}))$ increases monotonically with $\varpi(t_{l_2} - t_{l_1})$ in the range of $(0, \pi/2)$. Now we can rewrite PART II as

$$\text{PART II} = \sum_{k_2-k_1=1}^{p_{\min}-1}\gamma_{k_1,k_2}\left(\sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))\right). \tag{14}$$

Considering the two-tap ($L = 2$) channels, (14) can be further rewritten as

$$\text{PART II} = \sum_{k_2-k_1=1}^{p_{\min}-1}\gamma_{k_1,k_2} sin^2(\varpi t_1). \tag{15}$$

Then we have the following observation.

Observation 1: *Consider a given STTC-OFDM over the two-tap channels. Note that the positive constant γ_{k_1,k_2} is determined by the STTC and the value of PARTII increases with ϖt_1 in the range of $(0, \pi/2)$. Hence, the minimum determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x},\widehat{\mathbf{x}})\right)|_{p_{\min}}$ increases with the maximum tap delay t_1.*

Consider the given STTC-OFDM with the positive constant coefficients γ_{k_1,k_2} over the channels with L taps, where $L > 2$. To simplify the analysis, we assume that the maximum delay of the channel t_{L-1} is fixed first. Then we have the following lemma.

Lemma 1: *Since the time interval $t_{l_2} - t_{l_1}$ is within the open set $(0, \pi/2)$, where $l_2, l_1 \in \{1, ..., L-2\}$, and $l_2 > l_1$, the cost function $\sum_{|k_1-k_2|=1}^{p_{\min}-1}\gamma_{k_1,k_2}\left(\sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))\right)$ is a non-decreasing function of time differences. The points that the maximum value of this cost function are located at the extreme points (This results from the application of the extreme points [14].)*

Proof: Consider the objective function

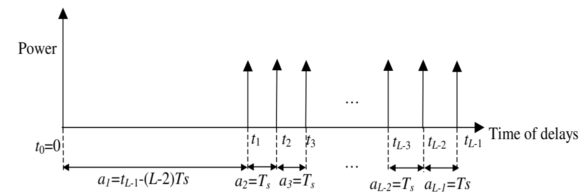
$$f(\Delta\mathbf{t_z}) = \sum_{|k_1-k_2|=1}^{p_{\min}-1}\gamma_{k_1,k_2}\left(\sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))\right),$$

where $\Delta\mathbf{t_z} = t_{l_2} - t_{l_1}, \mathbf{z} = 1, 2, ..., Z$, and Z is the total number of all the possible time differences $t_{l_2} - t_{l_1}$. We thus have the following non-linear programming problem:

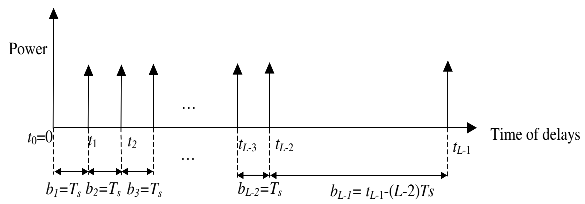
maximizing $f(\Delta\mathbf{t_z})$,
subject to $\Delta\mathbf{t_z} \in [T_s, T_{L-1} - T_s]$, and $f(\Delta\mathbf{t_z}) \in (0, 1)$.

Let $\Delta\hat{\mathbf{t}}_\mathbf{z}$ be a maximizing solution for the problem $\{max f(\Delta\mathbf{t_z}) : \mathbf{t_z} \in \mathbf{S}\}$, provided that $\Delta\hat{\mathbf{t}}_\mathbf{z} \in \mathbf{S}$ and $f(\Delta\hat{\mathbf{t}}_\mathbf{z}) \geq f(\Delta\mathbf{t_z})$, where $\mathbf{S} = \{\Delta\hat{t}_z : \Delta\hat{t}_z \leq T_{L-1} - T_s\}$ is a special case of a polyhedral set [14, p. 54]. In such a case, according to **Theorem 3.4.7** [14, p. 107], we say that a maximum solution $\Delta\hat{\mathbf{t}}_\mathbf{z}$ exists and $\Delta\hat{\mathbf{t}}_\mathbf{z}$ is an extreme point of $\mathbf{S}$. Since the time interval $t_{l_2} - t_{l_1}$ is within the open set $(0, \pi/2)$ and the objective function is an non-decreasing function of time differences, theoretically, the maximum solutions are given by $\Delta\mathbf{t_1} = ... = \Delta\mathbf{t_Z} = T_{L-1} - T_s$ [14, p. 55].

$$\det\left(\widetilde{\mathbf{D}}_{\alpha}(\mathbf{x},\widehat{\mathbf{x}})\right)|_{p_{\min}}=\underbrace{\frac{1}{\alpha^{n_T L}}+\sum_{i=1}^{n_T}\sum_{k=1}^{p_{\min}}\beta\left|\boldsymbol{\Delta}_i(k)\right|^2+...}_{\text{PART I}}+\underbrace{\sum_{k_2-k_1=1}^{p_{\min}-1}\gamma_{k_1,k_2}\left(\sum_{l_1,l_2=0}^{L-1}sin^2(\pi\left(k_2\text{-}k_1\right)\left(t_{l_2}\text{-}t_{l_1}\right)\boldsymbol{\Delta}f)\right)}_{\text{PART II}},\tag{10}$$



Channel Delay Distribution in (16)



Channel Delay Distribution in (17)

Fig. 1. Two sets of extreme points for the channel delay distributions with equal gain taps

We thus apply the above maximum solutions (extreme points) to the STTC-OFDM systems. Apparently, under the assumption of $t_{l_2} - t_{l_1} \geq T_s, t_0 = 0$, and the fixed t_{L-1}, the only possible sets of extreme points in the STTC-OFDM are

$$t_l = t_{L-1} - (L-1-l)T_s,\ l \in 1, 2, ..., L-1, \tag{16}$$

and

$$t_l = \left\{ \begin{array}{ll} lT_s, & l = 1, ..., L-2, \\ t_{L-1}, & l = L-1. \end{array} \right\} \tag{17}$$

respectively. ■

In other words, the extreme points give the channel delay distribution, under which the minimum determinant $\det\left(\widetilde{\mathbf{D}}_{\alpha}(\mathbf{x},\widehat{\mathbf{x}})\right)|_{p_{\min}}$ of the given STTC-OFDM has the largest value, if the maximum delay t_{L-1} is fixed. The channel taps with the uniform powers and the channel delay distributions in (16) and (17) are illustrated in Fig. 1. Now, we have the following properties.

Property 1: *For a given STTC-OFDM over the channels with the same maximum delay t_{L-1}, the minimum determinants in (10) under the channel delay distributions of (16) and (17) should be same.*

Proof: Let C and D denote the values of $\sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2}-t_{l_1}))$ under the channel delay distributions of (16) and (17), respectively. As shown in Fig. 1, defining $a_{l+1} = t_{l+1} - t_l$, under the delay distribution of (16) and $b_{l+1} = t_{l+1} - t_l$ under the delay distribution of (17), where $l \in \{0, 1, ..., L-1\}$, we have

$$a_l = \left\{ \begin{array}{ll} t_{l-1} - (l-2)T_s, & l = 1, \\ T_s, & l = 2, ..., L-1. \end{array} \right\} \tag{18}$$

and

$$b_l = \left\{ \begin{array}{ll} t_{l-1} - (l-2)T_s, & l = L-1, \\ T_s, & l = 1, ..., L-2. \end{array} \right\} \tag{19}$$

respectively. Then, C and D are rewritten as

$$\begin{aligned} C &= \sum_{l=1}^{L-1}\sin^2(\varpi a_l) + \sum_{l=1}^{L-2}\sin^2(\varpi(a_{l+1}+a_l)) \\ &\quad +... + \sin^2(\varpi(a_1 + ... + a_{L-1})), \end{aligned} \tag{20}$$

and

$$\begin{aligned} D &= \sum_{l=1}^{L-1}\sin^2(\varpi b_l) + \sum_{l=1}^{L-2}\sin^2(\varpi(b_{l+1}+b_l)) \\ &\quad +... + \sin^2(\varpi(b_1 + ... + b_{L-1})), \end{aligned} \tag{21}$$

respectively.

In (18) and (19), we can see that a_l and b_l, where $l = 1, 2, ..., L-1$, take the same set of values but in different orders. Thus, it is clear that

$$\begin{aligned} \sum_{l=1}^{L-1}\sin^2(\varpi a_l) &= \sum_{l=1}^{L-1}\sin^2(\varpi b_l), \\ \sum_{l=1}^{L-2}\sin^2(\varpi(a_{l+1}+a_l)) &= \sum_{l=1}^{L-2}\sin^2(\varpi(b_{l+1}+b_l)), \\ &\vdots \\ \sin^2(\varpi(a_1+...+a_{L-1})) &= \sin^2(\varpi(b_1+...+b_{L-1})). \end{aligned}$$

Then, we have

$$C = D.$$

In this case, for the given STTC-OFDM, the minimum determinants in (10) should be same. ■

Property 2: *For a given STTC-OFDM under the channel delay distributions of (16) or (17), the corresponding minimum determinant in (10) increases with the maximum delay t_{L-1}.*

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Proof: Let $t_{L-1}(1)$ and $t_{L-1}(2)$, where $t_{L-1}(1) > t_{L-1}(2)$, denote the maximum delays of two different channels, which are assumed to have the same multi-path powers and the channel delay distributions in (17). It can be seen that the minimum determinant of (10) depends on the value of $\sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))$. In this case, defining $A = \sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))$ with delays of $t_1, t_2, ..., t_{L-1}(1)$, we have

$$A = \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(1) - t_l)) + \sum_{l=0}^{L-3} \sin^2(\varpi(t_{L-2} - t_l)) + ... + \sum_{l=0}^{1} \sin^2(\varpi(t_2 - t_l)) + sin^2(\varpi t_1).$$

Similarly, defining $B = \sum_{l_1,l_2=0}^{L-1} sin^2(\varpi(t_{l_2} - t_{l_1}))$ with delays of $t_1, t_2, ..., t_{L-1}(2)$, we have

$$B = \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(2) - t_l)) + \sum_{l=0}^{L-3} \sin^2(\varpi(t_{L-2} - t_l)) + ... + \sum_{l=0}^{1} \sin^2(\varpi(t_2 - t_l)) + sin^2(\varpi t_1).$$

Then,

$$A - B = \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(1) - t_l)) - \sum_{l=0}^{L-2} \sin^2(\varpi(t_{L-1}(2) - t_l)).$$

As we have $t_{L-1}(1) > t_{L-1}(2)$ and $\varpi(t_{l_2} - t_{l_1}) \in (0, \pi/2)$, it is clear that

$$\sin^2(\varpi(t_{L-1}(1) - t_l)) > \sin^2(\varpi(t_{L-1}(2) - t_l)), \qquad (22)$$

where $l \in \{0, 1, ..., L-2\}$. Hence, we have

$$A > B.$$

■

This means the minimum determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x}, \widehat{\mathbf{x}})\right)|_{p_{\min}}$ increases with the maximum delay t_{L-1} under the channel delay distribution of (17). Similarly, according to **Property 1**, we can prove that the above conclusion is true for the STTC-OFDM under the channel delay distribution of (16).

V. Simulations

The code performance of STTC-OFDMs over frequency selective fading channels with various channel delay distributions is evaluated by simulations. In the simulations, the OFDM system is assumed to have a bandwidth of 1 MHz and 256 OFDM subcarriers. The subcarrier separation $\triangle f$ is 3.9KHz. The OFDM frame duration is $256\mu s$ and a guard interval is 40μs. The quasi-static frequency selective fading channels with equal gain taps but different delay distributions are assumed. Two transmit and three receive antennas are employed in the STTC-OFDMs.

Fig. 2 shows the minimum determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x}, \widehat{\mathbf{x}})\right)|_{p_{\min}}$ of 4 and 8-state 4-PSK STTCs [11] in the OFDM system over the two-tap channels with different delays t_1-t_0, where $t_1 \in$ [5, $39\mu s$] and $t_0 = 0$. In this case, we have $\alpha \approx 1$ and $p_{\min} = 2$. Note that $\varpi t_1 \in$ [0.06, 0.48]. According to Observation 1 in SECTION 4, the minimum determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x}, \widehat{\mathbf{x}})\right)|_{p_{\min}}$ increases with the delay t_1, as illustrated in Fig. 2.

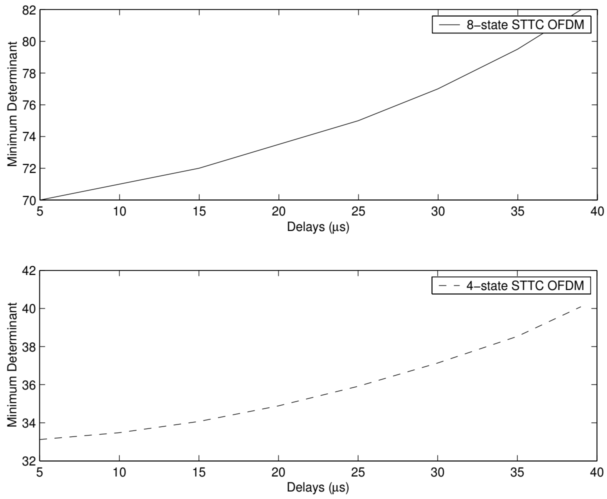

Fig. 2. Minimum determinant $\det\left(\widetilde{\mathbf{D}}_\alpha(\mathbf{x}, \widehat{\mathbf{x}})\right)|_{p_{\min}}$ of 4 and 8-state 4-PSK STTC-OFDMs in frequency selective fading channels with two equal gain taps and various delays.

The frame error rate (FER) performance of the 8 and 64-state 4-PSK STTCs [11] in the OFDM system over the two-tap channels with delays of t_1 =$5\mu s$ and t_1 =$39\mu s$ is shown in Fig. 3. Note that the 8-state ($p_{\min} = 2$) and 64-state ($p_{\min} = 4$) codes have $\varpi t_1 \in$ [0.06, 0.48] and $\varpi t_1 \in$ [0.18, 1.4], respectively. It is shown that the 8 and 64-state codes in the fading channel with a delay of $39\mu s$ outperform the corresponding codes in the channel with a delay of $5\mu s$ by 0.5 dB and 1.3 dB at the FER of 10^{-3}, respectively, which agrees with the previous discussion.

Fig. 3 also shows that the 64-state STTC is more sensitive to various channel delays than the 8-state one, as the 64-state code has a larger value of $p_{\min}$ and therefore a larger number of positive additive terms γ_{k_1,k_2} and $(\sum_{l_1,l_2} \sin^2(\varpi t_1))$ in (14).

Fig. 4 shows the performance of 16-state 4-PSK STTC [11] in the OFDM system over three-tap frequency selective fading channels with different delay distributions. Note that $p_{\min} = 3$ for the 16-state code and the positive coefficients γ_{k_1,k_2} defined in PART II are constant for the given code. From Fig. 4, we can see that the STTC-OFDM under both the channel delay distributions of $(t_0 = 0\mu s, t_1 = 35\mu s, t_2 = 39\mu s)$ and $(t_0 = 0\mu s, t_1 = 4\mu s, t_2 = 39\mu s)$, corresponding to (16) and (17), respectively, has the same performance and outperforms the one under the channel delay distribution of $(t_0 = 0\mu s, t_1 = 20\mu s, t_2 = 39\mu s)$ by 0.4 dB at the FER of 10^{-3}. In addition, it is shown that the STTC-OFDM under the channel delay distributions (17) with the maximum delay of $t_2 = 39\mu s$ outperforms the one under the delay distribution (17) with the maximum delay of $t_2 = 20\mu s$ by 1.1 dB at the FER of 10^{-3}. The simulation results are all consistent with

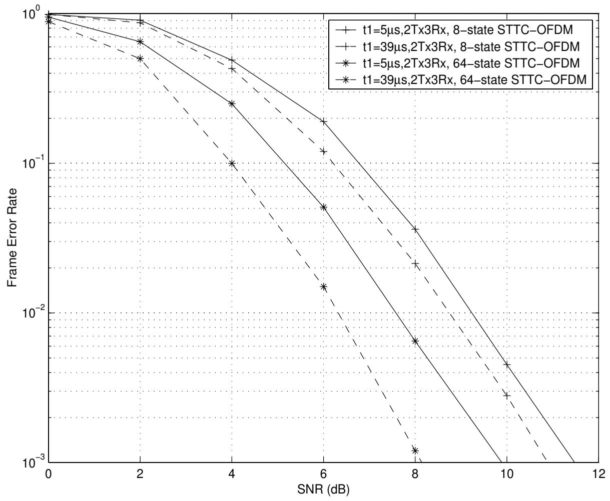

Fig. 3. FER performance of 8 and 64-state 4-PSK STTC-OFDMs in frequency selective fading channels with two equal gain taps and various delays.

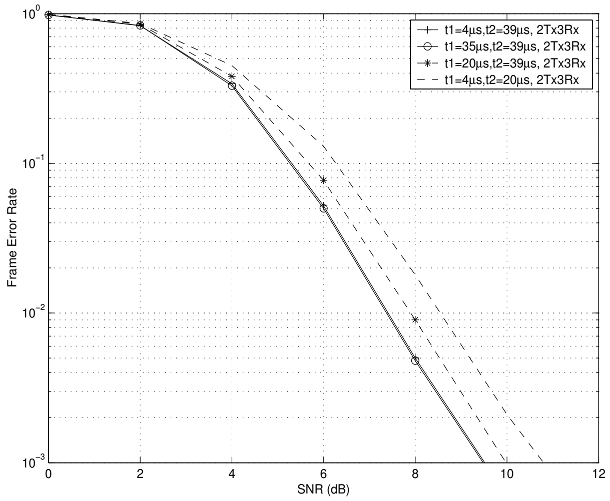

Fig. 4. FER performance of 16-state 4-PSK STTC-OFDM in frequency selective fading channels with three equal gain taps and various delays.

the analysis in Section 4.

Fig. 4 shows the performance of 16-state 4-PSK STTC [11] in the OFDM system over three-tap frequency selective fading channels with different delay distributions. Note that $p_{\min} = 3$ for the 16-state code and the positive coefficients γ_{k_1,k_2} defined in PART II are constant for the given code. From Fig. 4, we can see that the STTC-OFDM under both the channel delay distributions of $(t_0 = 0\mu s, t_1 = 35\mu s, t_2 = 39\mu s)$ and $(t_0 = 0\mu s, t_1 = 4\mu s, t_2 = 39\mu s)$, corresponding to (16) and (17), respectively, has the same performance and outperforms the one under the channel delay distribution of $(t_0 = 0\mu s, t_1 = 20\mu s, t_2 = 39\mu s)$ by 0.4 dB at the FER of 10^{-3}. In addition, it is shown that the STTC-OFDM under the channel delay distributions (17) with the maximum delay of $t_2 = 39\mu s$ outperforms the one under the delay distribution (17) with the maximum delay of $t_2 = 20\mu s$ by 1.1 dB at the FER of 10^{-3}. The simulation results are all consistent with the analysis in Section IV.

VI. Conclusion

In this paper, we consider the STTC-OFDM systems with no interleavors over quasi-static frequency selective fading channels. In order to provide a robust system design, we presented the performance analysis of STTC-OFDMs under various channel conditions in terms of the coding gain. In particular, the effect of various channel delay distributions on the code gain is investigated. Through this analysis, we point out two extreme conditions that produce the largest minimum determinant for a STTC-OFDM over multiple-tap channels. The analysis also proves that the corresponding coding gain increases with the maximum tap delay. The performance of STTC-OFDM under various channel conditions is evaluated by simulation. It is shown that 1) the minimum determinant of STTC in OFDM systems increases with the maximum tap delay of the channel; 2) the STTC-OFDM under two extreme channel conditions outperforms that under other channel conditions; and 3) the high memory order STTCs are more sensitive to the channel delays since they have a larger value of error event length p_{min}. Hence, we can see that all the simulation results are consistent with the performance analysis.

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