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The Approximate Solution of Linear Fuzzy Fredholm Integral Equations of the Second Kind by Using Iterative Interpolation

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Abstract**—**in this paper, we propose a numerical method for the approximate solution of fuzzy Fredholm functional integral equations of the second kind by using an iterative interpolation. For this purpose, we convert the linear fuzzy Fredholm integral equations to a crisp linear system of integral equations. The proposed method is illustrated by some fuzzy integral equations in numerical examples.

Keywords**—**Fuzzy function integral equations, Iterative method, Linear systems, Parametric form of fuzzy number.

I. Introduction

THE concept of integration of fuzzy functions was introduced by Dubois and Prade [3] for the first time and alternative approaches were later suggested by Goetschel and Voxman [6], Kaleva [7], Matloka [12], Nanda [13] and others. One of the first applications of fuzzy integration was given by Wu and Ma [16], who investigated the fuzzy Fredholm integral equations of the second type. In recent years some methods were introduced to solve fuzzy Fredholm integral equations. In this paper, we propose fuzzy iterative interpolation for solving the following fuzzy integral equation.

$$\widetilde{F}(t) = \widetilde{f}(t) + \lambda \int_a^b k(s,t)\widetilde{F}(s)ds, \quad a \le x \le b \qquad (1)$$

Where $k(s,t)$ is an arbitrary crisp kernel function over the square $a \le s,t \le b$, $\lambda > 0$ and $\widetilde{f}$ is a fuzzy function. In section 2, we briefly present the necessary preliminaries. In section 3, we propose the numerical method for solving the fuzzy Fredholm integral equations of the second kind based on the interpolation scheme. Some numerical examples are given in section 4.

II. Preliminaries

A. Definition 2.1

For a given set of support points (x_i, f_i), $i = 0,1,...,n$, we define the iterative interpolation scheme as follow,

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$$p_i(x) = f_i,$$

$$p_{i_0 i_1 \ldots i_k}(x) = \frac{(x - x_{i_0}) p_{i_1 \ldots i_k}(x) - (x - x_{i_k}) p_{i_0 \ldots i_{k-1}}(x)}{x_{i_k} - x_{i_0}},$$

Where $p_{i_0 i_1 \ldots i_k}$ is the Lagrange polynomial with less or equal

degree of k $(0 \le k \le n)$. The $p_{i_0 i_1 \ldots i_k}$ is equal with f in $x_{i_0}, x_{i_1}, ..., x_{i_k}$, where f is the exact solution.

B. Definition 2.2 [8]

A fuzzy number is a map $u = R \to I = [0,1]$ which satisfies

i) u is upper semi continues,

ii) $u(x) = 0$ outside some interval $[c,d] \subset R$,

iii) There exist real numbers a and b such that $c \le a \le b \le d$ where,

(1) $u(x)$ is monotonic increasing on $[c, a]$

(2) $u(x)$ is monotonic deccreasing on $[b, d]$

(3) $u(x) = 1, \quad a \le x \le b.$

The set of all such fuzzy numbers is represented by E^1

C. Definition 2.3 [1]

An arbitrary fuzzy number in parametric form is presented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, which satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0,1],
2. $\overline{u}(r)$ is a bounded left continuous non-increasing function over [0,1],
3. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

A crisp number a is simply represented by $\underline{u}(r) = \overline{u}(r) = a$, $0 \le r \le 1$. For arbitrary $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r))$ and $k \in R$, we define addition and multiplation by k as

- $u = v$ if and only if $\underline{u}(r) = \underline{v}(r)$ and $\overline{u}(r) = \overline{v}(r)$
- $u + v = (\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)),$

- $ku = \begin{cases} (k\underline{u}, k\overline{u}), & k \ge 0, \\ (k\overline{u}, k\underline{u}), & k < 0. \end{cases}$

D. Definition 2.4.

The $n \times n$ linear system

$$\begin{cases} a_{11}x_1 + a_{12}x_1 + \cdots + a_{1n}x_n = y_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2, \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = y_n, \end{cases} \tag{2}$$

Where, the given matrix of coefficients $A=(a_{ij})$, $1 \le j \le n$ and $1 \le i \le n$ is a real $n \times n$ matrix, the right- hand-side $y_i \in E^1$, $1 \le i \le n$, with the unknown $x_j \in E$, $1 \le j \le n$ is called a fuzzy linear system (FLS).

E. Definition 2.5.

A fuzzy number vector $(x_1, x_2, ..., x_n)^t$ given by $x_j = (\underline{x}_j(r), \overline{x}_j(r))$; $1 \le j \le n$, $0 \le r \le 1$, is called a solution of the fuzzy linear system (2) if

$$\begin{cases} \underline{\sum_{j=1}^n a_{ij}x_j} = \sum_{j=1}^n \underline{a_{ij}x_j} = \underline{y}_i, \\ \overline{\sum_{j=1}^n a_{ij}x_j} = \sum_{j=1}^n \overline{a_{ij}x_j} = \overline{y}_i. \end{cases} \quad \text{for } 1 \le i \le n$$

If, for a particular i, $a_{ij} > 0$, for all j we simply get:

$$\sum_{j=1}^n a_{ij}\underline{x}_j = \underline{y}_i, \qquad \sum_{j=1}^n a_{ij}\overline{x}_j = \overline{y}_i, \ 1 \le i \le n.$$

In general, an arbitrary equation for either $\underline{y}_i$ or $\overline{y}_i$ may include a linear combination of a $\underline{x}_j$'s and $\overline{x}_j$'s. Consequently, in order to solve the system given by (2), one must solve a crisp $2n \times 2n$ linear system where the right-hand side column is the function vector $(\underline{y}_1, \underline{y}_2, ..., \underline{y}_n, -\overline{y}_1, -\overline{y}_2, ..., -\overline{y}_n)^t$. We get the $2n \times 2n$ following linear system,

$$\begin{cases} s_{11}\underline{x}_1 + \cdots + s_{1n}\underline{x}_n + s_{1,n+1}(-\overline{x}_1) + \cdots + s_{1,2n}(-\overline{x}_n) = \underline{y}_1, \\ \vdots \\ s_{n1}\underline{x}_1 + \cdots + s_{nn}\underline{x}_n + s_{n,n+1}(-\overline{x}_1) + \cdots + s_{n,2n}(-\overline{x}_n) = \underline{y}_n, \\ s_{n+1,n}\underline{x}_1 + \cdots + s_{n+1,n}\underline{x}_n + s_{n+1,n+1}(-\overline{x}_1) + \cdots + s_{n+1,2n}(-\overline{x}_n) = -\overline{y}_1, \\ \vdots \\ s_{2n,1}\underline{x}_1 + \cdots + s_{2n,n}\underline{x}_n + s_{2n,n+1}(-\overline{x}_1) + \cdots + s_{2n,2n}(-\overline{x}_n) = -\overline{y}_n. \end{cases} \tag{3}$$

Where, S_{ij} are determined as follows:

$$a_{ij} \ge 0 \Rightarrow s_{ij} = a_{ij},\ s_{i+n,j+n} = a_{ij},$$
$$a_{ij} < 0 \Rightarrow s_{i,j+n} = -a_{ij},\ s_{i+n,j} = -a_{ij}, \tag{4}$$

and any s_{ij} which is not determined by (4) is zero. By using matrix notation we get,

$$SX = Y, \tag{5}$$

Where, $S = (s_{ij}) \ge 0$, $1 \le i \le 2n$, $1 \le j \le 2n$ and

$$X = \begin{bmatrix} \underline{x}_1 \\ \vdots \\ \underline{x}_n \\ -\overline{x}_1 \\ \vdots \\ -\overline{x}_n \end{bmatrix}, \quad Y = \begin{bmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_n \\ -\overline{y}_1 \\ \vdots \\ -\overline{y}_n \end{bmatrix}.$$

The structure of S implies that $s_{ij} \ge 0$, $1 \le i \le 2n$, $1 \le j \le 2n$ and that

$$S = \begin{pmatrix} B & C \\ C & B \end{pmatrix}.$$

F. Definition 2.6. [2, 10, 11]

For arbitrary fuzzy number $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$, the function which is shown as follow is the distance between u and v for $p \ge 1$.

$$D_p(u,v) = \left[\int_0^1 \left|\underline{u}(r) - \underline{v}(r)\right|^p dr + \int_0^1 \left|\overline{u}(r) - \overline{v}(r)\right|^p dr\right]^{\frac{1}{p})}$$

III. THE MIAN IDEA

In this section, we first replace Eq.(1) by the following equations

$$\underline{F}(t;r) = \underline{f}(t;r) + \lambda \int_a^b \underline{k(s,t)F(s;r)}ds \tag{6}$$

$$\overline{F}(t;r) = \overline{f}(t;r) + \lambda \int_a^b \overline{k(s,t)F(s;r)}ds \tag{7}$$

Where ,

$$\underline{k(s,t)F(s;r)} = \begin{cases} k(s,t)\underline{F}(s;r) & k(s,t) \ge 0, \\ k(s,t)\overline{F}(s;r) & k(s,t) < 0. \end{cases} \tag{8}$$

$$\overline{k(s,t)F(s;r)} = \begin{cases} k(s,t)\overline{F}(s;r) & k(s,t) \ge 0, \\ k(s,t)\underline{F}(s;r) & k(s,t) < 0. \end{cases} \tag{9}$$

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In substituted method to calculate $\underline{k(s,t)F(s;r)}$ and $\overline{k(s,t)F(s;r)}$, we apply the Lagrange interpolation by $n+1$ distinct points $a_0 < s_1 < s_2 < \ldots < s_n < b$ of $[a,b]$ as follows,

$$\underline{k(s,t)F(s;r)} = \sum_{l_j(s)\geq 0} \underline{k(s,s_j)F(s_j;r)} l_j(s) + \sum_{l_j(s)<0} \underline{k(s,s_j)F(s_j;r)} l_j(s) \tag{10}$$

$$\overline{k(s,t)F(s;r)} = \sum_{l_j(s)\geq 0} \overline{k(s,s_j)F(s_j;r)} l_j(s) + \sum_{l_j(s)<0} \overline{k(s,s_j)F(s_j;r)} l_j(s) \tag{11}$$

Where,

$$l_j(s) = \prod_{\substack{k=0\\k\neq j}}^{n} \left(\frac{s-s_k}{s_j-s_k}\right).$$

By integrating (10) and (11) from a to b

$$\int_a^b \underline{k(s,t)F(s;r)}ds = \sum_{h_j\geq 0} \underline{k(s,s_j)F(s_j;r)} h_j + \sum_{h_j<0} \overline{k(s,s_j)F(s_j;r)} h_j \tag{12}$$

$$\int_a^b \overline{k(s,t)F(s;r)}ds = \sum_{h_j\geq 0} \overline{k(s,s_j)F(s_j;r)} h_j + \sum_{h_j<0} \underline{k(s,s_j)F(s_j;r)} h_j \tag{13}$$

Where,

$$h_j = \int_a^b l_j(s)ds.$$

We get a system of linear equations by substituting Eq. (12) in (6) and (13) in (7), for $i = 0(1)n$, as follows

$$\underline{F}(s_i;r) = \underline{f}(s_i;r) \\ \lambda \sum_{h_j\geq 0} \underline{k(s_i,s_j)F(s_j;r)} h_j + \lambda \sum_{h_j<0} \overline{k(s_i,s_j)F(s_j;r)} h_j, \tag{14}$$

$$\overline{F}(s_i;r) = \overline{f}(s_i;r) \\ \lambda \sum_{h_j\geq 0} \overline{k(s_i,s_j)F(s_j;r)} h_j + \lambda \sum_{h_j\geq 0} \underline{k(s_i,s_j)F(s_j;r)} h_j. \tag{15}$$

The Eq. (14) and (15) gives a $(2n+2)\times(2n+2)$ crisp system of equations that we can obtain $\overline{F}(s_i;r)$ and $\underline{F}(s_i;r)$ by solving it. Now, by replacing these obtained values in the iterative interpolation polynomial, we can achieve the approximate value of the exact solution. In this case, we consider the following definition.

- *Definition 3.1.*

Assume that $\overline{F}(t;r)$ and $\underline{F}(t;r)$ in the points of $t_0,\ldots,t_n$ have been defined, t_i and t_j are two distinct points of $t_0,\ldots,t_n$ as $t_i < t_j$.

$$\underline{p}(t) = \begin{cases} \dfrac{(t-t_i)\underline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\overline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t_i < t_j < t, \\ \dfrac{(t-t_i)\underline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\underline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t_i < t < t_j, \\ \dfrac{(t-t_i)\overline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\underline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t < t_i < t_j, \end{cases}$$

$$\overline{p}(t) = \begin{cases} \dfrac{(t-t_i)\overline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\underline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t_i < t_j < t, \\ \dfrac{(t-t_i)\overline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\overline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t_i < t < t_j, \\ \dfrac{(t-t_i)\underline{p}_{0,1,\ldots,j-1,j+1,\ldots k}(t) - (t-t_j)\overline{p}_{0,1,\ldots,i-1,i+1,\ldots k}(t)}{(t_j-t_i)}, & t < t_i < t_j, \end{cases}$$

Where, $\underline{p}(t) = \underline{p}_{i_0,\ldots,i_k}(t)$ and $\overline{p}(t) = \overline{p}_{i_0,\ldots,i_k}(t)$

IV. NUMERICAL EXAMPLES

- *Example 4.1* [1]

We consider the following fuzzy Fredholm integral equation

$$\underline{f}(t;r) = rt + \frac{3}{26} - \frac{3}{26}r - \frac{1}{13}t^2 - \frac{1}{13}t^2 r,$$

$$\overline{f}(t;r) = 2t - rt + \frac{3}{26}r + \frac{3}{13}t^2 r - \frac{3}{26}t^2 - \frac{3}{13}t^2,$$

and kernel

$$K(s,t) = \frac{s^2+t^2-2}{13}, \quad 0 \leq s,t \leq 2, \quad \lambda = 1$$

and $a=0$, $b=2$. The exact solution in this case is given by

$$\underline{F}(t;r) = rt, \qquad \overline{F}(t;r) = (2-r)t.$$

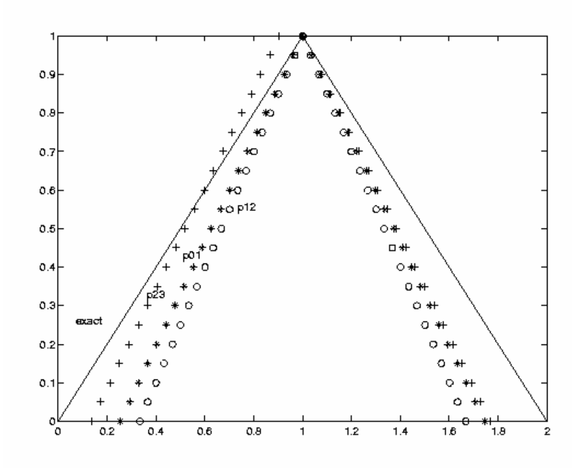

Fig. 4.1.a: the comparison between the approximate solutions $p_{i,i+1}, i = 0,1,2$ with $n = 3$ and the exact solution

TABLE I
THE DISTANCE OF THE EXACT SOLUTION AND THE APPROXIMATE SOLUTION FOR $n = 3$

i	t	$D_2(\widetilde{F}(1), \widetilde{p}_{i,i+1}(1))$	$D_2(\widetilde{F}(1), \widetilde{p}_{i,\ldots,i+2}(1))$	$D_2(\widetilde{F}(1), \widetilde{p}_{i,\ldots,i+3}(1))$
0	0			
		0.2100401063		
1	2/3		0.2564886573	
		0.2719715102		0.2466989373
2	4/3		0.2373094917	
		0.1510140954		
3	2			

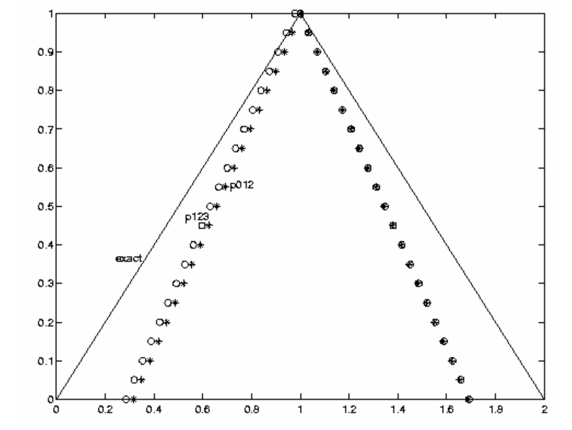

Fig..4.1.b: the comparison between the approximate solutions $p_{i,i+1,i+2}, i = 0,1$ with $n = 3$ and the exact solution

• *Example 4.2* [1, 4]
Consider the following fuzzy Fredholm integral equation

$$\underline{f}(t;r) = \sin(\frac{t}{2})(\frac{13}{15}(r^2 + r) + \frac{2}{15}(4 - r^3 - r)),$$
$$\underline{f}(t;r) = \sin(\frac{t}{2})(\frac{2}{15}(r^2 + r) + \frac{13}{15}(4 - r^3 - r)),$$

and kernel

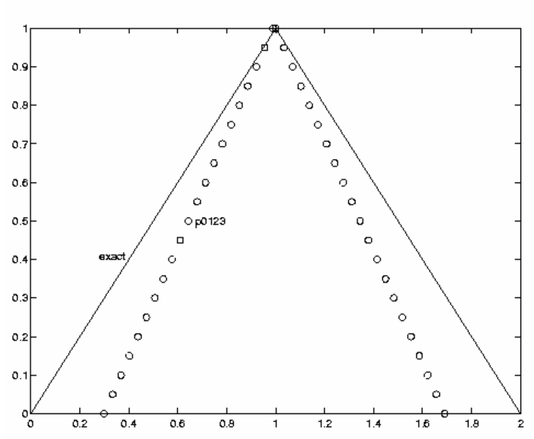

Fig. 4.1.c: the comparison between the approximate solutions $p_{i,i+1,i+2,i+3}, i = 0$ with $n = 3$ and the exact solution

$$K(s,t) = 0.1\sin(s)\sin(\frac{t}{2}), \quad 0 \le s, t \le 2\pi, \quad \lambda = 1$$

and $a = 0$, $b = 2\pi$. The exact solution in this case is given by

$$\underline{F}(t;r) = (r^2 + r)\sin(\frac{t}{2}), \quad \overline{F}(t;r) = (4 - r^3 - r)\sin(\frac{t}{2}).$$

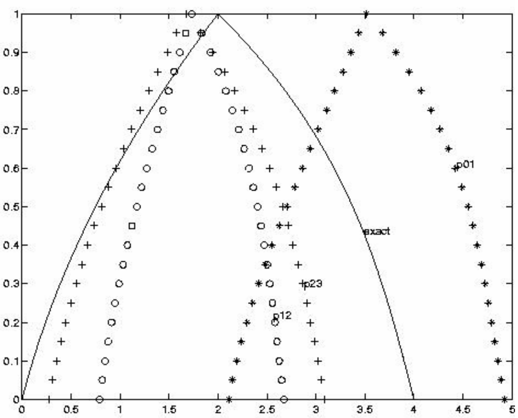

Fig. 4.2.a: the comparison between the approximate solutions $p_{i,i+1}, i = 0,1,2$ with $n = 3$ and the exact solution

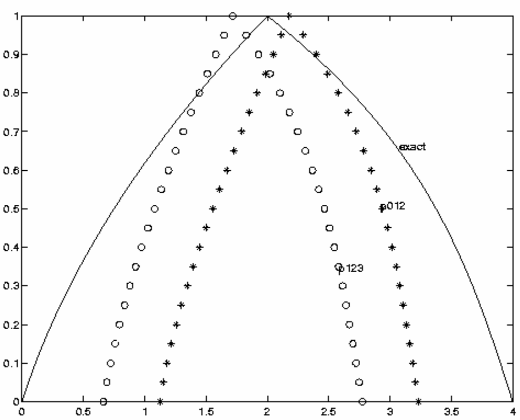

Fig. 4.2.b: the comparison between the approximate solutions $p_{i,i+1,i+2}, i = 0,1$ with $n = 3$ and the exact solution

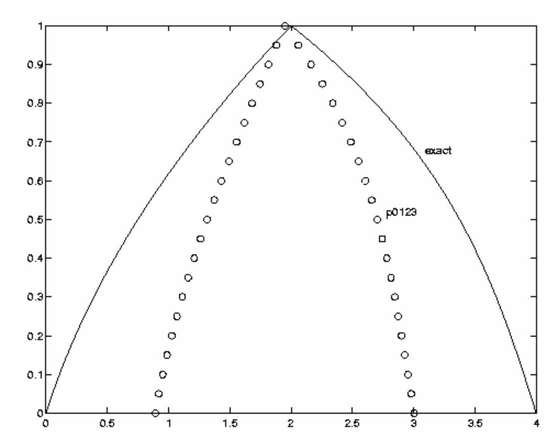

Fig. 4.2.c: the comparison between the approximate solutions $p_{i,i+1,i+2,i+3}, i = 0$ with $n = 3$ and the exact solution

TABLE II
THE DISTANCE OF THE EXACT SOLUTION AND THE APPROXIMATE SOLUTION FOR $n = 3$

i	t	$D_2(\widetilde{F}(\pi), \widetilde{p}_{i,i+1}(\pi))$	$D_2(\widetilde{F}(\pi), \widetilde{p}_{i,\dots,i+2}(\pi))$	$D_2(\widetilde{F}(\pi), \widetilde{p}_{i,\dots,i+3}(\pi))$
0	0			
		2.259333089		
1	$\frac{2\pi}{3}$		0.9283319588	
		1.074956182		0.8951302347
2	$\frac{4\pi}{3}$		0.9799819170	
		0.7256774309		
3	2π			

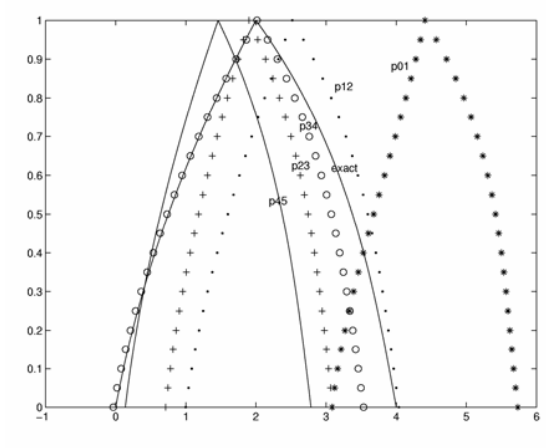

Fig. 4.2.d: the comparison between the approximate solutions $p_{i,i+1}, i = 0,1,2,3,4$ with $n = 5$ and the exact solution

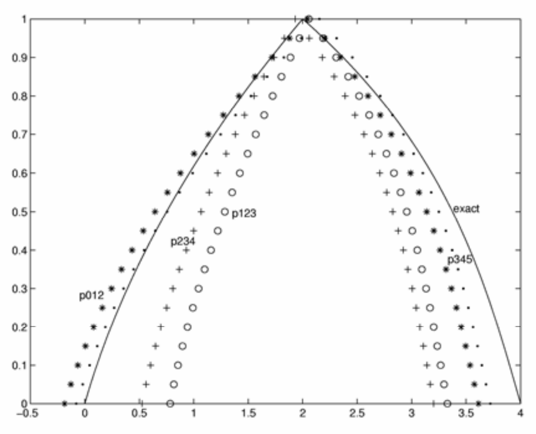

Fig. 4.2.e: the comparison between the approximate solutions $p_{i,i+1,i+2}, i = 0,1,2,3$ with $n = 5$ and the exact solution

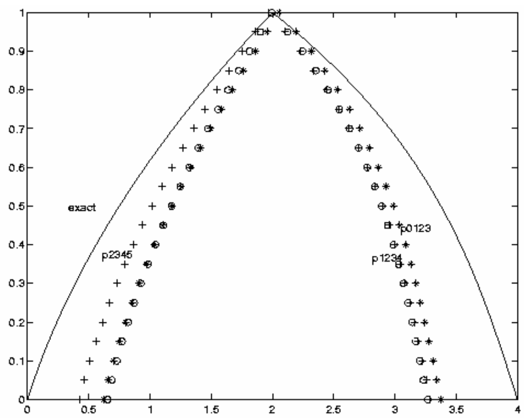

Fig. 4.2.f: the comparison between the approximate solutions $p_{i,i+1,i+2,i+3}, i = 0,1,2$ with $n = 5$ and the exact solution

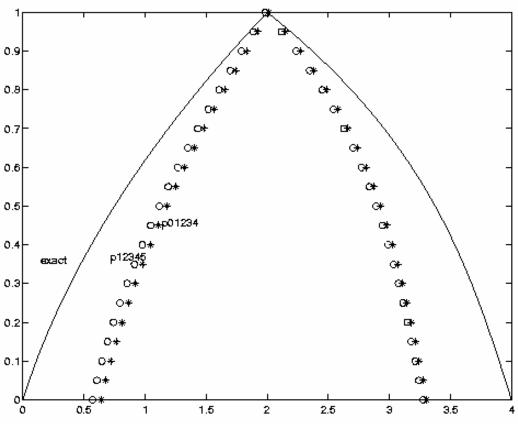

Fig. 4.2.k the comparison between the approximate solutions $p_{i,i+1,i+2,i+3,i+4}, i = 0,1$ with $n = 5$ and the exact solution

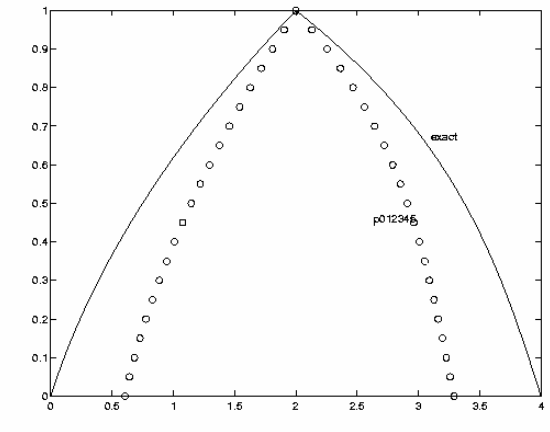

Fig. 4.2.I the comparison between the approximate solutions $p_{i,i+1,i+2,i+3,i+4,i+5}, i = 0$ with $n = 5$ and the exact solution

TABLE II
THE DISTANCE OF THE EXACT SOLUTION AND THE APPROXIMATE SOLUTION FOR $n = 3$

i	t	$D_2(\widetilde{F}, \widetilde{p}_{i,i+1})$	$D_2(\widetilde{F}, \widetilde{p}_{i \ldots i+2})$	$D_2(\widetilde{F}, \widetilde{p}_{i \ldots i+3})$	$D_2(\widetilde{F}, \widetilde{p}_{i \ldots i+4})$	$D_2(\widetilde{F}, \widetilde{p}_{i \ldots i+5})$
0	0					
		4.686462605				
1	$\frac{2\pi}{5}$		0.2727627638			
		0.8741343678		0.5897888721		
2	$\frac{4\pi}{5}$		0.6921217462		0.6311281170	
		0.7783754819		0.6573830445		0.6229692921
3	$\frac{6\pi}{5}$		0.6449272953		0.6165836167	
		0.3081461179				
4	$\frac{8\pi}{5}$		0.1758373651	0.5565625488		
		3.235716593				
5	2π					

III. CONCLUSION

In this work, for solving the fuzzy Fredholm integral equation of the second kind, we change the integral equation into two crisp integral equations. For the numerical solution of these equations, we apply an iterative interpolation with different r-cuts that is between zero and one. Each of them gives a $(n+1)\times(n+1)$ system of equations. Consequently, a linear $(2n+2)\times(2n+2)$ system of equations is constructed. By solving this system, we can estimate the value of function in the support points. Then, by replacing these values in the iterative interpolation polynomial, we can approximate the exact solution of the integral equation. Consequently, one can use this method to approximate the solution of a fuzzy Fredholm integral equation of the second type easily.

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