Simulation and Experimentation of Multibody Mechanical Systems with Clearance Revolute Joints

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Abstract—Clearance in the joints of multibody mechanical systems such as linkage mechanisms and robots is a main source of vibration, and noise of the whole system, and wear of the joints themselves. This clearance is an inevitable matter and cannot be eliminated, since it allows the relative motion between joint components and make them assemblage. This paper presents an experimental verification of the obtained simulation results of a slider–crank mechanism of one clearance revolute joint. The simulation results are obtained with the aid of CAD and dynamic simulation softwares, which is an effective method of simulation multibody systems with clearance joints and have many advantages. The comparison between both simulation and experimental results shows that the simulation results are so close to the experimental ones which proves the accuracy and efficiency of this method of modeling and simulation of mechanical systems with clearance joints.

Keywords—CAD and dynamic simulator softwares, Clearance joints, Experimental results, Slider–crank mechanism.

I. INTRODUCTION

JOINTS are one of the main components that are used for constructing and building of mechanical systems. They are used to connect two or more links and allow relative motion between them. They are modeled classically as an ideal or perfect. For instance, the journal and the bearing of a revolute joint are considered to be always concentric during the motion of the mechanism. However, there is always a gap between them which allows the journal to move freely and produces chaotic movements within the bearing boundaries during the motion of the mechanism. Hence, modeling of clearance joints becomes an important matter in order to study the clearance effect on the dynamic performance of mechanical systems, obtain simulation results for the kinematic and dynamic variables of the system closer to that of a real system, identify the vibration and noise levels resulted from certain values of clearances at the joints, or determine the maximum values of clearances at the joints to produce vibration and noise below a certain limits, etc. All of that attract the attention of many researchers to study how to model and simulate mechanical systems with clearance joints.

Flores et al [1] studied the effect of the friction between the journal and the bearing wall on the response of a mechanism using a modified coulomb’s friction law. They also studied the effect of lubricated joint on the kinematic and dynamic results.

They used slider-crank mechanism with only one revolute joint with clearance as a case study. Flores [2] studied the effect of wear occurring in revolute joints in which the amount of clearance is not of constant value as it changes over the whole mechanism life according to the theory of tribology. He combined this model with his previous model for revolute joint clearance, to simulate a slider–crank mechanism having only one real joint. Flores [3] investigated the dynamic response of a multibody system with multiple clearance joints. Different tests are performed to parametrically quantify the effect of the clearance size, the crank input speed, and the number of clearance joints on the dynamic performance of such a system. Liu et al [4] developed a FEM model to approximately represent cylindrical joints with clearance. They compared the FEM results with those obtained using Hertz model and Persson theory to study their limitations and constraints. Mukras et al [5] suggested a procedure to analyze a planar multibody system considering wear at its revolute joints. The used analysis was carried out by modeling multibody systems with clearance revolute joints. They combined this model with Archard’s wear model used to compute the wear as a function of the evolving dynamics and tribological data. This procedure was verified by comparing the predicted wear from the theoretical model with that occurred with an experimental slider-crank mechanism with a clearance-joint between the crank and the connecting rod. Park and Kwak [6] made an optimal design formulation to reduce the effects of undesirable dynamics due to joint clearance. A slider-crank mechanism with one clearance-joint was used as a demonstrative example. Rhee and Akay [7] studied the dynamic response of a four bar mechanism with one clearance-joint. The motion of the mechanism rocker-arm pin at the ground connection was modeled using a Lagrangian approach. Zhu and Ting [8] made the uncertainty analysis of planar and spatial robots with clearance-joints. The used models are based on the probability density function which expresses the motion of the endpoint of a planar robot manipulator. Zhang and Huang [9] made a robust tolerance design for function generation mechanisms with joint clearance. Their model enables them to quantify the effect of uncertainties on the accuracy of function generation mechanism. Their model enables them to choose the optimal tolerances for individual components in order to minimize the assembly cost and satisfying the required mechanism accuracy in the same time. Dupac and Beale [10] investigate the effect of slider clearance in flexible linkage mechanism with crack. The impact at slider joint is modeled by restitution coefficient. The results pointed out that clearance and imperfect links change the dynamical behavior of the system and their effect cannot be neglected. Bai and Zhao [11] make a dynamic analysis for space robot manipulator with joint clearance. The contact force in joint clearance is modeled using nonlinear spring-damper element and the frictional force is modeled using coulomb's friction law. A two flexible link robot with...
one clearance joint is used to demonstrate the suggested methodology. Megahed and Haroun [12, 13] studied the effect of multiple clearance revolute joints on the dynamic performance of Multibody systems using CAD and dynamic simulator software packages. The contact force approach is used to model the normal contact force between revolute joint components. A slider–crank mechanism with single and two clearance joints working in horizontal and vertical planes is used as an application example. They [14] used the same methodology again to study the effect of clearance spherical joints on the dynamic performance of Multibody systems. A spatial four bar mechanism with two spherical joints with clearance is used as a case study. The method of studying the effect of joint clearance using CAD and dynamic simulators softwares proposed by Megahed and Haroun [12, 13] has many advantages over simulation of mechanical systems by writing computer codes. It enables us to consider more than one clearance joint in the system, while this matter is difficult by using computer coding. In addition to that, modeling and simulation is much easier, and changing system configurations to simulate for different cases or simulate different types of mechanisms needs less effort and less simulation time than computer coding method which needs an individual code for each mechanism with certain configurations. Analysis of the dynamic performance of mechanical systems with clearance joints using CAD and dynamic simulator softwares has another important advantage. Most of mechanism designers or engineers use solid modeling and animation softwares for designing mechanical systems or mechanisms such as SolidWorks/CosmosMotion. Hence, our method of simulating mechanical system with clearance joints using CAD and dynamic simulator softwares will enable them to include the effect of joint clearance in their design easily, rather than make a separate computer code, which needs a good knowledge of multibody dynamics, methods of modeling clearance joints and also a strong mathematical background to form an effective integration technique which is used for simulating mechanical systems with high accuracy and less simulation time. The proposed method of using CAD and dynamic simulator softwares for analyzing the dynamic performance of mechanical systems with clearance joints has many advantages. However, an experimental verification of the simulation results obtained by this method should be done to in order to ensure its accuracy and efficiency. In this experiment a mechanical system or mechanism with at least one clearance joint should be considered, and one or more of the kinematic or dynamic variables should be measured and compared with that obtained from simulation result. Hence, in this work a slider–crank mechanism with one clearance joint is considered as the experimental mechanism. The slider acceleration is measured by an accelerometer, and the measured signal is transmitted to PC via an external data acquisition system. The measured signal is online monitored and recorded on the PC. A comparison between the experimental and simulation results is done to validate the simulation results, and ensure the accuracy of the proposed method. This paper is organized as follows, after introduction section 2 describes the modeling method of Clearance revolute joint. Section 3 presents the equation of motion of multibody systems according to the multibody dynamics approaches.

Section 4 comes to show the developed computational algorithm for simulating multibody systems with clearance joints. Section 5 shows the simulation results of slider–crank mechanism while section 6 presents the experimental procedures and comparison between simulation and experimental results. Ends with section 7 which outline the conclusions of this work.

II. MODELING OF CLEARANCE REVOLUTE JOINTS

The existence of clearance at any revolute joint adds two extra degrees of mobility to the mobility index of the whole system. Hence, a mechanical system which contains at least one clearance joint becomes dynamically driven system [15], in which the kinematic equations only cannot be used to fully determine all the system coordinates. Instead, the dynamic equations in addition to the kinematic equations should be used. Hence, modeling of clearance joints depends primarily on how to calculate the interaction forces between joint components during the motion of the mechanism. The components of the revolute joint or journal–bearing joint are in the form of two aligned short cylindrical surfaces with slightly different radii equal to the value of clearance inserted into each other. Therefore, the interaction force between the joint components takes normal and tangential components during the contact or impact periods as shown in Fig. 1.

![Fig. 1](image_url)

**Fig. 1 (a) revolute clearance-joint (b) joint contact forces [13, 14]**

While the contact forces approach zero when the journal moves freely inside the bearing boundaries. Both cases of contact are expressed after:

\[ F = 0 \quad \text{If} \quad \delta < 0 \]  \hspace{1cm} (1)

\[ F = F_N + F_T \quad \text{If} \quad \delta \geq 0 \]

Where \( \delta \) is the relative penetration depth (Fig. 1). According to the contact condition, \( \delta \) is given by:

\[ \delta = e - c \]  \hspace{1cm} (2)

Where \( e \) is the eccentricity of the journal center relative to the bearing center (Fig. 1-a) and \( c \) is the value of the joint clearance.
The tangential contact force $F_T$ mainly is the frictional force between the journal and the bearing. In this work Coulomb’s friction law is used to model the tangential contact force. $F_T$. Interested readers on how to calculate the tangential force $F_T$ according to Coulomb’s friction law see [12, 13]. However, the normal contact force is modeled according to the continuous contact force model [12, 13]. The mathematical model used in this work for the continuous contact force model is the Hertz model [16] modified by Lankarani and Nikravesh [17]. This model assumes that when the journal makes a contact or an impact into the bearing surface a normal force to the plane of contact or collision is produced. The normal force equation consists of two force terms, the first is the elastic force term which is a function of the journal penetration $\delta$ and the second is the energy dissipation term which is a function of the penetration velocity as expressed by:

$$ F_N = K \delta^\alpha + D \delta^\star $$

(3)

The first term $K \delta^\alpha$ represents the elastic force and the second term $D \delta^\star$ accounts for the energy dissipation, $K$ is the generalized stiffness parameter, $D$ is the hysteresis damping coefficient and $\delta^\star$ is the relative impact velocity. The exponent $\alpha$ depends on the contact surfaces materials and equals to 1.5 for metallic contacts.

In this work simulation of the slider – crank mechanism is done by SolidWorks/CosmosMotion software package. This software gives many facilities to make a complete solid modeling of the mechanism parts, assembly them, and simulate for the results. Simulation of mechanical systems by this software is done by the embedded ADAMS simulation engine. Hence, in order to simulate the mechanism with clearance joints using SolidWorks/CosmosMotion software according to the continuous contact force model, an adaptation is made to the continuous model [12, 13] to obtain the necessary parameters of the ADAMS functions. The following are the equations and procedure of obtaining the parameters of the contact force equations that should specified to SolidWorks/CosmosMotion to simulate for the results. More Interested readers on the original equations of the continuous model and how to adapt them with ADAMS methodology see [12,13].

The generalized stiffness parameter $K$ depends on the geometry of the contacting surfaces and their physical properties, which is given by [16]:

$$ K = \frac{4}{3(\sigma_i + \sigma_j)} \sqrt{\frac{R_i R_j}{R_i + R_j}} $$

(4)

In which the material parameters $\sigma_i$ and $\sigma_j$ are given by:

$$ \sigma_{z} = \frac{1 - \nu^2}{E_{z}} \quad (z = i, j) $$

(5)

Where $V_z$ and $E_z$ are the Poisson’s Ratio and Young’s modulus associated with each body respectively. The radius of curvature is taken as positive for convex surfaces and negative for concave surfaces. The hysteresis damping coefficient can be expressed by the following form

$$ D = \begin{cases} 
0 & \delta \leq 0 \\
C_{\text{max}} \left( \frac{\delta}{d_{\text{max}}} \right)^2 \left( 3 - 2 \frac{\delta}{d_{\text{max}}} \right) & 0 < \delta < d_{\text{max}} \\
C_{\text{max}} & \delta \geq d_{\text{max}} 
\end{cases} $$

(9)

Where $D$ is the damping coefficient. $C_{\text{max}}$ is the maximum value of the damping coefficient, and $d_{\text{max}}$ is the maximum value of penetration depth.

The value of maximum penetration depth $d_{\text{max}}$ is taken according to the recommended values for the journal – bearing materials from CosmosMotion library. However the value of the maximum damping coefficient $C_{\text{max}}$ is estimated as follows

- First the value of $C_{\text{max}}$ is taken according to the recommended values for journal – bearing materials from CosmosMotion library, then run simulation according to this value.
- After running simulation and an average value of the penetration velocity ($V_{\text{avg}}$) just before impacts (at the instants of $\delta = 0$) is calculated and the maximum damping coefficient $C_{\text{max}}$ are calculated using (Eq. 10 and Eq. 11) as follows:

$$ H_{\text{avg}} = \frac{3K(1-r^2)}{4V_{\text{avg}}} $$

(10)

$$ C_{\text{max}} = H_{\text{avg}} d_{\text{max}}^\alpha $$

(11)

- More accurate values of $C_{\text{max}}$ are obtained by updating the value of $V_{\text{avg}}$ using the resulted value of $C_{\text{max}}$ at the preceding iteration till the value of $C_{\text{max}}$ is stabilized or meet the required accuracy. In our simulation we stopped when the integer part of the value is stabilized.

III. EQUATION OF MOTION OF MULTIBODY SYSTEMS

ADAMS uses the multibody dynamics approach for simulation multibody mechanical systems. The following is a brief description of multibody dynamics methodology for simulating multibody systems.

The position of a rigid body $i$ (Fig 2) is defined by the position and orientation of body reference frame as follow:
\[ q_i = [r_i \ \phi_i] \]  \hspace{1cm} (12)

Where \( r_i = [R_x, R_y, R_z] \) are the translation coordinates represented by the Cartesian coordinates, and \( \phi_i \) are the rotational coordinates which could be described by Euler angles \( \phi_i = [\phi, \theta, \psi] \), or Euler parameters \( \phi_i = [s_E, p_E, q_E, r_E] \). The velocity and acceleration of the body \( i \) in terms of \( \omega_i \) and \( \phi_i \) respectively.

\[ \dot{q}_i = [\dot{r}_i \ \omega_i]^T \]  \hspace{1cm} (13)
\[ \ddot{q}_i = [\ddot{r}_i \ \cdot \omega_i]^T \]  \hspace{1cm} (14)

The equation of motion of a rigid body \( i \) in terms of Cartesian coordinates is given by:

\[ M_i^{\dddot{q}_i} = Q^e_i \]  \hspace{1cm} (15)

Where \( Q^e_i \) is a vector of all external forces acting on the body. It is worth mentioning that when the body is included in planar multibody system or mechanism works in vertical plane, the gravitational force of the body acts as an external force to it and should be added to the vector of external forces \( Q^e_i \). However, in case of horizontal plane motion the direction of the gravitational force of the body acts perpendicular to the plane of motion; hence it does not included in the vector of external forces \( Q^e_i \). Interaction forces at the clearance joints which calculated according to the proposed models for both dry and clearance joints are considered as external forces on the associated links.

The kinematic equations in multibody dynamics are described in the form of harmonic allegorical constraint equations as follows

\[ C(q,t) = 0 \]  \hspace{1cm} (16)

Where \( C(q,t) \) is a vector of all joints constraints' and specified motion trajectories in the multibody system. The equation of each constrain depends on the type of joint and the degrees of freedom that it allows.

As stated before, the multibody system or mechanism in the presence of clearance joints is a dynamically driven system in which the kinematic equations as well as the dynamic equations are used to obtain the generalized coordinated of the system. Using the Lagrange multipliers technique the constraint equations are added to the equations of motion of all bodies in the system and reduced to the following form:

\[ \begin{bmatrix} M & C_q^T \\ C_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q^e \\ Q_d \end{bmatrix} \]  \hspace{1cm} (17)

Where:

- \( M \) is the system mass matrix that contains all mass matrices of all bodies in the system
- \( C_q \) is the generalized Jacobian matrix of the kinematic constraints for any number of bodies,
- \( \lambda \) is a vector of Lagrange multipliers, and it is used to calculate the reaction forces at the connected joints
- \( Q^e \) is a vector of all external forces acting on the bodies of the system
- \( Q_d \) is vector that groups all the terms of the acceleration constraint equations that depend on the velocities only, and it takes the following form

\[ Q_d = -C_n - (C_q \dot{q})_q \dot{q} - 2C_{\phi} \dot{q} \]  \hspace{1cm} (18)

Where \( C_n \) is the twice differentiation of the vector of constraint equations \( C(q,t) \) with respect to time

Solving Eq. 17 which combines the kinematic and dynamic equations of all bodies of a multibody system will returns a vector of Lagrange multipliers \( \lambda \) and a vector of generalized acceleration \( \ddot{q} \). Lagrange multipliers vector \( \lambda \) is used to obtain reactions at the system joints. The generalized acceleration vector \( \ddot{q} \) is integrated twice to obtain generalized velocity vectors \( \dot{q} = [\dot{q}_1 \ \dot{q}_2 \ \cdots \ \dot{q}_n] \) and generalized position vector \( q = [q_1 \ q_2 \ \cdots \ q_n] \). The resulting kinematics of the system at a certain time step are used for calculating at the next time step forces at the clearance joints which are included in the vector of generalized external forces, the vector \( Q_d \) that groups all terms of acceleration coordinates, and the constraint Jacobean matrix \( C_q \). All of those are used again to calculate the system kinematics for the next time step by solving Eq. 17 again. This procedure is done until reach the final simulation time.

IV. COMPUTATIONAL ALGORITHM

The flow chart of the computational algorithm used for simulation of multibody mechanical system with clearance joints by SolidWorks/CosmosMotion software is described in Fig. 3.

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**Fig. 2** Body i local coordinates

**Fig. 3** Simulation of multibody mechanical system with clearance joints by SolidWorks/CosmosMotion software is described in...
The computational procedures are based on three main assumptions. The first is that the value of clearance at each joint remains constant during the motion of the mechanism. This means that no wear occurs at the joints. The second is that all mechanism links are rigid; the dimensions of the links are constant during the motion of the mechanism. The last assumption is that the clearance joints are dry joint, in which no lubricant exists at the journal – bearing gap.

V Simulation of Slider – Crank Mechanism

As stated before a slider – crank mechanism with one clearance joint (Fig. 5) is used in this work to validate our method of modeling and simulation of multibody mechanical system with clearance joints, which we have presented in our previous work. We choose slider – crank mechanism because it is a so famous mechanism and its performance is familiar to most of people who works in mechanism design. We consider also one clearance joint only in the mechanism although our modeling and simulation method is applicable for a system with multiple clearance joints. Because considering more than one clearance joint produce a higher peaks in the performance curves [12, 13] which needs a special sensors that can measure a signals of very high amplitudes. These sensors are out of our facilities.

A. Mechanism Data

The dimensions, masses and inertias of the links of the mechanism used in simulation and experimentation are listed in table 1. The parameters of the clearance-joints are listed in table 2 while integration parameters are listed in table 3. The crank, which is the driving link, rotates with a constant angular velocity of 300 rpm ccw. However, the crank takes a transient period of 0.2 sec to move from standstill till reach a constant angular velocity of 300 rpm.

The transient period in simulation is taken equal to that in experimentation for the sake of comparison between both finally. This period (0.2 sec.) has a strong relation with the speed control system of the experimental mechanism which is discussed in details in the section of experimentation. The initial configuration of the mechanism is taken with the slider

![Fig. 3 Computation algorithm flowchart](image-url)
is full extended in the X-axis, the journal and the bearing of each joint are concentric. The all mechanism parts are made from steel, while the bearing of the clearance joint made from brass.

![Slider – crank mechanism with one clearance revolute joint](image)

**Fig. 5** Slider – crank mechanism with one clearance revolute joint

**TABLE I**

<table>
<thead>
<tr>
<th>Mechanism Parts</th>
<th>Length [m]</th>
<th>Mass [kg]</th>
<th>Moment of Inertia [kg.m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crank</td>
<td>0.08</td>
<td>2.37</td>
<td>0.01201</td>
</tr>
<tr>
<td>Connecting rod</td>
<td>0.32</td>
<td>0.72</td>
<td>0.0006926</td>
</tr>
<tr>
<td>Slider</td>
<td>1.00</td>
<td>2.28</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Clearance Joint Parameters</th>
<th>Stainless steel Young’s modulus</th>
<th>Brass Young’s modulus</th>
<th>Static Friction Coeff.</th>
<th>Static Friction Vel.</th>
<th>Max. Penetration</th>
<th>Restitution coeff.</th>
<th>Kinetic Friction Vel.</th>
<th>Kinetic Friction Coeff.</th>
<th>Clearance</th>
<th>Bearing diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>207 Gpa</td>
<td>97 Gpa</td>
<td>0.08</td>
<td>0.0001 m/s</td>
<td>0.00001 m</td>
<td>0.55</td>
<td>0.01016 m/s</td>
<td>0.05</td>
<td>0.5 mm</td>
<td>15.00 mm</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Integration Parameters</th>
<th>Max. number of iteration</th>
<th>Min. time step</th>
<th>Accuracy</th>
<th>Jacobian pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>0.0000 s</td>
<td>0.0000 s</td>
<td>100%</td>
</tr>
</tbody>
</table>

**B Results and Discussion**

The slider – crank mechanism is simulated in two different cases. The first when all joints are ideal or perfect. The second when a 0.5 mm clearance only exists on the joint connect the slider and connecting rod (s – cr joint). Analysis of the performance of the slider – crank mechanism is done through obtaining simulation results of slider displacement (Fig. 6), slider velocity (Fig. 7), slider acceleration (Fig. 8), and the applied torque to the crank (Fig. 9). In the slider velocity curve (Fig. 7) a stairs shapes appear sometimes. These stairs refer to a constant velocity of the slider followed by a sudden change. This actually occurs when the slider moves freely without any external force or restriction of the mechanism through the clearance of the joint (free mode) then an impact happen and, the velocity changes instantaneously (impact mode). These stairs are reflected to high peaks on the slider acceleration curve (Fig. 8), and crank torque curve as the impact force at the s – cr joint is transmitted directly to the crank through the rigid connecting rod, and as a result the crank torque increases suddenly to maintain the crank moves with constant angular velocity.

![Slider displacement at $\omega = 300$ rpm](image)

**Fig. 6** Slider displacement at $\omega = 300$ rpm

![Slider velocity at $\omega = 300$ rpm](image)

**Fig. 7** Slider velocity at $\omega = 300$ rpm

![Slider acceleration at $\omega = 300$ rpm](image)

**Fig. 8** Slider acceleration at $\omega = 300$ rpm

![Applied crank torque at $\omega = 300$ rpm](image)

**Fig. 9** Applied crank torque at $\omega = 300$ rpm

The contact force at the s – cr joint is shown in Fig. 10. High peaks appear sometimes. These peaks indicate that impacts happen at those times. The impacts mostly happen...
when the velocity of the mechanism at the s – cr joint place reach its highest value and tends to decrease, then the slider continue moves with its previous velocity through the clearance of the joint until make an impact. This strong impact is followed by a series of small impacts until the slider is stabilized and begins to follow the mechanism motion again (contact mode). The different modes of contact between the journal and the bearing of the clearance joint during the motion of the mechanism can be shown from Fig. 11, which represents the motion of the journal center relative to bearing center. When the contact mode occurs the journal center lies approximately on 0.5mm circle. When free mode occurs the journal center lies inside the 0.5mm circle, while the points lies outside 0.5 mm circle refer to the impact mode in which the journal penetrate on the bearing surface.

VI  EXPERIMENTATION OF SLIDER – CRANK MECHANISM

The efficiency and accuracy of the proposed method of modeling and simulation of mechanical systems with clearance joints using CAD and dynamic simulation softwares is ensured in this work by comparing simulation results of slider acceleration with the experimental one measured by an accelerometer. The slider acceleration is chosen only to be measured to validate the simulation results among other kinematic and dynamic variables such as contact force at the clearance joint which needs a very small sensor that can be inserted into a very small gap of about 0.5mm and withstand a high impact forces, which is really difficult. In addition to that, validating the simulation results of slider acceleration only can fulfill the requirements of checking the accuracy and efficiency of the proposed method of modeling clearance joints, since slider acceleration mathematically depends on the calculated forces at clearance joints. Hence, accurate values of slider acceleration means accurate values of calculated forces at clearance joints by the proposed method.

A Test Rig Design

The test rig is divided into three parts (1) Mechanism; slider – crank mechanism with a 0.5mm clearance exists on the s – cr joint (2) Speed control system; a speed controller should be used to maintain a constant input speed to the mechanism (3) Measuring instruments; A system of online measuring of slider acceleration and recording the measuring data, the following are a detailed explanation of each.

1. Mechanism Data

The mechanism used in this experiment is a slider – crank mechanism with one clearance joint. The clearance exists on the joint connected the crank and connecting rod (s-cr joint). The dimensions and inertias of the mechanism links are the same as the mechanism of the simulation case for the sake of comparing between the experimental and simulation results.

Figs. 12 and 13 show the test rig design. The crank which is the driving link is in the form of circular disc with 4 holes away from the disc center with 50, 60, 70, and 80 mm central distances. Those holes are made to allow us to change the crank length to do many experiments. However, in this experiment the length of 80 mm only used. The Mass moment of inertia is relatively high to act as a flywheel to reduce the effect of changing the crank speed due to changing of the load torque. In order to ensure a smooth motion of the slider, the prismatic joint is made of a ball sleeve and very smooth guides. The design of the test rig allows us to test slider – crank mechanism with positive and negative offset. However, the experiment is carried out on slider – crank mechanism with zero offset. The sleeve of the s – cr joint is changeable so we can use different sleeves to obtain different values of clearance at this joint, in addition to using it nearly perfect or with negligible clearance. The journal and the bearing of the revolute joints of the mechanism are made from steel and brass respectively, in order to avoid rust and keep their surfaces smooth.

2. Speed Control System

The crank which is the driving link is actuated by Ac electric motor of 750 watt and 1450 rpm nominal speed. The speed of the motor is controlled by an inverter. The most important point of the motor control is to maintain the motor speed constant while the load torque is changing. The commercial available inverters in the market support one or more control mode. Those modes are: V/f control, V/f control with pulse generator feedback, vector control, and flux vector control. In our case the inverter operates according to vector...
control mode which gives speed accuracy about ±0.2%. Beside the control system, the speed fluctuation is intended to be decreased to the minimum limit by changing the some of the mechanism design features. The reduction of fluctuation is reduced by (1) using a high motor power more than the maximum power required which decreases the slip of the motor, (2) the relatively high inertia of the crank help in reducing the crank speed fluctuation.

3. Measuring Instruments

The slider acceleration is measured using an accelerometer as stated before. The used accelerometer model is Kyowa AS-100HA, with a rated capacity of $980.7 \text{ m/s}^2$. This accelerometer is connected with a sensor interface or external data acquisition system (PCD-300A). This interface have four channels which enable us to measure 4 different readings from different sensors at the same time. In our case only one channel is used. The maximum sampling frequency of the interface is 5 kHz, which means that it can get 5000 readings per second or one reading every 0.0002 second. This is an important factor in choosing the data acquisition system, since in our case we have a high peaks in acceleration which happened instantaneously or in a very short period of time. This device with this sampling rate will enable us to locate and detect those peaks accurately. Fig. 14 represents a scheme of the measuring system.

The sensor interface is directly connected to PC via USB cable. The control software (PCD-30A) should be installed on a PC having windows (98/ME/2000/XP) operating system. This software is a LabVIEW software compatible with the interface configurations. The software enables us to get online measuring of the sensor readings and record them in one or separated files. Each file can store a maximum reading of 100,000 readings. Therefore, if we use a sampling rate of 5000 readings/sec, each file will store readings of 20 seconds, after that a new file will be opened to record readings for the next period and so on.

The experiment is carried out on the slider – crank mechanism under study in two stages. The First, when there is a very small or negligible clearance at all mechanism joints (by changing the sleeve of the joint as mentioned before). At this case, the performance of the mechanism or more specifically the experimental slider acceleration is compared with the simulation results of that of the ideal mechanism. The second stage, when a noticeable clearance of 0.5mm exists on (s – cr joint). At this case the experimental slider acceleration is compared with that of simulation results of one clearance joint mechanism. The second stage is the important part of the experiment to validate the efficiency and accuracy of the proposed method of modeling joint clearance. However, we intend to perform an experiment on the ideal mechanism first, in order to ensure the accuracy of the measuring system, and control system, as well as detect any problem in the test rig before running the desired experiment.

1. Experimental Results (Without Clearance)

Fig.15 shows simulation and experimental slider acceleration. The steady state response of all of the application examples in the previous work [12, 13, 14] is the main important part of simulation. However, in case of comparing the simulation results with the experimental, we have to focus on the transient response of the mechanism. The reason is that in all application examples the input speed is constant from starting of the mechanism till the end of simulation time. However, it is not the actual case when running a physical mechanism although we are using a speed control system. The speed of the motor cannot reach to constant value of 300 rpm from rest in zero time. The speed control system allows us to set a value for the acceleration time to increase the motor speed linearly. One characteristic of the speed control is by decreasing the acceleration time the speed control accuracy decreases at the transient response, and increasing it to reasonable value, the speed control accuracy increases.

In this experiment, the mechanism is run many times at each time the acceleration time is increased, and also the mechanism is simulated using SolidWorks/CosmosMotion.
with the same acceleration times. A simulation time of 0.18 second is found enough to obtain accurate results of the transient response, so an acceleration time of 0.2 second is used in our simulation and experimentation.

The resulted value of restitution coefficient can be easily obtained by the proposed procedure stated in section 2 using Eq. 10 and Eq. 11. However, the Restitution coefficient \( r \) is a difficult factor to be estimated and strongly affect the value maximum damping coefficient \( C_{\text{max}} \).

The value of the Coefficient of Restitution is depending not only on the surface and body materials, but it also depends on the geometry of both. For example the value of Restitution coefficient changes from ball to cylinder or cube and from large cylinder to small cylinder and so on, and also the natural of the surface. Therefore, the value of Restitution coefficient of the journal – bearing combination of the clearance joint in our case is determined experimentally from the test rig. This is done by running simulation different times, at each time a different value of maximum damping coefficient \( C_{\text{max}} \) is used and the simulation result is compared with the experimental one until reach to the accurate value of \( C_{\text{max}} \) at which the simulation result is nearest to the experimental one. Then the value of restitution coefficient is reversely calculated from the obtained \( C_{\text{max}} \). Table 4 shows some of the iterations and the maximum deviation between theoretical and experimental results at each. The resulted value of restitution coefficient in our case is 0.55.

Since friction parameters are difficult to be estimated, in this experiment the surfaces of the clearance joint components are made very smooth and the friction coefficients are taken very small in simulation (Table 2). Therefore, error or wrong estimation of the friction coefficients does not lead to a large error in the simulation results.

### TABLE IV

<table>
<thead>
<tr>
<th>Trail</th>
<th>Max. Damping Coeff. ( C_{\text{max}} ) ( (\text{N} \text{l/(m/s)}) )</th>
<th>Max. percentage error %</th>
<th>Coeff. Of Restitution ( (r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25283</td>
<td>24%</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>25000</td>
<td>27%</td>
<td>0.722</td>
</tr>
<tr>
<td>3</td>
<td>26500</td>
<td>18%</td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>27000</td>
<td>15.5%</td>
<td>0.62</td>
</tr>
<tr>
<td>5</td>
<td>27500</td>
<td>12%</td>
<td>0.592</td>
</tr>
<tr>
<td>6</td>
<td>27700</td>
<td>10.75%</td>
<td>0.564</td>
</tr>
<tr>
<td>7</td>
<td>27960</td>
<td>7%</td>
<td>0.55</td>
</tr>
</tbody>
</table>

3. Comparison Between Simulation and Experimental Results

Fig. 16 shows the simulation and experimental results of the slider acceleration. It can be observed that: the experimental and simulation results are close to each other. The maximum percentage error is about 7%, which is an acceptable error to demonstrate the accuracy and efficiency of our proposed method of modeling and simulation of multibody mechanical systems with clearance joints.
This error may be due to other mechanical inaccuracies in the mechanism such as the existence of a very small clearance in other mechanism joints, geometrical tolerances and so on. The accuracy of the measuring instruments may participate also in this error.

There is also time lag in between the simulation and experimental with about 0.01 seconds. This lag may be due to the time lag in the response of the speed control system.

V. CONCLUSION

In this work an experiment is carried out to validate the efficiency and accuracy of the presented methodology of modeling and simulation of multibody mechanical systems with dry clearance joints using CAD and dynamic simulator softwares. This method of simulation has many advantages and easy to be used. Hence, an experimental verification of the simulation should be done to ensure its accuracy and efficiency. A slider – crank mechanism with one dry clearance revolute joint working in vertical plane is used in this experiment. The input motion to the mechanism is a constant angular velocity motion applied to the crank through an electrical motor. The motor speed is controlled through an inverter directly connected to the motor. The idea of the experiment to validate the simulation results is to simulate the experimental mechanism using our simulation method, one of the resulted dynamic or kinematic variables have to be compared with the corresponding one of the simulation results. Slider acceleration is chosen to be that variable. The acceleration is measured by an accelerometer and a continuous reading or online measuring of the acceleration is transmitted to the computer via an external data acquisition system.

The experiment is carried out in two stages. The first one when no or negligible value of clearance exists on the mechanism joints, which is considered the case of ideal joints. The second stage when a 0.5 mm clearance exists on the joint connected slider and connecting rod (s – cr joint). The first stage of the experiment is essential to be done to check accuracy of the measuring system, control system, and detect any problem in the test rig before running the desired experiment.

For the simulation purpose in the second stage the clearance joint parameters need to be identified before running simulation. All the parameters have been estimated through the equations of the continuous contact force model except for the coefficient of restitution which is difficult to be estimated through theoretical equations. Hence, a series of iterations are done by running the simulation different times and comparing with the experimental results. The best value of the restitution coefficient is the value that makes the simulation results nearest to the experimental results.

Finally the comparison between the experimental slider acceleration and the simulation results shows the efficiency and accuracy of our method of modeling and simulation of mechanical systems with dry clearance joints

VI. ABBREVIATIONS

s – cr joint is the joint connected the slider and connecting rod in slider – crank mechanism

REFERENCES